Experimental verification of the GDH Sum Rule

A survey including the extension of the GDH integral to virtual photons

Klaus Helbing

Klaus.Helbing@physik.uni-erlangen.de

University of Erlangen

Outline

- The GDH Sum Rule for real photons
  - Setup at ELSA and MAMI
  - Results for the proton
  - Resonance structure, high-energy behavior
  - Future experiments
- The GDH integral for virtual photons
  - Formalism for generalization
  - Status of predictions
  - Results from JLAB AND HERMES

The GDH Sum Rule at $Q^2 = 0$

$$\int_0^\infty d\nu \, \frac{\sigma_{3/2}(\nu) - \sigma_{1/2}(\nu)}{\nu} = \frac{2\pi^2\alpha}{m^2}\,\kappa^2$$

- Connects static observables of the nucleon to dynamics.
- Links QED with strong interactions: excitation spectrum.
- Gerasimov, Drell and Hearn (1966): Dispersion theory, rated SR purely academic since no exp. test at horizon.
- Hosoda, Yamamoto (1966): Current algebra, holds only if proton Dirac particle, exp. straight forward to test.
- Iddings (1965): No explicit mention …but already generalized version!

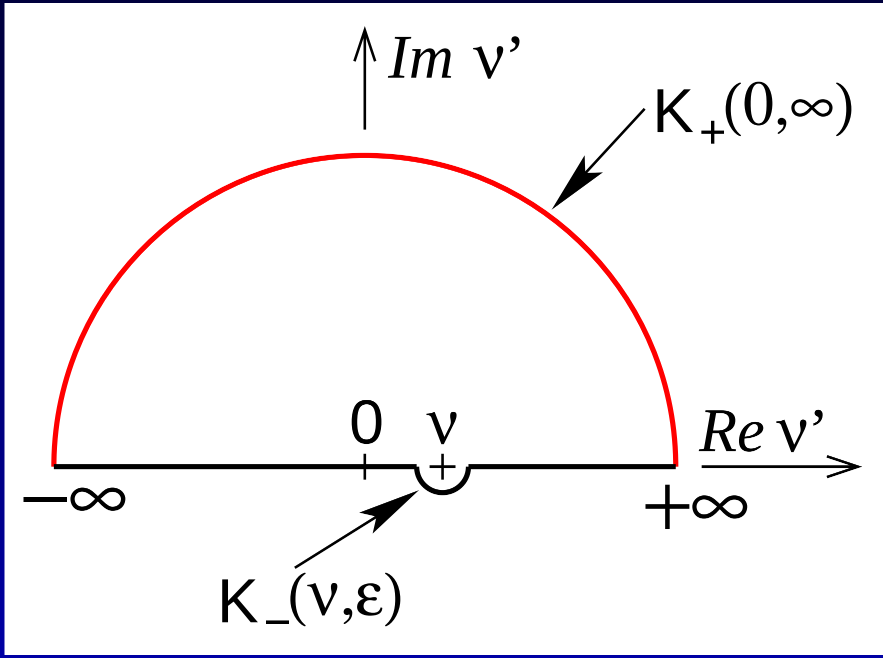
Derivation: Fundamental bases

- Lorentz invariance, gauge invariance $\longrightarrow$ Compton-forward amplitude
  $f(\nu) = f_1(\nu)\vec{e_2^*} \cdot \vec{e_1} + f_2(\nu) i\, \vec{\sigma} \cdot (\vec{e_2^*} \times \vec{e_1})$
- Unitarity $\longrightarrow$ Optical Theorem:
  $\Im m\ f_2(\nu)/\nu = (4\pi)^{-1} \cdot (\sigma_{1/2} - \sigma_{3/2})$
- Lorentz, gauge invariance $\longrightarrow$ Low-Theorem:
  $f_2(\nu)/\nu \overset{\nu \to 0}{=} -\alpha/2m^2 \cdot \boldsymbol{\kappa^2}$
- Causality + **"No-subtraction"** $\longrightarrow$ Kramers-Krönig dispersion relation:
  $\Re e\, f_2(\nu)/\nu = \frac{2}{\pi}\, \mathcal{P} \int\limits_0^\infty d\nu'\, \nu' \frac{\Im m\ f_2(\nu')/\nu'}{\nu'^2 - \nu^2}$

No-subtraction hypothesis

Statement on spin-flip Compton-forward amplitude.

Integration path for Kramers-Krönig dispersion relation:



$$\text{No-subtraction} \Leftrightarrow \int_{K_+(0,\infty)} d\nu' \, \frac{f_2(\nu')/\nu'}{\nu' - \nu} = 0$$

Violation of No-subtraction

…leads to weird behavior:

- Amplitude: $\lim f_2(\nu) \propto \nu$
- Differential cross section:

$$\lim_{\nu\to\infty} \frac{1}{d\Omega} \left(d\sigma_{3/2} - d\sigma_{1/2}\right)\Big|_{\theta=0} = \infty$$

…while Regge-Theory tells

- Imaginary part of ampl.: $\lim \frac{1}{\nu} \Im m f_2(\nu) = 0$
- Total cross section:

$$\lim_{\nu\to\infty} \left(\sigma_{3/2}^{\text{tot}} - \sigma_{1/2}^{\text{tot}}\right) = 0$$

Possible modifications discussed

- Anomalous charge density commutator (Chang, Liang, Workman)
  $\longrightarrow$ Cancels in infinite-momentum limit (Pantförder).
- Low-Theorem true only in lowest order of electromagnetic coupling (Low).
  - Explicitly holds up to second order (Roy, Cheng).
  - “Exactly correct in any known theory” (Gell-Mann, Goldberger).
- Photoproduction of gravitons (Goldberg)
  $\longrightarrow$ restoration by string excitations.

Modifications (cont.)

- Fixed $J = 1$ Regge pole (Abarbanel, Goldberger)
  - Froissart theorem: No fixed poles at $J \leq 1$ in pure hadronic processes;
    usually assumed to hold in electro-weak processes.
  - Violates Landau-Yang?

$\Rightarrow$ **Today no stringent indication for modifications!**

Any possible modification would be related to violation of No-Subtraction Hypothesis

Experimental approach

$$\int_0^\infty \frac{d\nu}{\nu}\, \sigma_{3/2}(\nu) - \sigma_{1/2}(\nu) \quad = \quad \frac{2\pi^2\alpha}{m^2}\,\kappa^2$$

dynamic, measure = static, known (prec. 10^{-6})

- Determination of the GDH integrand at 2 accelerators:

  from resonance region **0.14 - 0.8 GeV** at MAMI

  to onset of Regge regime **0.70 - 3.0 GeV** at ELSA

- **Tagging-System:** determination of photon energy and flux
- **Polarized Target:** longitudinally polarized ^{1}H or ^{2}H
- **Møller-Polarimeter:** longitudinal electron polarization $\xrightarrow{\text{hel. transfer}}$ circular photon pol.
- **Detector:** DAPHNE (MAMI), GDH-Detector (ELSA)

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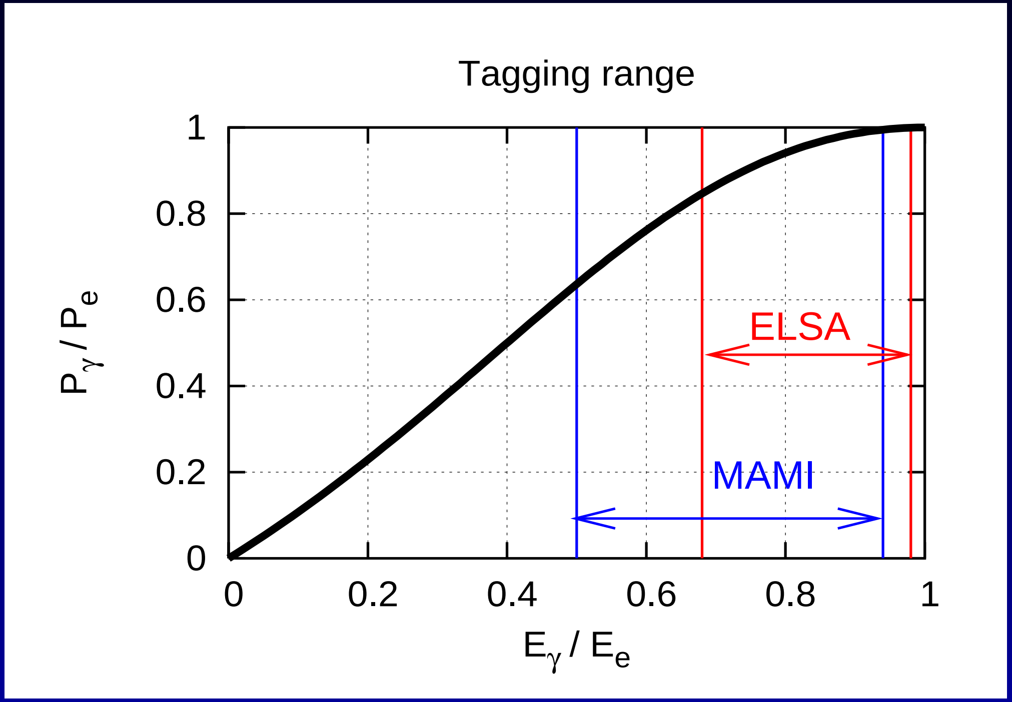
Helicity transfer



MAMI

Tagging range: ~ 50 - 94 %

2 primary energy settings:

0.4, 0.85 GeV

$E_\gamma = 140 - 800$ MeV

ELSA

Tagging range: 68 - 97 %

5 primary energy settings:

