

Software Development for Beam Cooling Simulation Including General Collider Physics

**Interim report
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Abstract

General attention at this stage of the work was devoted to development of the electron cooling model represented by the force vector and the tensor of diffusion coefficients for the real electron distribution function. The same model was realized for intrabeam scattering simulation at arbitrary shape of the distribution function.

In order to provide realistic comparison between simulation of antiproton beam dynamics and experiments at Fermilab Recycler ring a new model of electron beam with parabolic density distribution was developed. A new model of the ion synchrotron motion at rectangular RF barrier bucket was implemented.

Fast algorithm for calculation of the luminosity for individual ion at Gaussian and bi-Gaussian distributions was developed and tested.

A model for simulation of longitudinal stochastic cooling at RHIC is under development.

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1. Local models for IBS and electron cooling

Main goal of the local model for IBS process is to simulate the distribution function evolution without additional assumption about its shape. Electron cooling application leads to formation of bi-Gaussian distribution in the ion beam. For simulation of IBS process in this case the “core-tail” model is used in BETACOOOL now. However, this model includes a few free parameters that can not be calculated from the distribution as itself. The local model will give a possibility to benchmark the “core-tail” model and provide a choice of its parameters on the basis of the beam dynamics simulation.

The local model of the electron cooling is necessary to compare different electron distribution from the side of the cooling process efficiency.

To realise the local models for IBS and electron cooling the structure of the beam object in the code was modified. The bunch of the particle can be presented in the laboratory or in the particle rest frame. Corresponding modules for particle co-ordinate transformation from laboratory frame to beam frame and back were introduced. The friction force and diffusion tensor components are calculated in the particle rest frame. In the current version of the algorithm a kick of the ion momentum due to action of electron cooling is provided in the laboratory frame, kick of the ion momentum due to action of IBS – in the beam frame. All the others parts of the algorithms are the same. To reduce the simulation time in the case of IBS simulations the simplified optic structure of the ion ring is used. The total lattice is reduced to a few optic elements (10 – 20) that have the same properties from the side of IBS process.

The new models are based on the statement that intrabeam scattering (and ion scattering on electrons) is the local process and the ion interacts efficiently only with relatively small number of nearest particles. The particle density in the vicinity of the ion in this case is closed to uniform, the friction force and diffusion tensor components can be calculated using well known formulae from plasma physics.

In the frame of local algorithm the program finds in the total array of the particle a small local array and calculates local density and rms parameters of the particle distribution in this local array. The local parameters are used for calculation of the friction and diffusion components. To avoid systematic error in local density evaluation the program calculates number of particles inside a small cell surrounding the test ion. Dimensions of this cell are calculated from rms dimensions of the local array. Such an algorithm permits to adjust the cell dimensions to the local density of the particle distribution and provide an accurate calculation in a dense core and in tails of the distribution function.

The algorithm includes three parameters: total number of the particles in the array N , number of the local particles N_{loc} and dimensions of the cell used for local density evaluation.

The local particles are found in the beam frame; therefore for RHIC parameters the local array has a specific shape. Expected electron bunch length in the laboratory frame is about 1 mm, the ion bunch length is between 15 and 30 cm that corresponds in the beam frame to about 1 and more than 15 meters. The transverse dimensions of the electron and ion bunches are closed to each other in the cooling section and expected to be between 3 and 5 mm. The ion beam transverse dimensions in other optic elements are about 1-2 mm. Thus the bunch length is about two orders of magnitude larger than the transverse dimensions. Therefore the array of the local particle almost coincides with a longitudinal slice inside the bunch. The local rms transverse sizes are close to the global ones, when the rms length of the local array is less than the total

bunch length by the ratio between total and local particle number. The electron and ion bunches has approximately axial symmetry shape in the transverse plane.

To take into account these peculiarities of the particle distribution and provide fast and accurate algorithm the following procedure for the local density evaluation is used. The local density is calculated as a particle number located inside an elliptical cylinder of the length of $2\sigma_s$ and half-axis of the cross-section of $\alpha\sigma_x$ and $\alpha\sigma_y$ divided by the volume of this cylinder: $2\pi\alpha^2\sigma_s\sigma_x\sigma_y$.

