Control of Sextupolar Modes

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1.0 Theory

1.1 Control of the First Order Lie Generators

The one-turn map can be written

$$\mathcal{M} = \mathcal{A}^{-1} e^{:h_3 + h_4 + h_5 + \dots :} \mathcal{R}\mathcal{A},$$

$$h_3 \equiv \sum_{|\vec{l}| = n} h_{\vec{l}} h_x^{+i_1} h_x^{-i_2} h_y^{+i_3} h_x^{-i_4} \delta^{i_5}, \qquad \text{(EQ 1)}$$

$$h_x^{\pm} = \sqrt{2J_x} e^{\pm\phi_x} = \sqrt{2J_x}\cos(\phi_x) \pm \sqrt{2J_x}\sin(\phi_x) = x \mp ip_x$$

There are two chromatic terms

$$h_{11001} = \frac{1}{4}\sum_{i=1}^{N} [(b_{2i}L) - 2(b_{3i}L)\eta_{xi}]\beta_{xi}, \qquad h_{00111} = -\frac{1}{4}\sum_{i=1}^{N} [(b_{2i}L) - 2(b_{3i}L)\eta_{xi}]\beta_{yi} \qquad \text{(EQ 2)}$$

and 5 geometric

$$h_{10110} = h^*_{01110} = \frac{1}{4}\sum_{i=1}^{N}(b_{3i}L)\beta_{xi}^{1/2}\beta_{yi}e^{i\mu_{yi}}, \qquad h_{21000} = h^*_{12000} = -\frac{1}{8}\sum_{i=1}^{N}(b_{3i}L)\beta_{xi}^{3/2}e^{i\mu_{xi}},$$

$$h_{30000} = h^*_{03000} = -\frac{1}{24}\sum_{i=1}^{N}(b_{3i}L)\beta_{xi}^{3/2}e^{i3\mu_{xi}},$$

$$h_{10020} = h^*_{01200} = \frac{1}{8}\sum_{i=1}^{N}(b_{3i}L)\beta_{xi}^{1/2}\beta_{yi}e^{i(\mu_{xi}-\mu_{yi})},$$

$$h_{10200} = h^*_{01020} = \frac{1}{8}\sum_{i=1}^{N}(b_{3i}L)\beta_{xi}^{1/2}\beta_{yi}e^{i(\mu_{xi}+\mu_{yi})}$$

(EQ 3)

Starting from the $M \times N$ system

$$\overline{\Delta \boldsymbol{h}} = \boldsymbol{T}\overline{(\Delta \boldsymbol{b}_3 \boldsymbol{L})} \qquad \textbf{(EQ 4)}$$

where

$$\overline{\Delta \boldsymbol{h}} \equiv \left[\Delta \boldsymbol{h}_1, \Delta \boldsymbol{h}_2, \ldots, \Delta \boldsymbol{h}_{\boldsymbol{M}}\right]^{\mathrm{T}}, \qquad \overline{(\Delta \boldsymbol{b}_3 \boldsymbol{L})} = \left[(\Delta \boldsymbol{b}_3 \boldsymbol{L})_1, (\Delta \boldsymbol{b}_3 \boldsymbol{L})_2, \ldots, (\Delta \boldsymbol{b}_3 \boldsymbol{L})_{\boldsymbol{N}}\right]^{\mathrm{T}} \qquad \textbf{(EQ 5)}$$

the inverse is obtained by SVD

$$\overline{(\Delta \boldsymbol{b}_3 \boldsymbol{L})} = \boldsymbol{T}^{-1}\overline{\Delta \boldsymbol{h}} \qquad \textbf{(EQ 6)}$$

To summarize, the “sextupole response matrix” can be computed off-line from the linear optics. It can then be implemented on the control system for effective control of the individual Lie generators, i.e. “smart knobs”.

1.2 Measurement of the Lie Generators

The n-turn map is

$$\vec{x}_n = \mathcal{M}^n \vec{x}_0 \qquad \text{(EQ 7)}$$

For example, the generator

$$\begin{aligned} h_{30000} h_x^{+3} + \text{c.c.} &\equiv A_{30000} e^{i\phi_{30000}} h_x^{+3} + \text{c.c.} \\ &= 2A_{30000} (2J_x)^{3/2} \cos(3\phi_x + \phi_{30000}) \end{aligned} \qquad \text{(EQ 8)}$$

leads to

$$J_x(n) = J_x + \frac{3A_{30000}(2J_x)^{3/2}}{\sin(3\pi\nu_x)} \sin[\phi_{30000} + 3(\phi_x - \pi\nu_x + n2\pi\nu_x)] \qquad \text{(EQ 9)}$$

Conclusion, by collecting turn-by-turn data from two adjacent BPMs, the linear invariant can be computed, Fourier analyzed, and...

2.0 Implementation

1. Sextupole response matrix and check by Tracy-2 simulation. Yun/Johan
2. Smart knobs for ditto. Nikolay.
3. Off-line analysis of the collected data. Johan
4. On-line data analysis tool. Todd