1.0, 1.4, 1.9, 2.4, 3.0 GeV

$E_\gamma = 700 - 2950$ MeV

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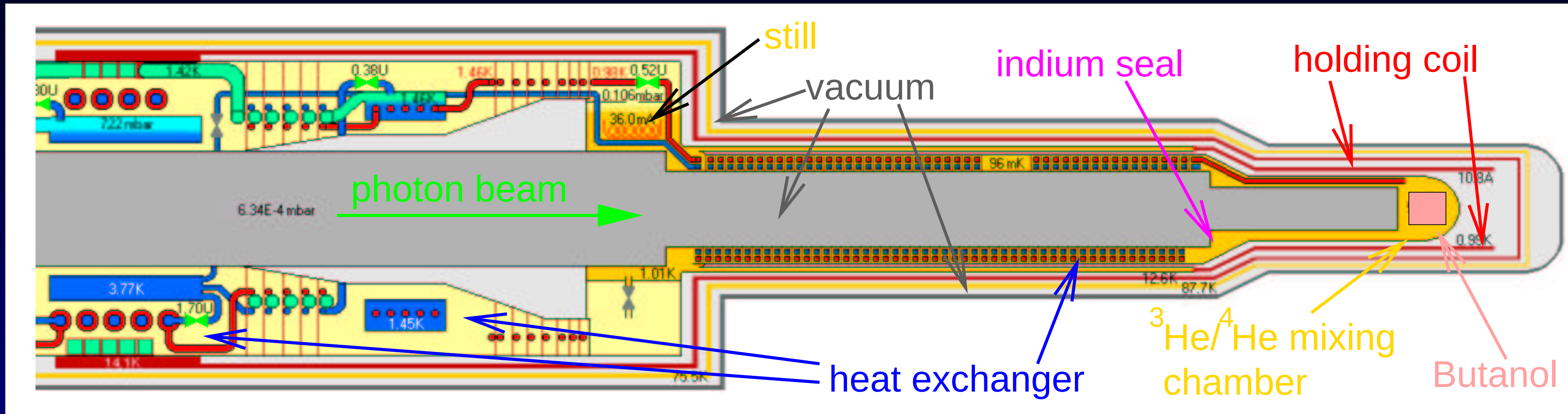
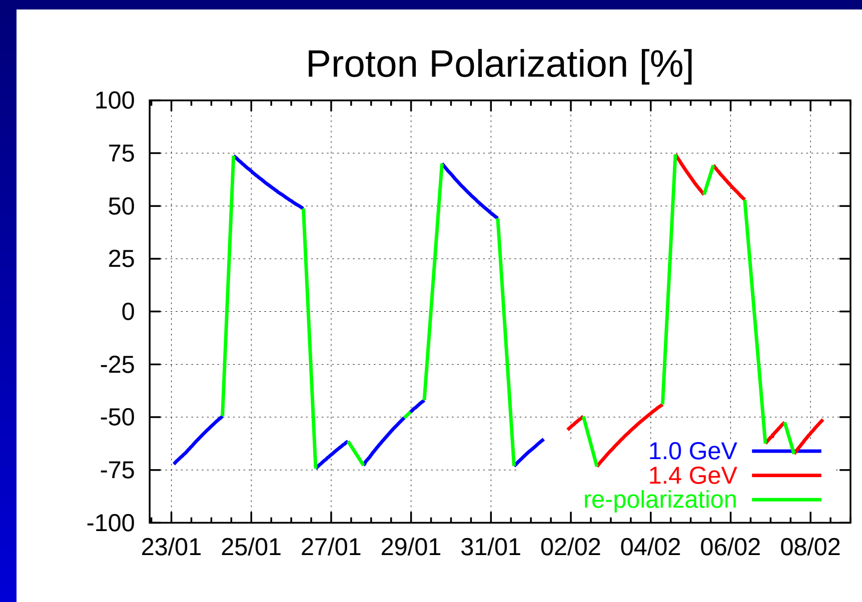
Polarized frozen spin target



- **Horizontal cryostat** $\rightarrow$ dead solid angle minimal.
- Material: **Butanol, ^{6}LiD**
- **Frozen spin** at 60 mK $\rightarrow$ low holding field (0.5 T).
- Super-conducting **internal holding coil** $\rightarrow$ low fringe field in detector.



$P_{\text{max}} \sim 85\%; \quad \tau_{\text{max}}^{\text{relax}} \sim 200$ h.

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Detector concepts

GDH-Detector

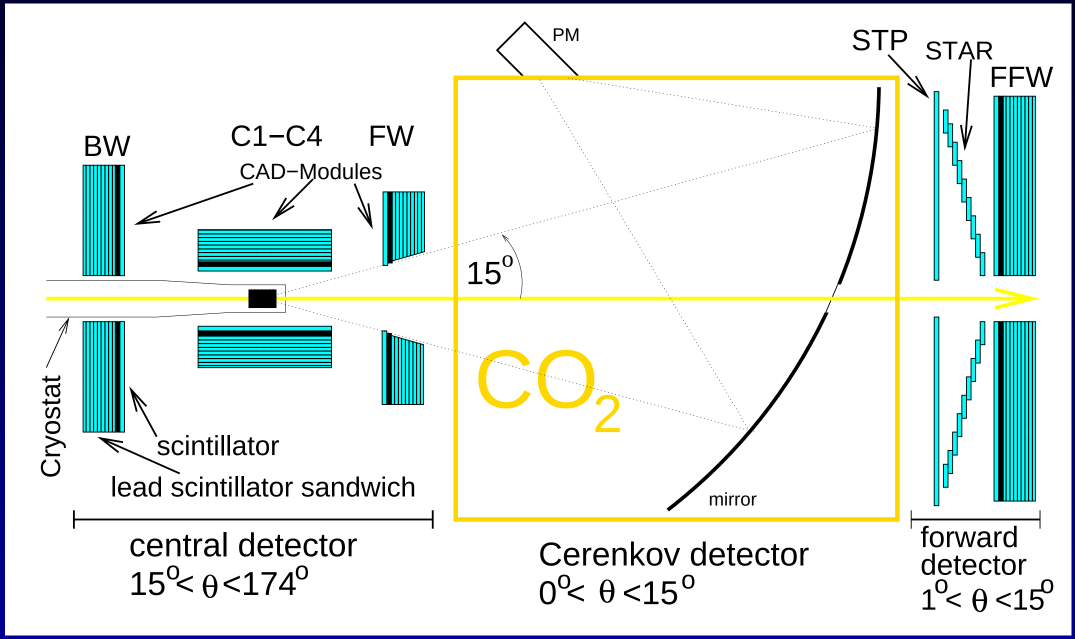
Built for higher energies with unexplored channels:

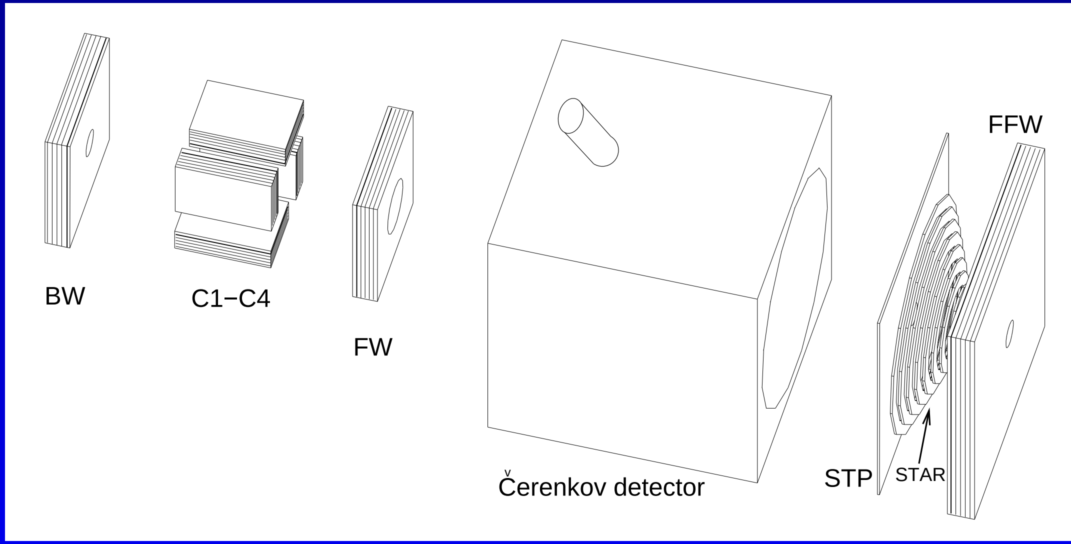
$\gamma p \rightarrow \pi^+ \pi^0 n, \Lambda K^+, \Sigma \ldots$

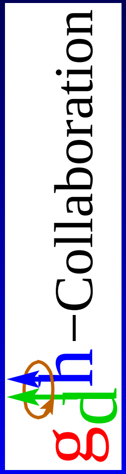
$\Rightarrow$ *inclusive measurement.*

$\Rightarrow$ Detection of charged particles and decay photons.

