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EMPIRICAL FORMULA FOR INCLUSIVE PROTON SPECTRA BETWEEN 10 AND 300 GeV[*]

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ABSTRACT

An empirical formula, which well represents the proton spectra in the inclusive reaction, p + Be → p + anything between 10 and 300 GeV, is constructed.

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I. Introduction

For many purposes, such as planning high energy experiments, designing high energy secondary beams, calculating radiation shields, designing total absorption nuclear-cascade detectors, calculating the energy deposition of secondary particles in superconducting magnets, etc., it is useful to construct simple analytical expressions for secondary productions in high energy collisions. Previous papers deal with pion, kaon and antiproton productions.[1,2] Constructed in this paper is an empirical formula which gives an excellent representation of the inclusive proton spectra in high energy collisions between 10 and 300 GeV.

II. Scaling Behavior of the Longitudinal Momentum Distribution

The longitudinal momentum distribution of the secondary protons appears to be consistent with the linear scaling of secondary pions described in the previous paper.[1] Namely, the longitudinal momentum spectra as the function of X is linearly proportional to the incident momentum p_i, where $X = p_\ell/p_m$, p_ℓ being the longitudinal momentum and p_m the maximum kinematically allowed value of p_ℓ or essentially p_i. Henceforth, we replace p_m by p_i. Such a behavior is illustrated in Fig. 1, in which the zero-degree production data from Dekkers et al.[3] are plotted as the function of X for three incident momenta, 11.8, 18.8 and 23.1 GeV/c. The curves are from Formula (1). It is seen that the spectrum moves up linearly with the incident momentum.

The longitudinal momentum distribution can therefore take the form

$$\frac{d^2\sigma}{dp\,d\Omega}\,(0^\circ) = A\,p_i\,\exp\,(B\,X^C) \qquad (1)$$

where A is the normalization constant, B and C are constants determining

Yes. Write [illegible]

I think should be C = 0.6324 see Fig. 1.

the scaling functions exp (B X^C). A least squares fitting program determined the constants: A = 1.30, B = 5.354 and C = 6.324. Later, the double differential cross section in units of mb/GeV/c/sr per nucleus will be converted into number of protons/GeV/c/sr by dividing by 227 mb, the p-Be absorption cross section.[4]

III. Normalization of Data from Various Experiments

Data from five more experiments were utilized in the subsequent analysis of the angular dependence of the momentum spectra. Unfortunately, of the six experiments, none has overlapping points to allow a direct comparison of the normalization.

The data from Lundy et al.[5] at 13.4 GeV/c and those from Allaby et al.[6] at 19.2 GeV/c allow one to make reasonably accurate extrapolations to zero degree production, that would then allow one to check the normalization by means of the scaling behavior discussed in the previous section. It was immediately noticed that the data of Lundy et al. are lower than those of Dekkers et al. by a factor of ~ 1.5, and the data of Allaby et al. are higher than those of Dekkers et al. by a factor of ~ 1.5. Therefore, as an average, the data of Dekkers et al. were chosen as the standard of normalization, although there is no guarantee that this is the correct choice. The difficulties with the normalization of this type of experiments are well known. Sometimes the data can be off by a factor of 2.5 to 5, as noted previously.[7]

The final values of the normalization factor for each experiments were determined during the least squares analysis after the forms of the angular dependence were chosen. The result is that the data of Lundy et al., Allaby et al., Baker et al.[8] and Aubert et al.[9] have to be renormalized by factors of 1.5, 0.67, 0.71 and 1.43 respectively. The external target efficiency of References 8 and 9 is taken as 37%. The internal target efficiency of Reference 10 is taken as 50% but the data are renormalized by a factor of 1.43.