Where σ_s , σ_x , σ_y are the rms longitudinal and transverse dimensions of the local array and α is numerical coefficient less than unity.

For benchmarking of the local model an array of electrons can be generated in the program in accordance with the following distributions:

- Gaussian,
- bi-Gaussian,
- uniform in transverse plane and Gaussian along the bunch.

A few new procedures was developed for the benchmarking: the program can output local density and the friction force components in a given position inside electron bunch, which are calculated from the local array or analytically. This permits to adjust parameters of the model, such as the particle numbers in global and in local arrays and the cell dimensions, to obtain required accuracy of the simulations. Results of the code benchmarking will be presented in the final report.

2. Algorithms for simulations of Fermilab experiments

To provide realistic simulations of electron cooling process in Recycler ring in Fermilab one needs to take into account properties of the electron beam and peculiarity of the antiproton synchrotron motion at barrier RF bucket application.

In previous simulations the model of electron beam with uniform density distribution in transverse cross-section was used. Real electron distribution has a shape closed to parabolic function of radial coordinate. To reproduce the parabolic density shape a new model of the electron beam was implemented into the program. Input parameters of the model are the beam radius a and the beam current I_e . The electron density n as a function of radial coordinate r is calculated in accordance with

$$n(r) = n_0 \left(1 - \left(\frac{r}{a} \right)^2 \right), \quad (2.1)$$

when $r < a$ and equal to zero in opposite case. The central density n_0 is calculated from the beam current as

$$n_0 = \frac{3I_e}{\pi a^2 v e}, \quad (2.2)$$

where v is the electron velocity and e – elementary charge. The linear variation of the transverse electron velocity spread across the beam is realized in this model also.

At the Fermilab Resycler a square wave barrier RF bucket is used. Theory of the square wave barrier bucket is well known. The RF voltage time dependence and phase trajectories of ions are sketched in the Fig. 2.1. When the ion passes through the cavity gap at voltage $\pm V_0$ it gains (losses) an equal amount of energy ZeV_0 , i.e.

$$\frac{d(\Delta E)}{dt} = \pm \frac{ZeV_0}{T_0}, \quad (2.3)$$

where ΔE is the energy deviation from synchronous one, T_0 – revolution period. The ion trajectory in the longitudinal phase space $(t-t_0, \Delta E)$ inside the bucket can be written in the following form:

$$(\Delta E)^2 = \begin{cases} A_E^2 & \text{if } |t-t_0| \leq T_2/2 \\ A_E^2 - \left(|t-t_0| - \frac{T_2}{2} \right) \frac{2\beta^2 E_0 ZeV_0}{T_0 |\eta|} & \text{if } T_2/2 \leq |t-t_0| \leq (T_2/2) + T_1 \end{cases} \quad (2.4)$$

where A_E is the maximum energy deviation from synchronous energy E_0 . The phase-space trajectory is composed of a straight line in the RF gap region and a parabola in the square RF wave region.