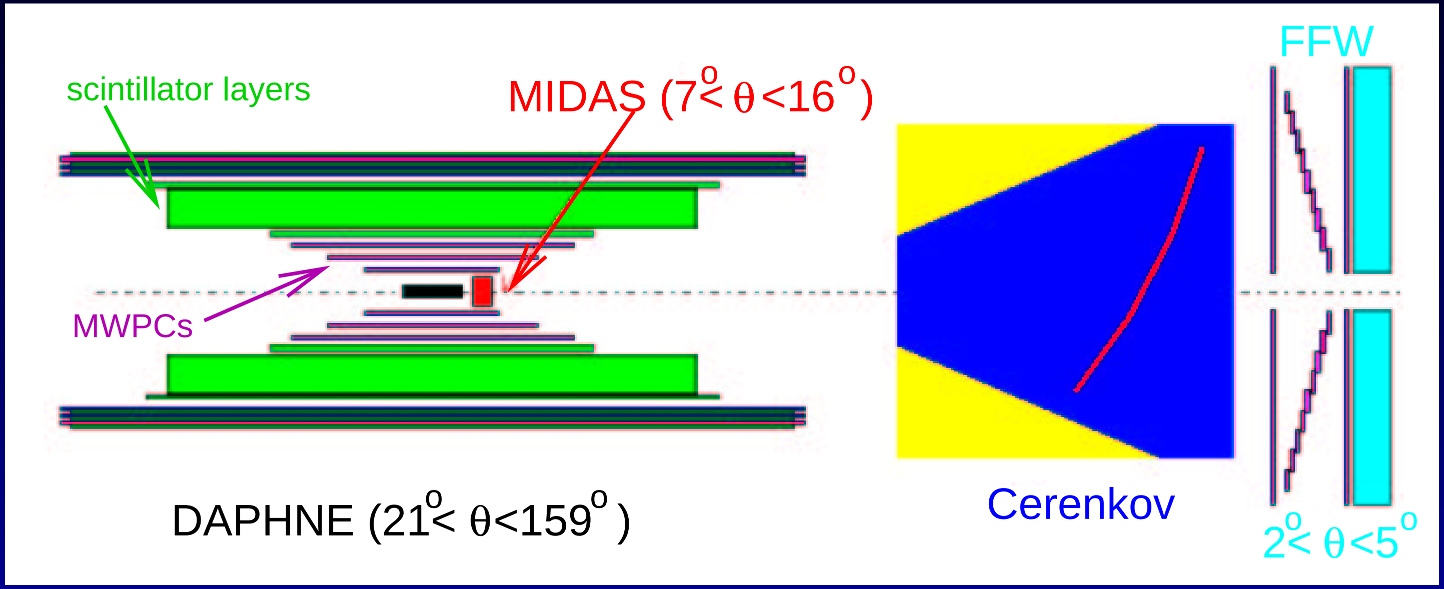
- **99%** efficiency for hadronic processes.
- **99.6%** solid angle coverage!
- Cerenkov detector: **99.99%** elm. veto!



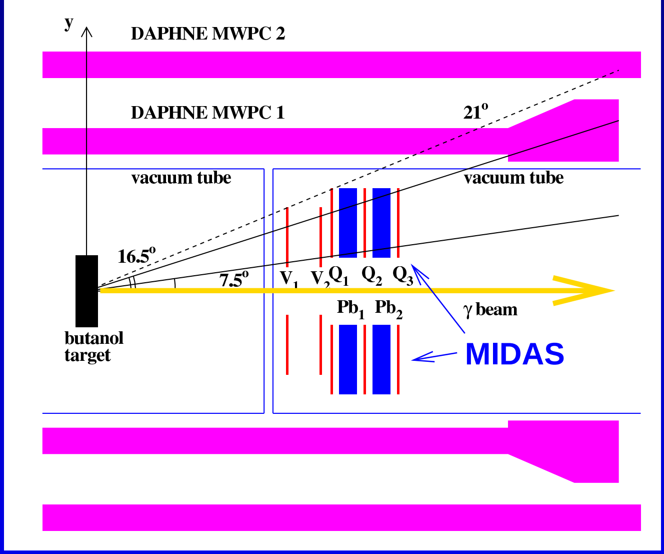




Detector concepts (cont.)



DAPHNE + Forward at MAMI



Small number of channels and low multiplicity states

⇒ charged particle tracking and identification

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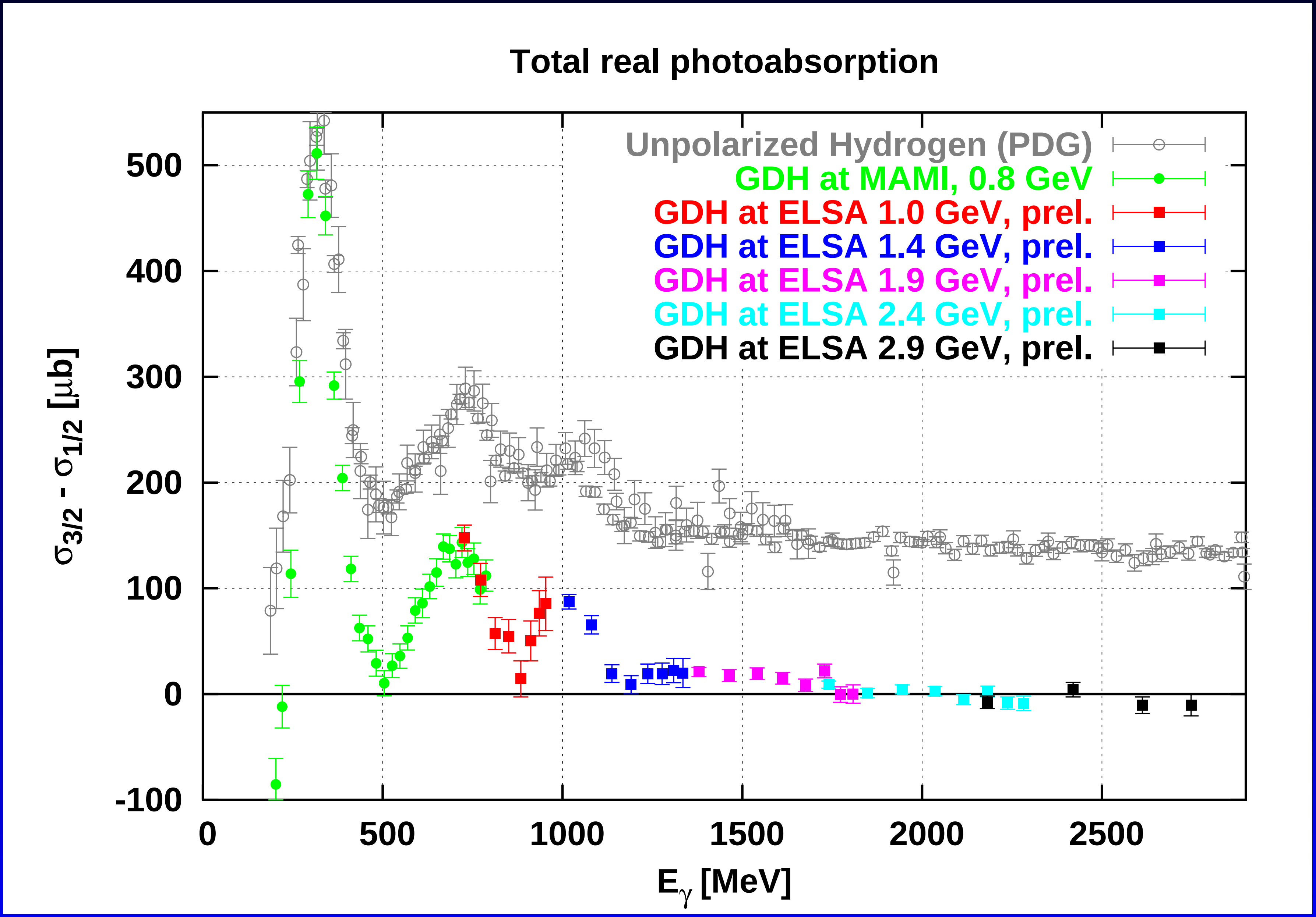
Polarized x-section difference