IV. Angular Dependence of the Momentum Spectra

Attempts were made but to no avail, to represent the angular dependence of the momentum spectra by a simple exponential of the transverse momentum distribution as has been done successfully in the case of pion production.[1] Instead, a somewhat complicated expression has to be constructed in order to fully reproduce the details of the data.

$$Y = \frac{d^2N}{dpd\Omega} = 0.005726\, p_i \exp \left\{5.354(p/p_i)^{\overset{0.6324}{\cancel{6.324}}} - D\theta^E(p^E - F \cos^G \theta) \right\} \times (1 - p/p_i)^{H\theta p/p_i} \tag{2}$$

gives the number of protons/GeV/c/sr, where p_i and p are incident and secondary momenta in GeV/c, θ is the production angle in radian. D and E determine the general feature of the transverse momentum distribution, while F and G take care of the crossing-over of the spectra at low momenta. The last characteristics, although not explicit from the limited p-Be data, is clearly demonstrated by the more detailed data of p-p collision[11] and is indispensable in the present fitting. The last term with the parameter H is required to obtain good fitting to the non-zero degree, large momentum data. This term has no effect on the longitudinal momentum distribution, because it becomes one when θ becomes zero. It also satisfies the kinematical constraint that the spectra vanish as p becomes p_i.

Least squares analyses were again carried out to determine the remaining parameters D through H, by minimizing the quantity

$$Q = \frac{1}{F} \sum_i \left(\frac{\log_{10} Y_i^e - \log_{10} Y_i^c}{\Delta Y_i^e / Y_i^e} \right)^2 \tag{3}$$

where the superscripts e and c refer to the experimental and calculated values, respectively, ΔY_i^e is the error pertaining to ith experimental datum, and F is the number of degrees of freedom. All data are assigned 20% errors except where the errors are explicitly provided by the experimenters. The result of the analysis is $Q = 0.23$, indicating an excellent fit with $D = 3.508$, $E = 1.15$, $F = 4.129$, $G = 37.34$ and $H = 20.27$.

V. Discussion

As can be seem from Fig. 1 through 9, formula (2) fits all available data quite well. Although it involves no deeper physics other than the scaling, formula (2) provides a simpler yet more accurate way to calculate the inclusive proton spectra than the computer program using the thermodynamical model.[12]

Since there are only minor differences in the inclusive spectra from light elements, formula (2) should be a good approximation for elements other than Be. However, for small angles, near the kinematic limit of the momentum, there are pronounced elastic or quasielastic peaks. They are not accounted for in the present formula.

References

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Figure Captions

Fig. 1. The zero-degree production data from Dekkers et al. (Ref. 3), plotted as the junction of X, indicate a linear scaling with incident momenta. The curves are from formula (1).

Fig. 2. The inclusive proton spectra from p + Be → p + anything, between 10 and 300 GeV where the data exist. The curves were generated with formula (2) after fitting to the data.