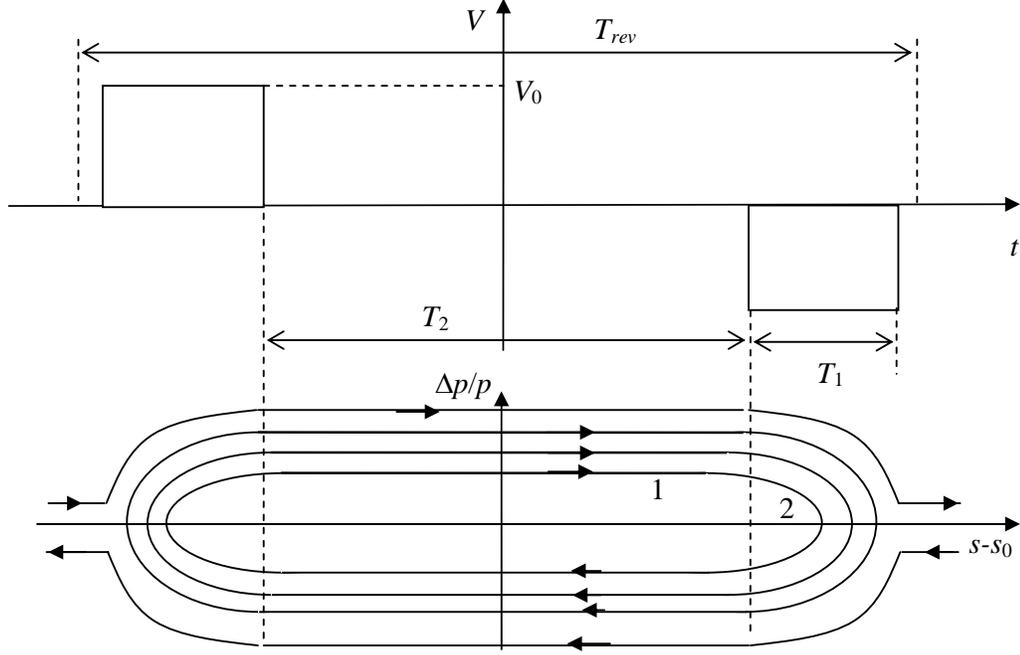


Fig. 2.1. RF voltage and particle trajectories in the longitudinal phase plane at square wave barrier-bucket. V_0 is the voltage height, T_1 is the pulse width, T_2 is the gap duration.

To implement the Barrier Bucket simulation into BETACOOOL code the equation of the synchrotron motion was rewritten in the phase plane ($s-s_0$, $\delta = \frac{\Delta p}{p}$). The equation is

$$\begin{cases} \frac{d(s-s_0)}{dt} = |\eta| \beta c \delta_A \\ \frac{d\delta}{dt} = 0 \end{cases} \quad (2.5)$$

at the part of the trajectory 1 (Fig. 2.1), and

$$\begin{cases} \frac{d(s-s_0)}{dt} = |\eta| \beta c \delta \\ \frac{d\delta}{dt} = -\frac{ZeV_0}{Cp_0} \end{cases} \quad (2.6)$$

at the part 2. Here δ_A is the amplitude of the momentum deviation, C is the ring circumference, p_0 – synchronous momentum.

The motion equations have an analytical solution in both parts of the phase trajectory. On the basis of the analytical solution the procedures for generation of a particle array matched with the RF shape, for change of the particle coordinates during synchrotron motion and for generation of new particle after the particle loss were prepared. The algorithm of the barrier bucket simulation is under benchmarking now. Detailed description and results of simulations will be presented in the final report.

3. Development of stochastic cooling simulation

First experiments at RHIC with longitudinal stochastic cooling application demonstrated its ability to compensate increase of the bunch length due to intrabeam scattering and exclude particle losses from the bucket. At these conditions one can expect increase of the emittance growth due to intrabeam scattering. To provide realistic simulation of the ion distribution function evolution due to common action of the longitudinal stochastic cooling and intrabeam scattering the development of the stochastic cooling simulation has been started.

Main peculiarities of the RHIC longitudinal stochastic cooling are sufficient non-linearity of a friction force and dependence of the diffusion on synchrotron amplitudes beginning from their zero value. The algorithm existing in Betacool presumes linear friction force and diffusion independent on the synchrotron amplitude. The development of the algorithm is realizing in three stages:

- at the first stage the model presumed cubic nonlinearity of the friction and constant diffusion was realized. Numerical parameters of the model are input into the program and used for fitting of experimental results. This model permits to estimate an influence of the longitudinal stochastic cooling on the transverse intrabeam growth rates and particle loss and simulate luminosity time dependence in the presence of the cooling. However it can not predict properties of the stochastic cooling at different parameters of the ion beam.
- the second stage presumes realization of more realistic model for the friction force calculation.
- and finally the diffusion will be calculated as a function of the particle synchrotron amplitude and the distribution shape.