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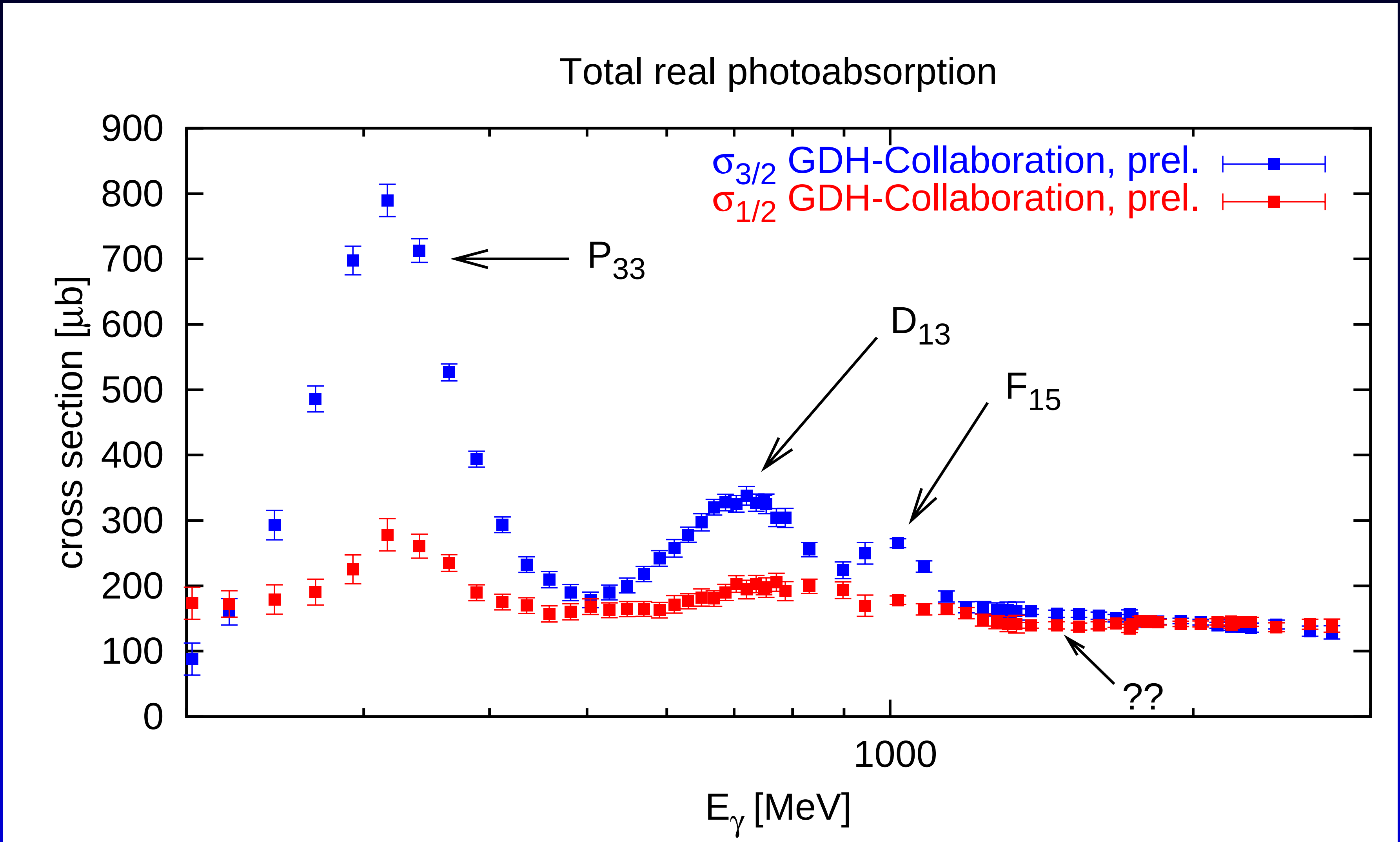
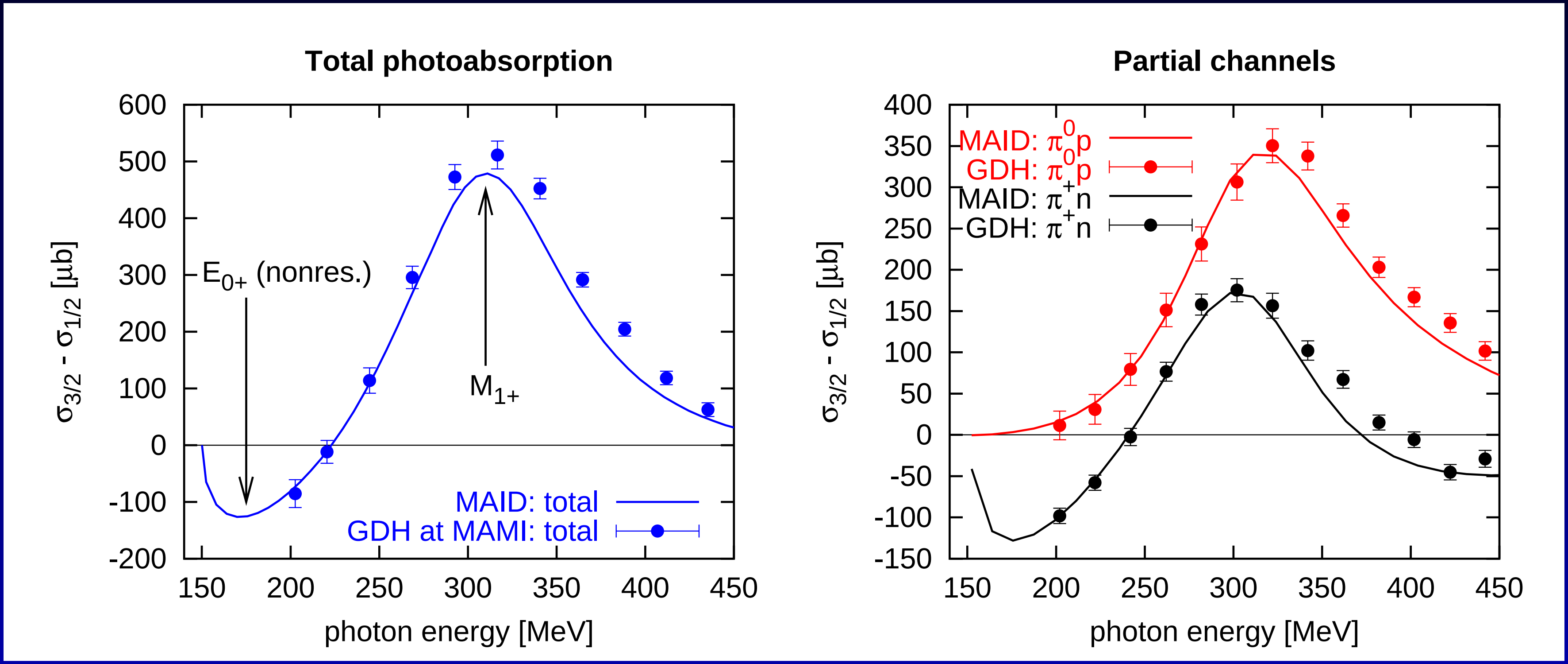
X-sections: Helicity 3/2 and 1/2



Major resonances have $J \geq 3/2$

$\Rightarrow \sigma_{3/2}$ carries most of the resonance structure

Delta resonance



Example for partial wave analysis: MAID

Unitary Isobar Model: Phenomenological model based on Born terms, vector mesons and multipoles P_{33}, P_{11}, D_{13}, S_{11}, F_{15}, D_{33}

Good agreement with data

$\pi^0 p$: charge at center of gravity

$\longrightarrow$ no dipole moment

$\longrightarrow$ no E_{0+} at thresh.

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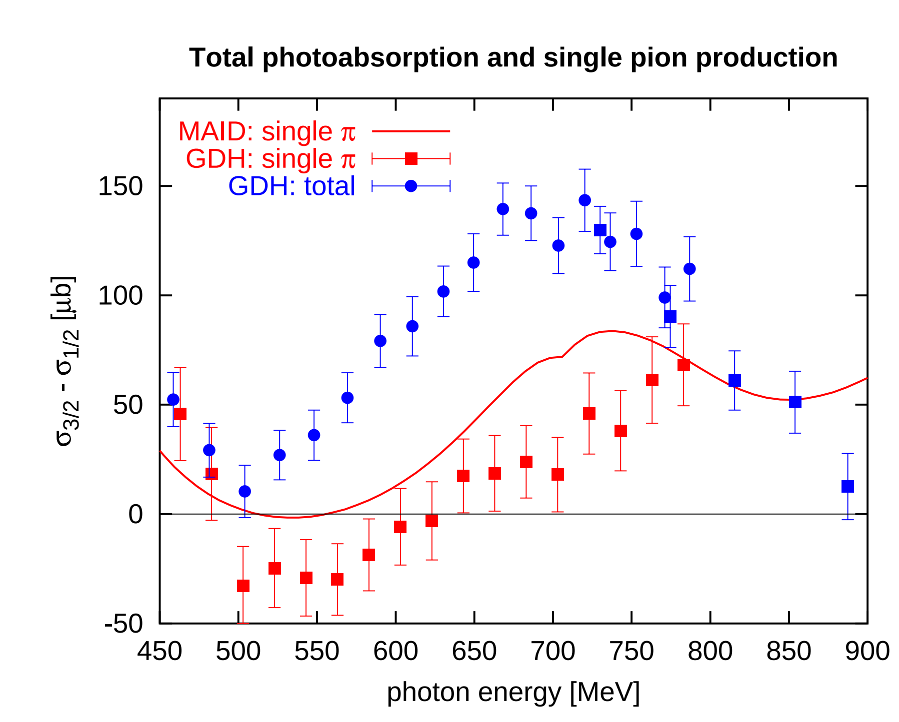
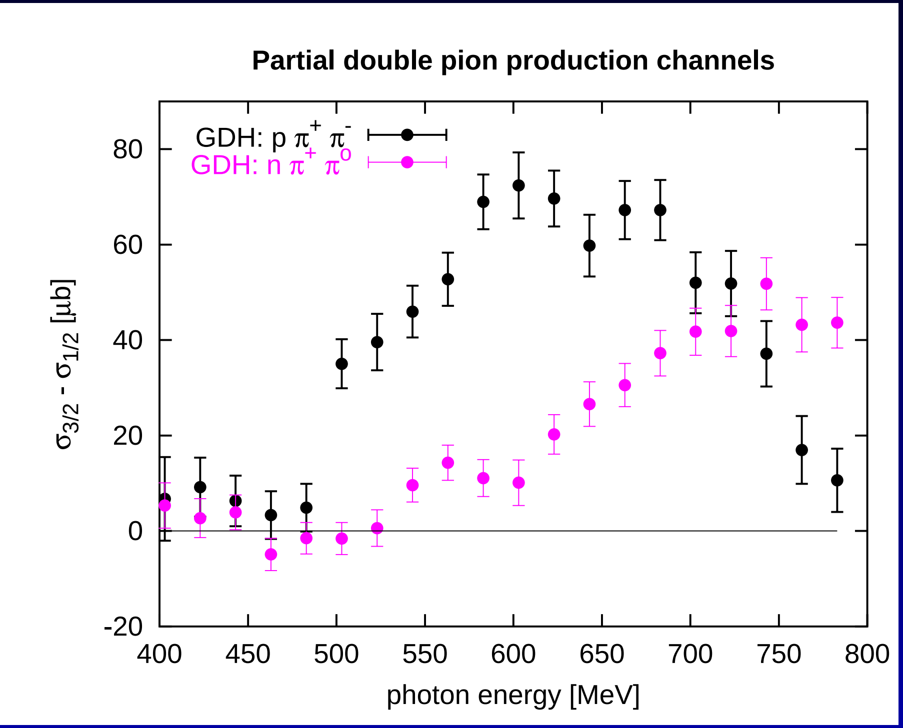
2nd resonance