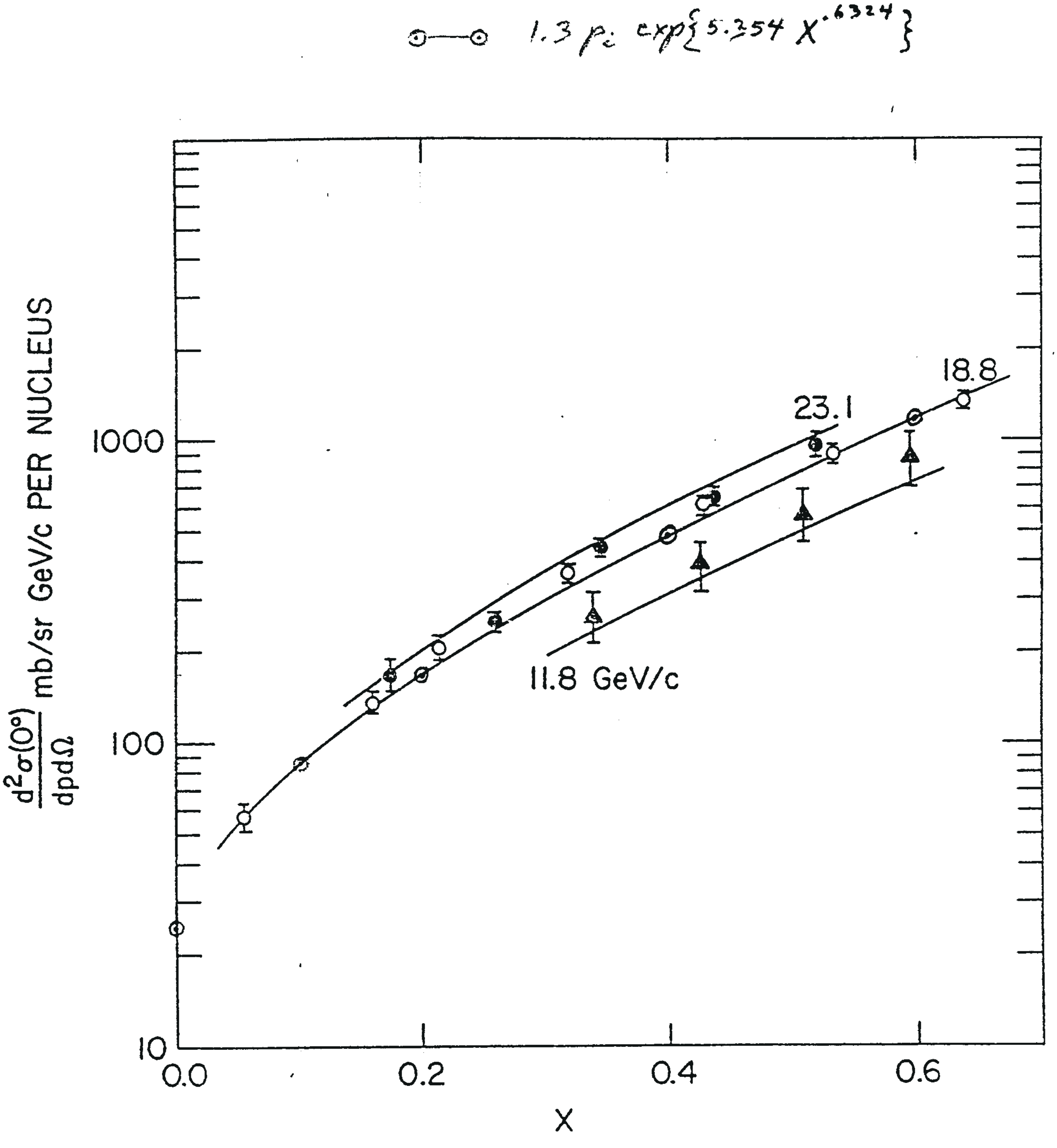
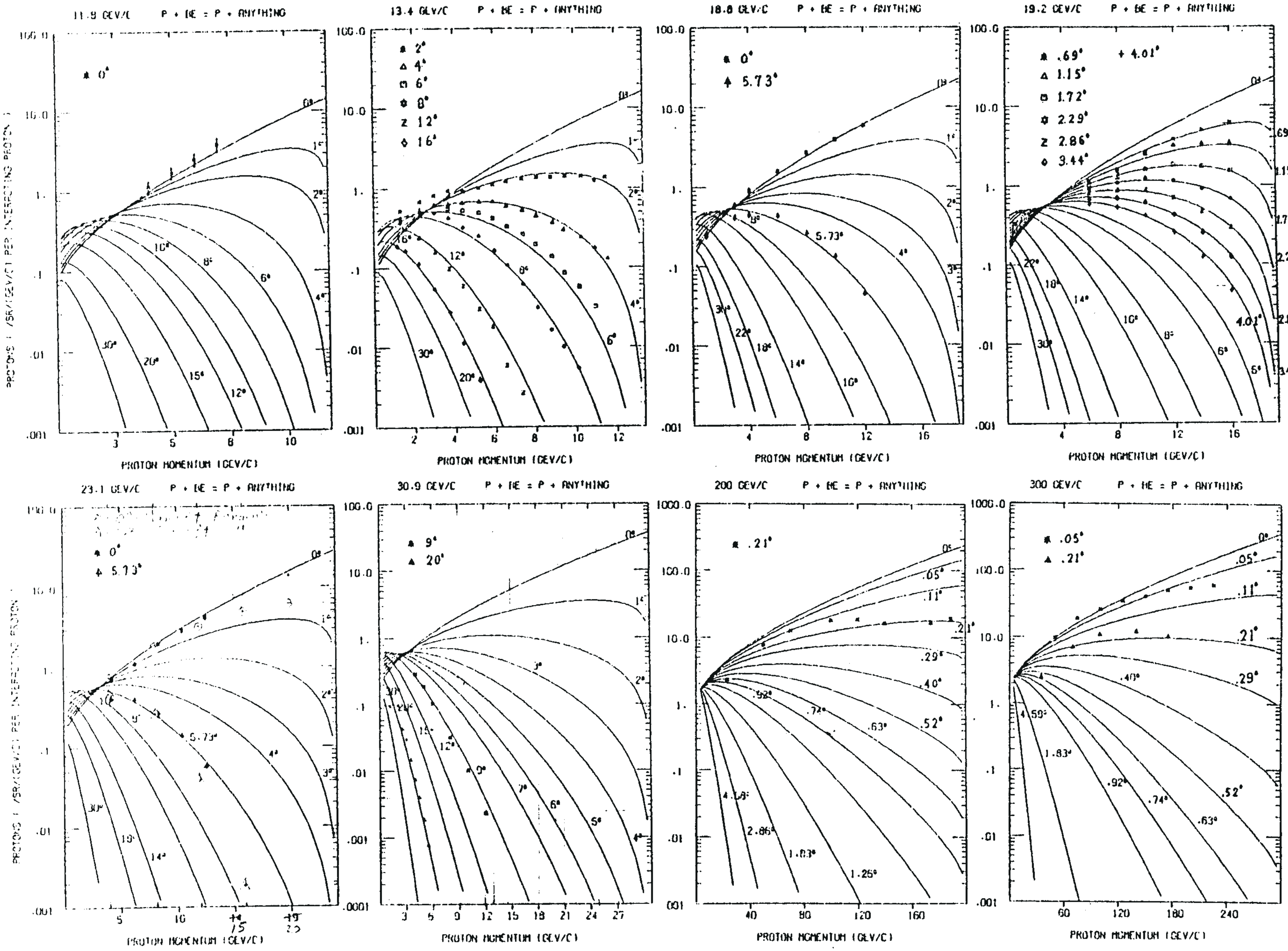
⊙—⊙ $1.3\, p_i \exp\{5.354\, X^{.6324}\}$
$\frac{d^2\sigma(0°)}{dp\,d\Omega}$ mb/sr GeV/c PER NUCLEUS
X
10
100
1000
0.0
0.2
0.4
0.6
23.1
18.8
11.8 GeV/c