In the frame of Model Beam algorithm the program solves Langevin equation for model particle from the particle array. The particle momentum during simulations is changed regularly by action of a friction force and randomly by diffusion. In the three dimensional case each component of the particle momentum is changed in accordance with the step of integration over time of Δt as:

$$P_i(t + \Delta t) = P_i(t) + F_i \Delta t + \sqrt{\Delta t} \sum_{j=1}^3 C_{i,j} \xi_j,$$

where F_i are the components of the friction vector, $C_{i,j}$ are connected with the diffusion tensor components in accordance with the equation:

$$\sum_{k=1}^3 C_{i,k} C_{j,k} = D_{i,j},$$

and ξ_j are independent random numbers distributed in accordance with Gaussian law at unit variance.

In the case of longitudinal stochastic cooling we have one dimensional task and the equation simplifies. Denoting relative momentum deviation as $\delta = \frac{\Delta p}{p}$ one can write

$$\delta(t + \Delta t) = \delta(t) + F \Delta t + \sqrt{\Delta t} C \xi. \quad (3.1)$$

If the diffusion due to thermal noise is negligible the coefficient C is determined by the Shottky noise diffusion.

In usual stochastic cooling the friction is a linear function of momentum deviation, and in the present algorithm for the stochastic cooling simulation in Betacool it is calculated as:

$$F = -\frac{2\delta}{\tau_0}$$

where τ_0 is so called “single-particle” cooling time. The “single particle” cooling time τ_0 is given by

$$\frac{1}{\tau_0} = 2A_1 e^2 \sqrt{n_p n_k} Z G_A W f_c \frac{\kappa}{E_0},$$

where $\kappa = \eta \frac{\gamma}{\gamma + 1}$, $f_c = \frac{f_{\min} + f_{\max}}{2}$ is the central frequency of the band. A_1 is a numerical factor of the order of unit which is determined from the loop length of the longitudinal electrodes, $n_{p,k}$ is the number of lambda quarter loops in pickup and kicker, G_A is the linear gain of the system from pickup to kicker.

At given parameters of the cooling system and storage ring this value does not depend on the particle number and particle distribution function and is a linear unction of the gain. If this correct for arbitrary design of the cooling system the cooling time can be expressed in term of relative gain g

$$\frac{1}{\tau_0} = C_\tau g$$

where C_τ is some constant determined by the system design. To take into account cubic nonlinearity of the friction force in the RHIC cooling system the action of the friction force can be calculated as:

$$\delta(t + \Delta t) = \delta(t) - C_\tau g \cdot (\alpha \delta(t) + \beta \delta^3(t)) \Delta t. \quad (3.2)$$

The coefficients α and β are determined by the system design.

In existing version of the algorithm the coefficient C in the equation (3.1) related with the Shottky noise power, is calculated as the function of the particle number N and rms momentum spread of the beam σ_p :

$$C = \sqrt{\frac{BN\gamma}{2E_0(\gamma + 1)}} \sigma_p.$$

The coefficient B is proportional to the gain square:

$$B = A_1 e^4 n_k n_p Z^2 \frac{|K|}{E_0} G_A^2 f_0 f_C W .$$

Introducing a constant determined by the system and ring parameters one can write:

$$C = \sqrt{C_{Sch} g^2 N \sigma_p} . \quad (3.3)$$

Finally the momentum variation can be expressed as

$$\delta(t + \Delta t) = \delta(t) - C_\tau g \cdot (\alpha \delta(t) + \beta \delta^3(t)) \Delta t + \sqrt{\Delta t} \sqrt{C_{Sch} g^2 N \sigma_p} \xi , \quad (3.4)$$

and four constants: C_τ , α , β , and C_{Sch} are the parameters of the algorithm that has to be calculated with other program or measured in experiment. In the simulations these constants can be used as fitting parameters: C_τ at optimum gain determines the cooling time, ratio between C_τ and C_{Sch} determines the threshold of instability, α and β determine the distribution shape in the equilibrium. This algorithm is realized now and its benchmarking is started.

At the next stages of the algorithm development the constants will be calculated from the system parameters.