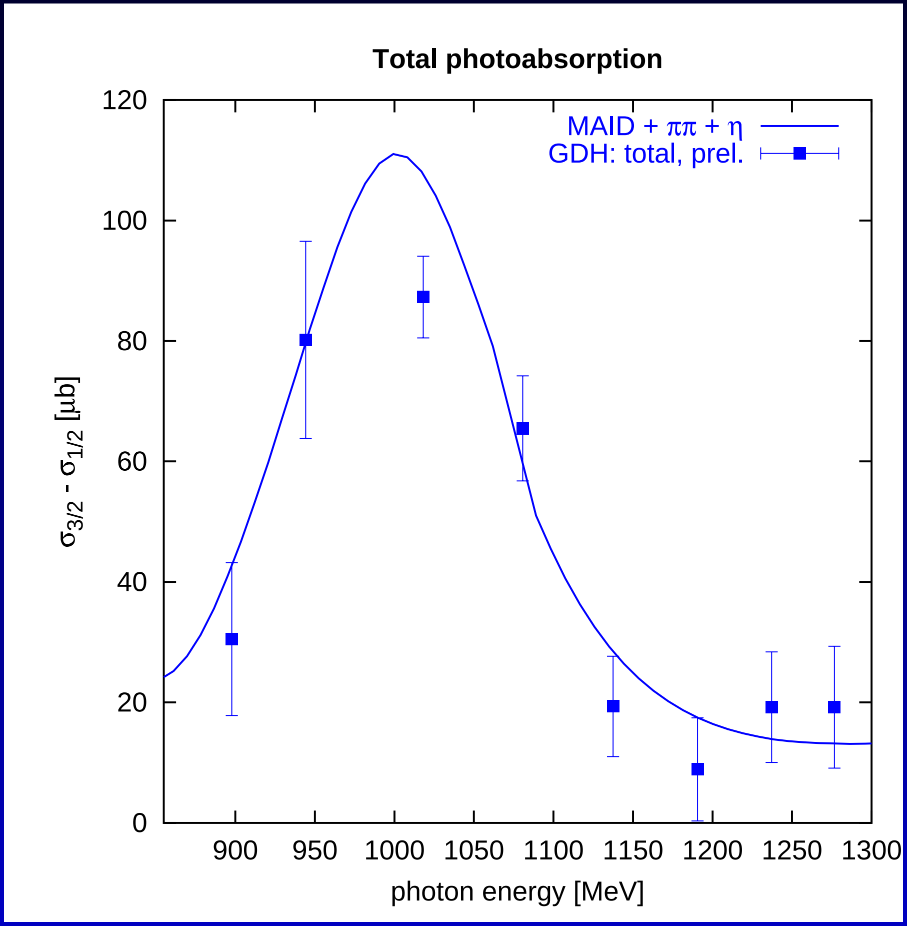




MAID overestimates single pion
$\rightarrow$ Data lead to refined D_{13} resonance couplings to $M_{2-}^{1/2}, E_{2-}^{1/2}$

All existing model predictions changed dramatically in view of data.

3rd resonance



MAID + 2π estimate: rough agreement

$\longrightarrow$ mainly resonant single pion production

Could be coincidence only

$\longrightarrow$ to be verified by data analysis!

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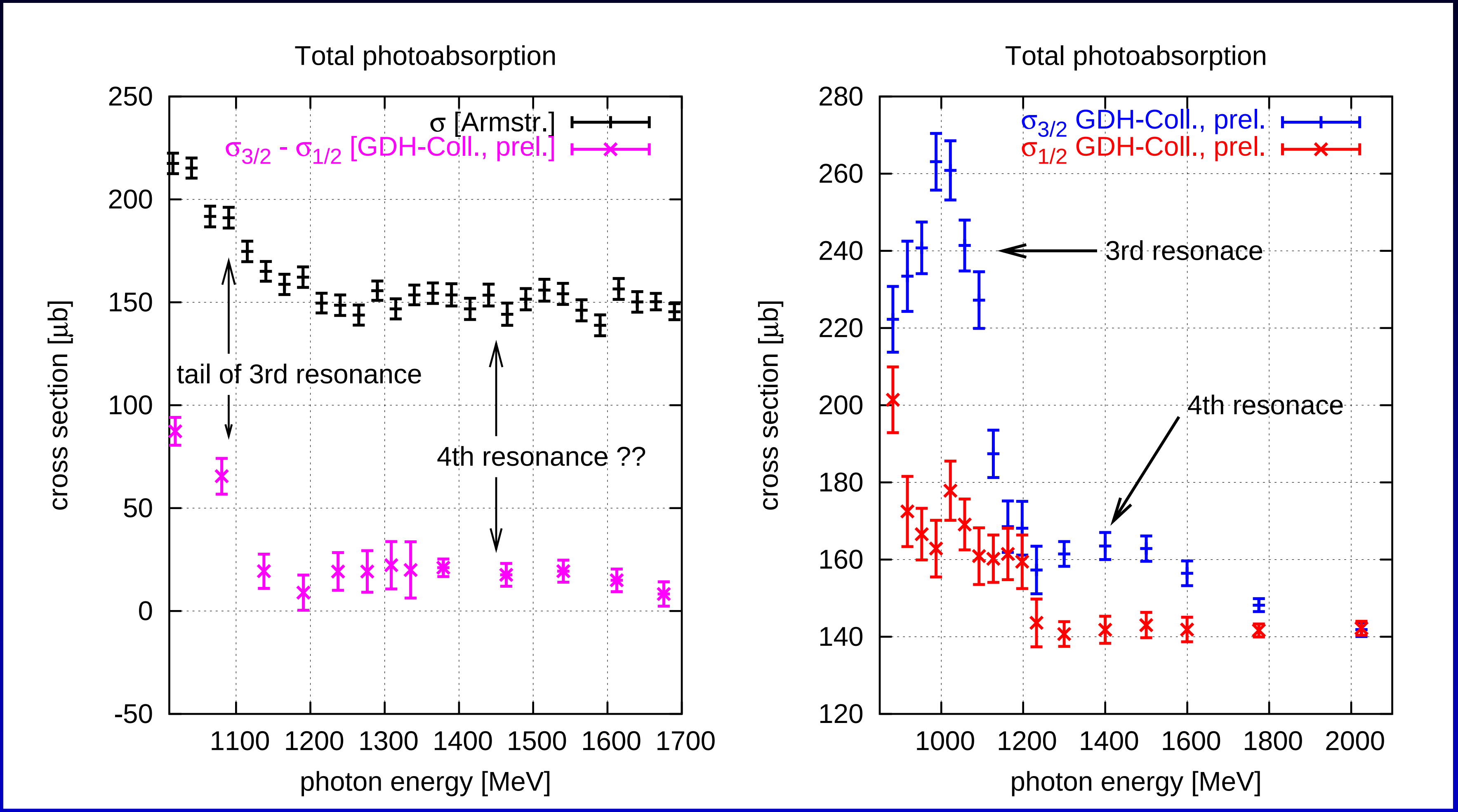
4th resonance



- no structure visible in unpolarized cross section
- clearly visible in separate helicity cross sections

Structure could be generated by F_{35}, F_{37} resonances.