Fig. 1


11.9 GEV/C P + BE = P + ANYTHING
13.4 GEV/C P + BE = P + ANYTHING
18.8 GEV/C P + BE = P + ANYTHING
19.2 GEV/C P + BE = P + ANYTHING
23.1 GEV/C P + BE = P + ANYTHING
30.9 GEV/C P + BE = P + ANYTHING
200 GEV/C P + BE = P + ANYTHING
300 GEV/C P + BE = P + ANYTHING
PROTONS / (SR/(GEV/C) PER INTERACTING PROTON)
PROTON MOMENTUM (GEV/C)
100.0
10.0
1.
.1
.01
.001
.0001
1000.0
0°
1°
2°
3°
4°
5°
6°
7°
8°
9°
10°
12°
14°
15°
16°
18°
20°
22°
24°
30°
5.73°
.69°
1.15°
1.72°
2.29°
2.86°
3.44°
4.01°
.05°
.11°
.21°
.29°
.40°
.52°
.63°
.74°
.92°
1.03°
1.25°
1.83°
2.86°
4.58°
4.59°
2
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160
180
240

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23.1 BEV/C P-BE. POSITIVE PION PRODUCTION

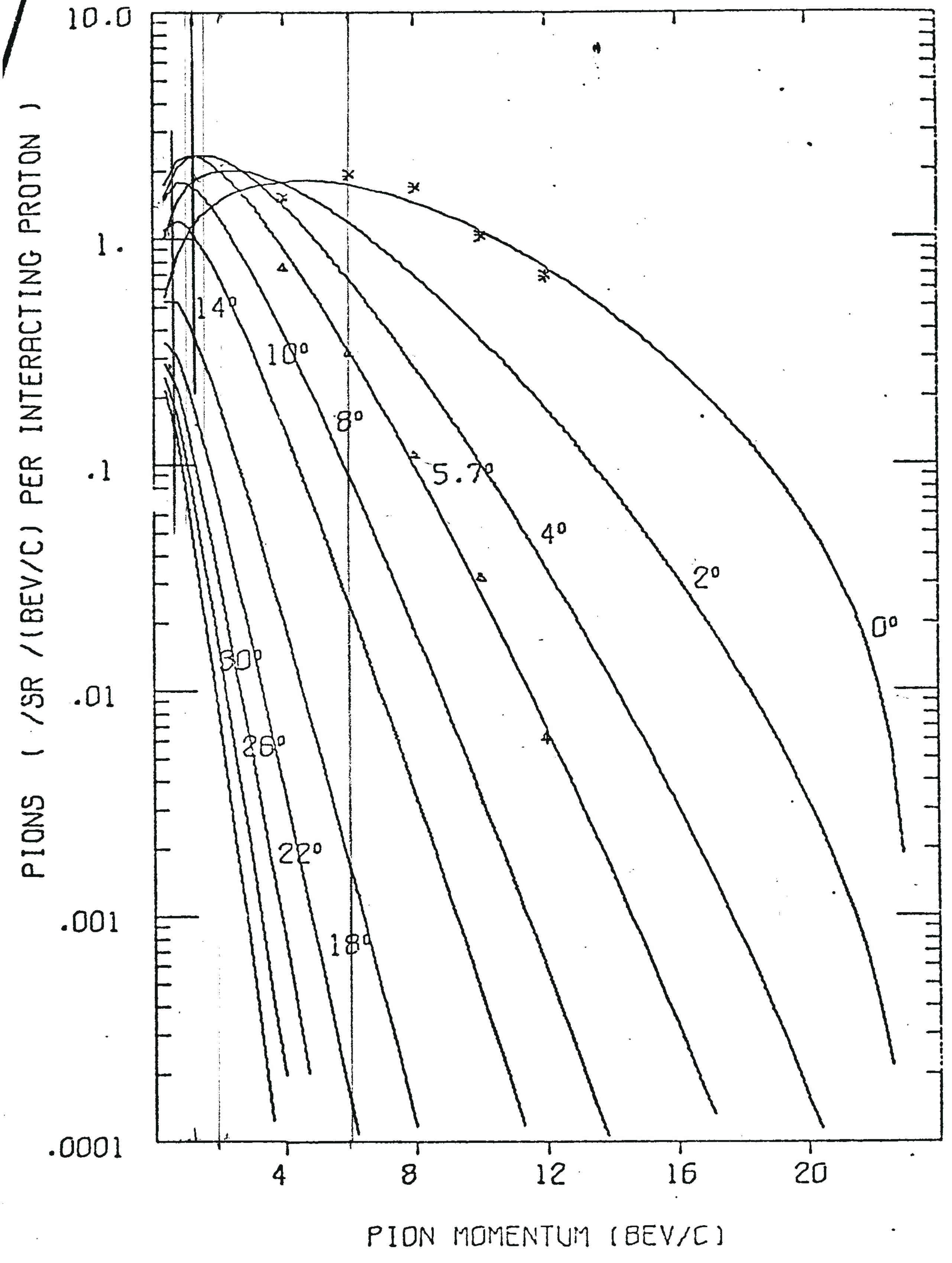

Fig. 7