Algorithm for realistic calculation of the friction force was proposed by M. Blaskevich. The shape of the friction force as a function of the revolution frequency offset can be calculated without account of the signal shielding in accordance with:

$$F = g \cdot \text{Im} \left\{ \sum_{k=25}^{15+25} G[h \cdot k \cdot \Delta \omega] \right\} , \quad (3.5)$$

where the frequency offset is given by

$$\Delta \omega = -\omega_0 \eta \frac{\Delta p}{p}$$

with $\omega_0 = 2\pi/T_0$, T_0 is the revolution period and η is the momentum slip factor of the ring, h is the harmonic number.

In the general case the sum in the (3.5) has to be done over all the harmonic number satisfying the condition

$$\frac{f_{\min}}{f_{rev}} \leq h \leq \frac{f_{\max}}{f_{rev}}$$

Here $f_{\min} = 5$ GHz and $f_{\max} = 8$ GHz are the minimum and maximum frequencies of the cooling system band, f_{rev} is the revolution frequency. However the total bandwidth of the RHIC cooling system is divided by 15 bands corresponding to the cavity number. Therefore the sum can be calculated for central frequency of each band only. Central frequency of the first cavity corresponds to the harmonic number equal to 2560.

The form factor G is given by

$$G(h \cdot \Delta\omega) = \exp(ih \cdot \Delta\omega T_d) [1 - \exp(ih \cdot \Delta\omega T_0)]^2, \quad (3.6)$$

where the time delay $T_d = \frac{2}{3}T_0$ at RHIC cooling system.

4. Luminosity calculation for individual ion

For correct calculation of the particle loss in the interaction point the algorithm for luminosity calculation for individual ion is necessary. In the current version of the program there are a few algorithms for luminosity calculation based on evaluation of local areal density. For benchmarking of these algorithms and for fast luminosity calculation in the case of Gaussian or bi-Gaussian distribution a new algorithm was developed. Here the luminosity for individual ion interacting with a Gaussian bunch is described. In the case of bi-Gaussian distribution the luminosity is calculated for a superposition of two Gaussian bunches.

In the frame shown in the Fig. 4.1 the bunch is moving along negative direction of z -axis. The bunch velocity is equal u , the ion velocity - v .

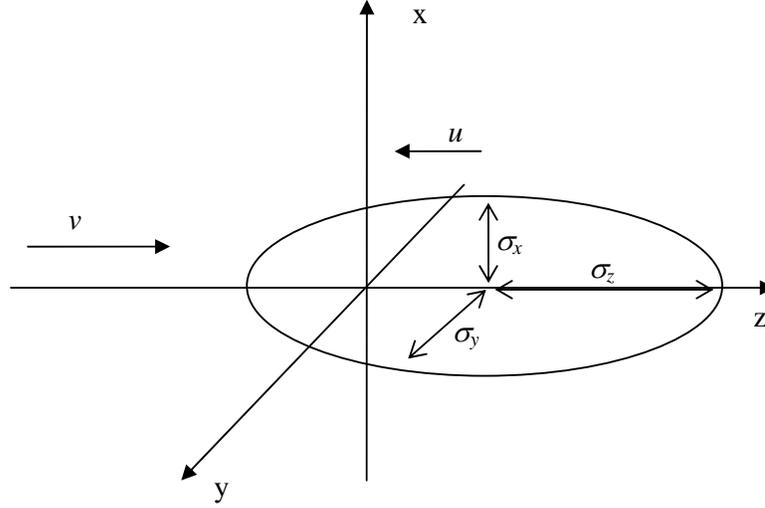


Fig. 4.1. Geometry of the collision. Collision point is located in the (0,0,0) point.