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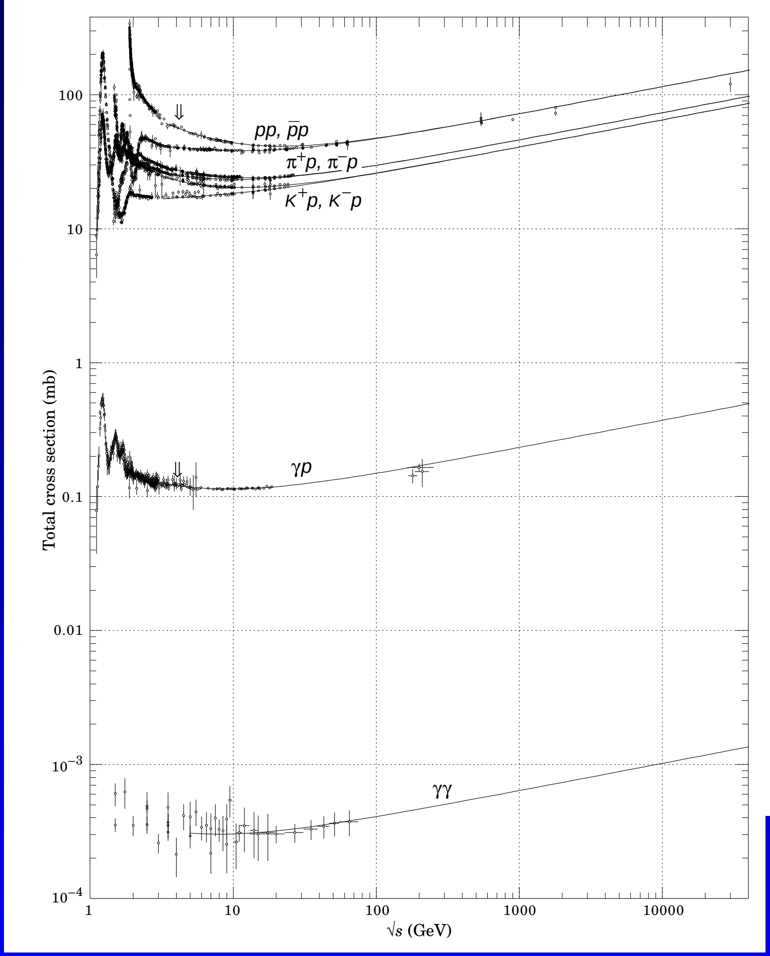
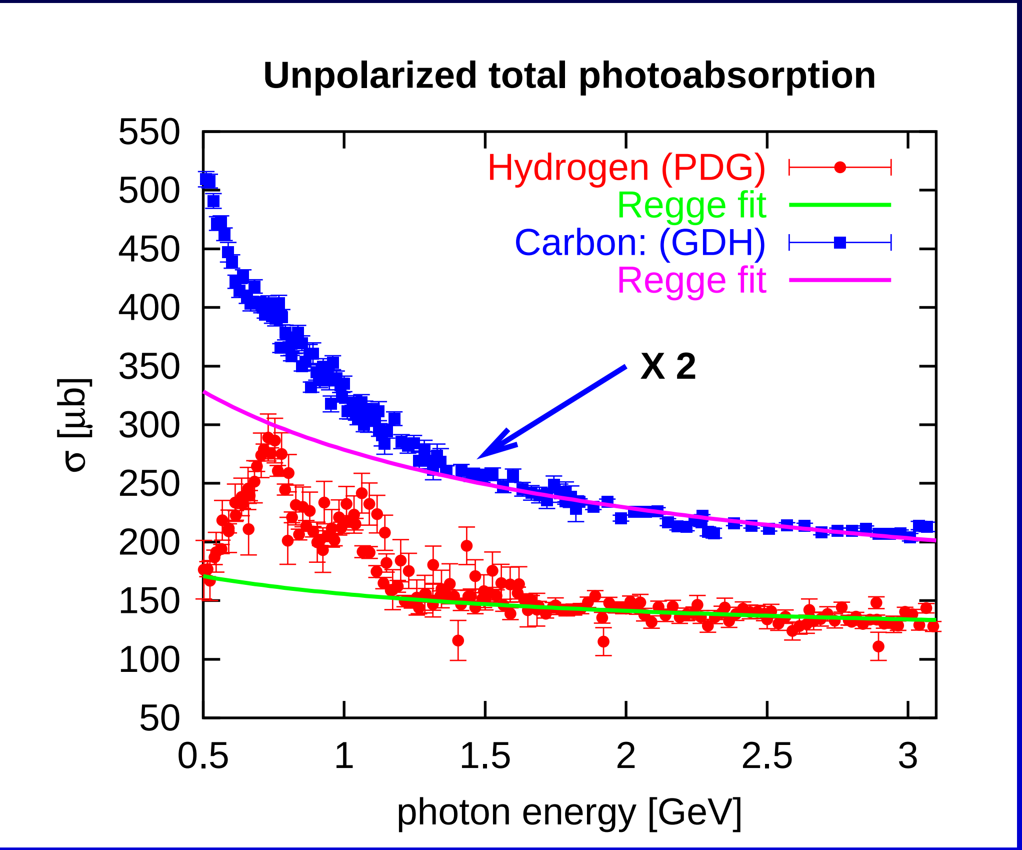
Unpolarized high energies

Regge: s large $\Rightarrow \sigma_T(s) \simeq c_1 \cdot s^{\alpha_R(0)-1} + c_2 \cdot s^{\alpha_P(0)-1}$

$\alpha_R(0) = 0.53$: ρ, ω trajectory; $\alpha_P(0) = 1.08$: Pomeron trajectory





Regge fit works down to $\sim$ 1.2 GeV!

Polarized: Regge trajectories

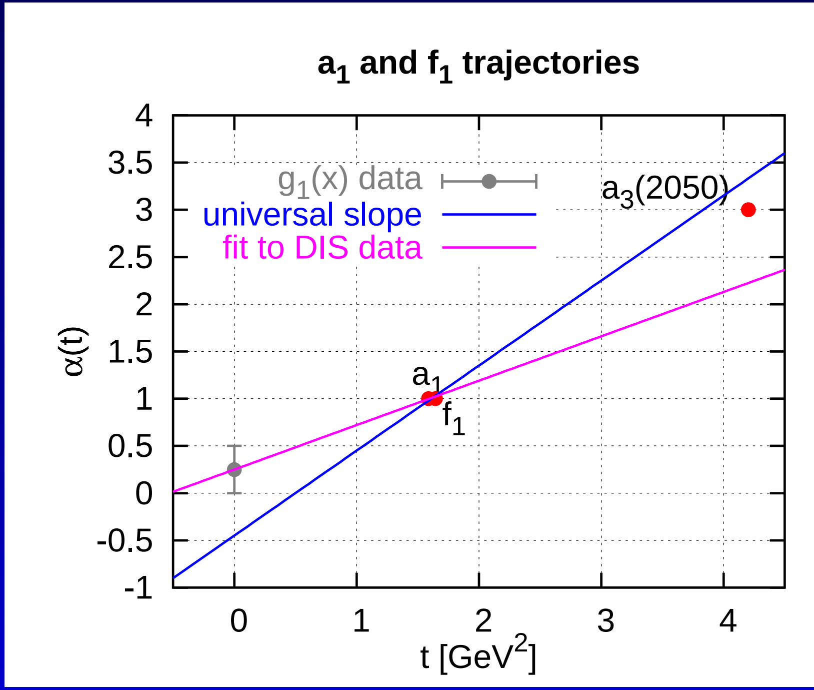
Virtual photons(Bass, Brisudová):

$$\sigma_{3/2} - \sigma_{1/2} = \left[c_1 \; s^{\alpha_{a_1}-1} \cdot I + c_2 \; s^{\alpha_{f_1}-1} + c_3 \frac{\ln s}{s} + \frac{c_4}{\ln^2 s} \right] F(s, Q^2)$$

Real photons:

- Pomeron cuts negligible
- $a_1(1260) : I^{GC} = 1^{-+}$
  $\Delta\sigma^{p-n} = c_1 \; s^{\alpha_{a_1}(0)-1}$
- $f_1(1285) : I^{GC} = 0^{++}$
  $\Delta\sigma^{p+n} = c_2 \; s^{\alpha_{f_1}(0)-1}$

$\Rightarrow \Delta\sigma \simeq c_1 \; s^{\alpha_{a_1}-1} \cdot I + c_2 \; s^{\alpha_{f_1}-1}$

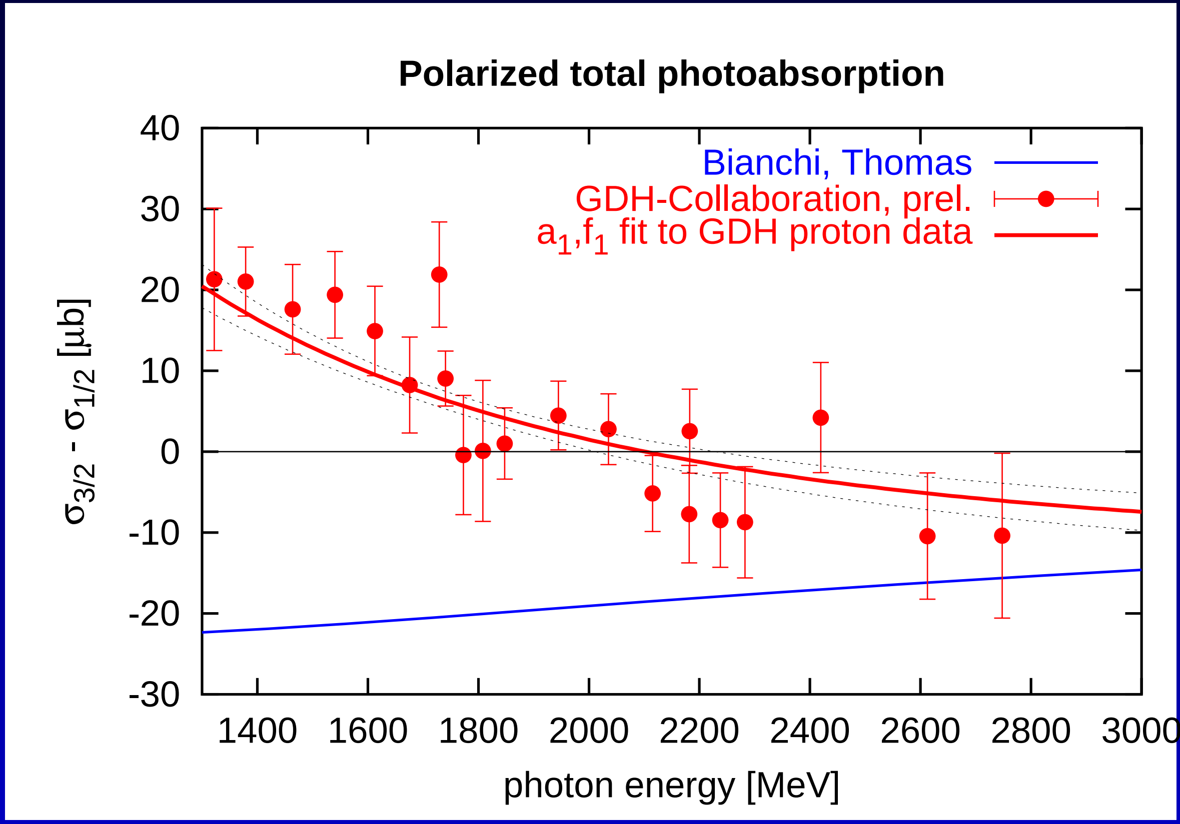


Regge fits to DIS data indicate: $\alpha_{f_1} \simeq -0.5$ but

$\alpha_{a_1} \simeq +0.2 \pm 0.4$ less precisely determined; +0.2 adopted here.

Application to polarized data

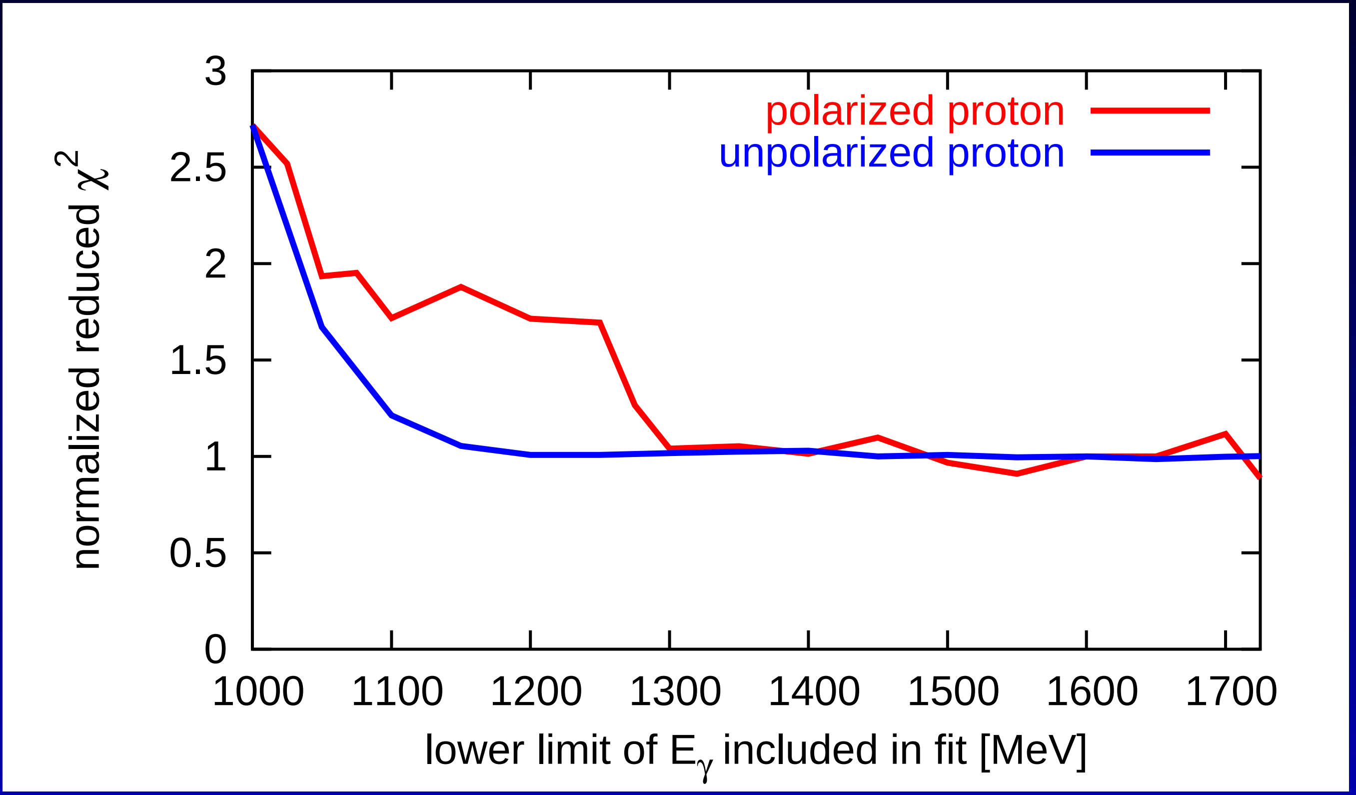
Proton: $\sigma_{3/2} - \sigma_{1/2} = +c_1(Q^2)\, s^{\alpha_{a_1}-1} + c_2(Q^2)\, s^{\alpha_{f_1}-1}$



Direct fit via $c_{1,2}(0)$ of real photon data works well and indicates sign change!

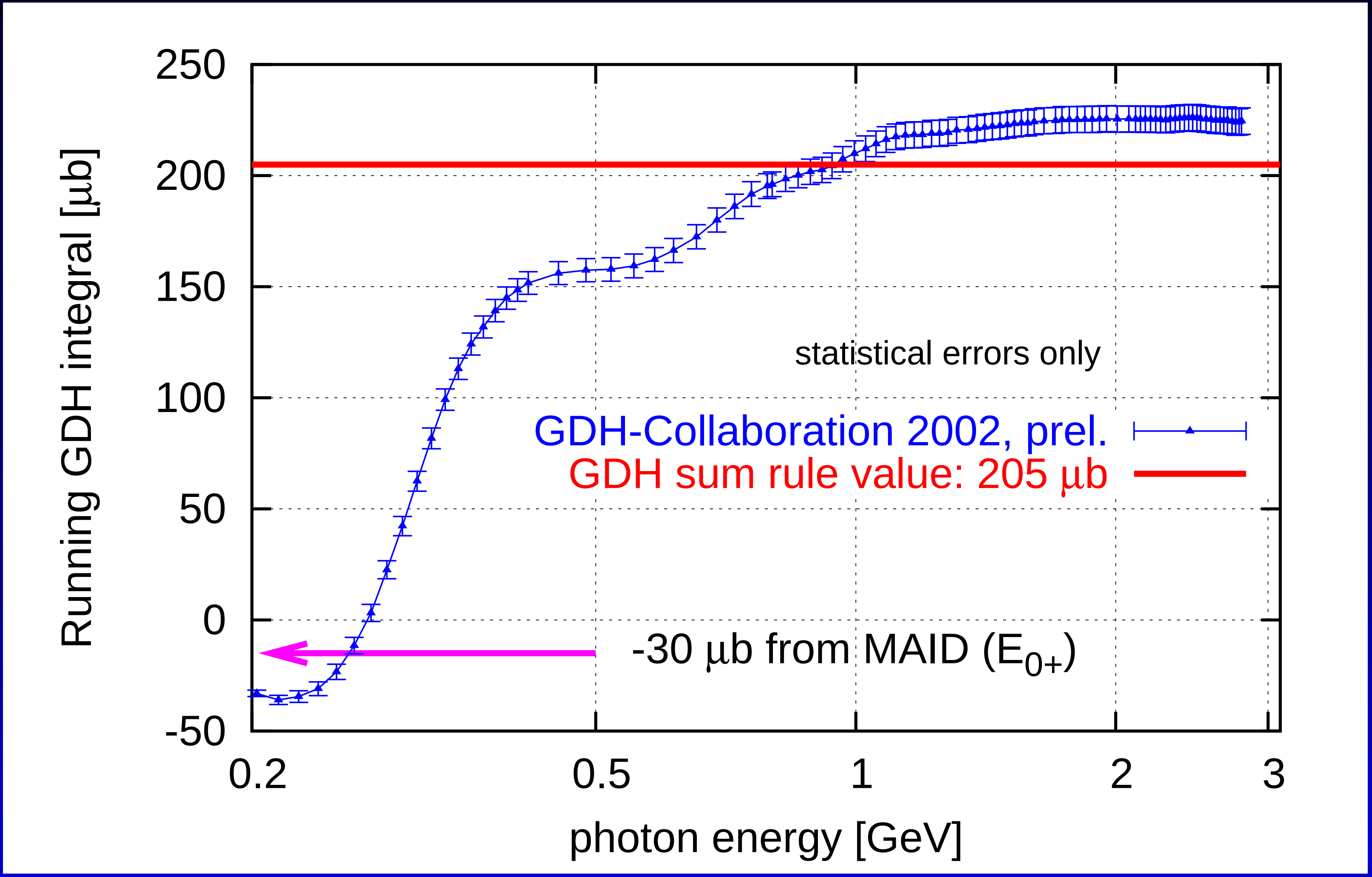
Extrapolation of $c_{1,2}$ to $Q^2 \to 0$ of Regge fits to DIS data fail.

Are we in the Regge regime?



- Unpolarized: works down to $\sim$ 1.2 GeV
- Polarized: Fit works down to $\sim$ 1.3 GeV
- Another indication for 4th resonance!

Running GDH integral



The GDH sum up to 2.8 GeV: $(\mathbf{225 \pm 5_{stat}})\mu$b (prelim.)

Contribution from $E_\gamma > 3$ GeV: -20 – -35 μb

α_{a_1} crucial $\longrightarrow$ to be fixed by ELSA neutron data.