Particle distribution inside the bunch is given by:

$$\rho[x(t), y(t), z(t)] = \frac{N}{\sqrt{2\pi}^3 \sigma_x(t) \sigma_y(t) \sigma_z(t)} \exp \left[-\frac{x^2(t)}{2\sigma_x(t)} - \frac{y^2(t)}{2\sigma_y(t)} - \frac{z^2(t)}{2\sigma_z(t)} \right]. \quad (4.1)$$

The luminosity is determined as

$$L = \int_S \rho(\vec{r}) ds, \quad (4.2)$$

where S is the ion trajectory. Rms dimensions of the bunch are changed in the vicinity of the collision point as:

$$\sigma_x^2(t) = \sigma_x^2(0) \left\{ 1 + \frac{u^2 t^2}{\beta_x^{*2}} \right\}, \quad \sigma_y^2(t) = \sigma_y^2(0) \left\{ 1 + \frac{u^2 t^2}{\beta_y^{*2}} \right\}, \quad \sigma_z(t) = \sigma_z(0), \quad (4.3)$$

where β_x^* , β_y^* are the beta-functions in the collision point. The integral (4.2) can be rewritten in the following form

$$L = \frac{(v+u)N}{\sqrt{2\pi}^3 \sigma_x(0)\sigma_y(0)\sigma_z(0)} \times \int_{-\infty}^{\infty} \frac{\exp\left\{-\frac{(x_0 + x'_0 z_0 + x'_0 vt)^2}{2\sigma_x^2(0)[1+u^2 t^2/\beta_x^{*2}]} - \frac{(y_0 + y'_0 z_0 + y'_0 vt)^2}{2\sigma_y^2(0)[1+u^2 t^2/\beta_y^{*2}]} - \frac{(z_0 + [v+u]t)^2}{2\sigma_z^2(0)}\right\}}{\sqrt{(1+u^2 t^2/\beta_x^{*2})(1+u^2 t^2/\beta_y^{*2})}} dt. \quad (4.4)$$

Or at substitution $\xi = \frac{z_0 + (u+v)t}{\sqrt{2\sigma_z^2(0)}}$ it can be expressed as

$$L = C \int_{-\infty}^{\infty} \frac{\exp\left\{-\frac{A_x^2(\xi)}{B_x(\xi)} - \frac{A_y^2(\xi)}{B_y(\xi)}\right\}}{\sqrt{B_x(\xi)B_y(\xi)}} \exp(-\xi^2) d\xi, \quad (4.5)$$

where

$$C = \frac{N}{2\sqrt{\pi}^3 \sigma_x(0)\sigma_y(0)},$$

$$A_x(\xi) = x_0 + x'_0 z_0 + x'_0 \frac{v}{u+v} \left(\sqrt{2\sigma_z^2(0)}\xi - z_0\right),$$

$$A_y(\xi) = y_0 + y'_0 z_0 + y'_0 \frac{v}{u+v} \left(\sqrt{2\sigma_z^2(0)}\xi - z_0\right), \quad (4.6)$$

$$B_x(\xi) = 1 + \frac{u^2}{(u+v)^2 \beta_x^{*2}} \left(\sqrt{2\sigma_z^2(0)}\xi - z_0\right)^2,$$

$$B_y(\xi) = 1 + \frac{u^2}{(u+v)^2 \beta_y^{*2}} \left(\sqrt{2\sigma_z^2(0)}\xi - z_0\right)^2.$$

The integral (4.5) is calculated using Gauss-Cristoffel method in accordance with:

$$L = \sum_{k=1}^N c_k F(x_k), \quad F(\xi) = C \frac{\exp\left\{-\frac{A_x^2(\xi)}{B_x(\xi)} - \frac{A_y^2(\xi)}{B_y(\xi)}\right\}}{\sqrt{B_x(\xi)B_y(\xi)}}, \quad (4.7)$$

where x_k are the roots of Hermit polinom of N -th order, c_k – coefficients, calculated in accordance with the formulae:

$$c_i = \int_{-\infty}^{\infty} h_i(x) \exp(-x^2) dx,$$

$$h_i(x) = \frac{\prod_{j=1..N, j \neq i} (x - x_j)}{\prod_{j=1..N, j \neq i} (x_i - x_j)}. \quad (4.8)$$

At $N = 8$ relative mistake in the integral evaluation is less than 10^{-5} until 6σ inclination of the ion co-ordinates from expectation.