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Real photons: outlook

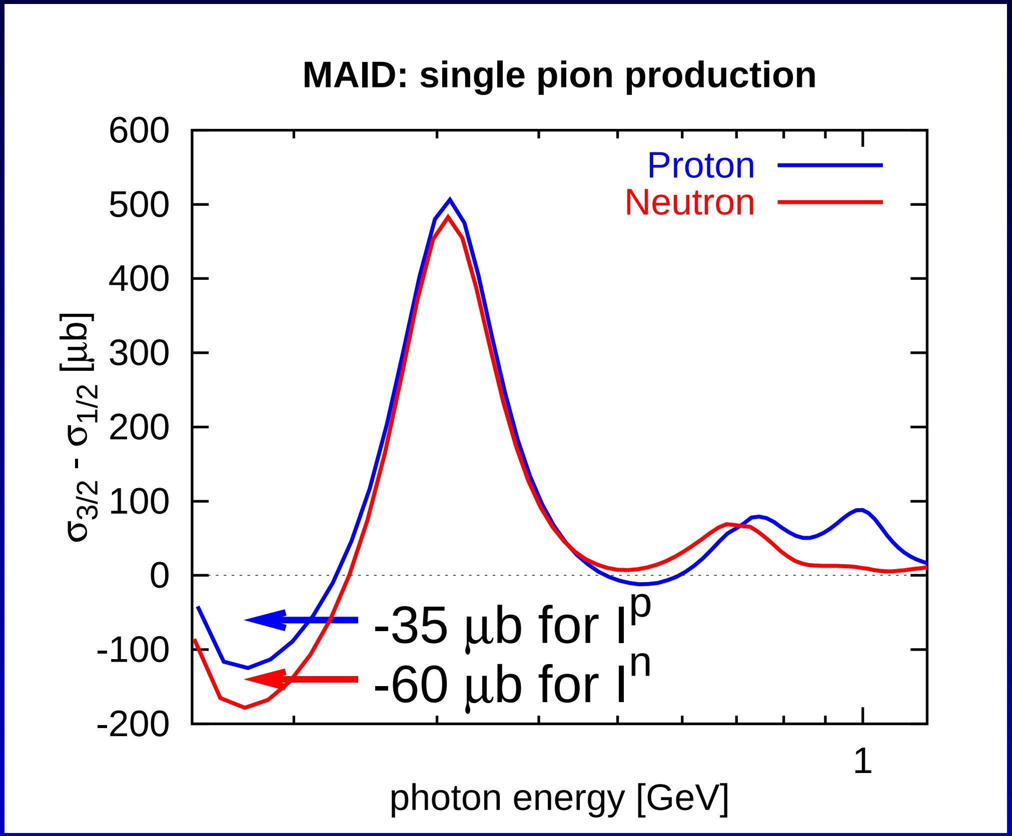
Laser backscattering facilities (more: Thu, A. Sandorfi):

- LEGS: $E_\gamma < 470$ MeV (first results at tip of Δ)
  HD-target tricky $\to$ more: Tue, S. Hoblit
- GRAAL: $E_\gamma < 550$ MeV (2003)
- SPRING-8: 1.8 – 2.8 GeV (2003/4)
  dynamic pol. of PE-foils

- ELSA: High statistics data up to 3 GeV on tape
  $\longrightarrow$ refine Regge fit for proton to consolidate extrapolation
- JLAB: Proposal for 2.5 – 6 GeV in preparation
  acceptance issues $\to$ frozen spin target under development
- SLAC: Approved exp. for 4 – 40 GeV (2005)
  dynamic NH_3, ND_3 target pol.; calorimeter; untagged coherent γ

Real photons: neutron prospect

SR predictions: $I^p_{\mathrm{GDH}} = 205\mu\mathrm{b} \leftrightarrow I^n_{\mathrm{GDH}} = 233\mu\mathrm{b}$



$\Rightarrow \sim 50\ \mu$b missing for I^n_{GDH} up to 500 MeV!

Either large difference for neutron at high energies or something exciting in resonances or isovector GDH Sum Rule fails !

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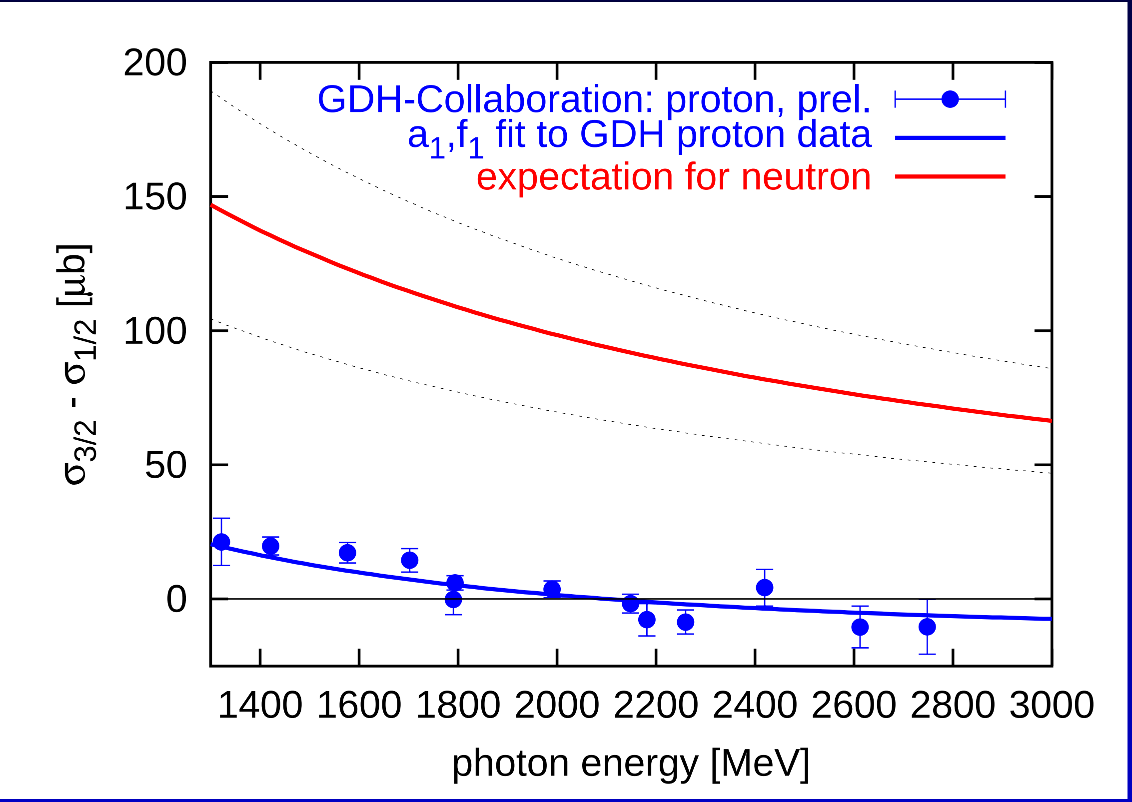
Neutron: high energies

Regge parameterization:

$$\Delta\sigma^n = -\,c_1 s^{\alpha_{a_1}-1} + c_2 s^{\alpha_{f_1}-1} \;(c_1, c_2 \text{ from proton})$$



Data at 1.9 GeV (fix α_{a_1}) and 1.2 GeV already on tape!

GDH program at MAMI will be resumed in 2003.

Stay tuned for further neutron results!

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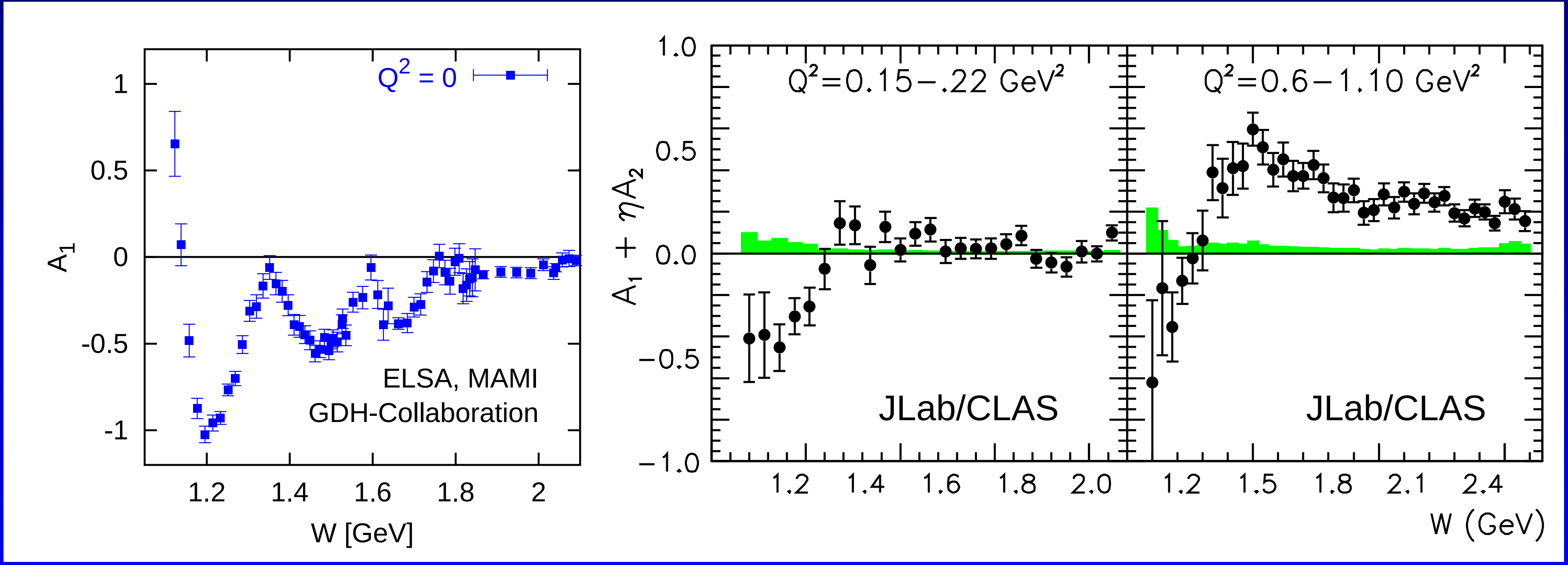
Polarized virtual absorption

$$\frac{d\sigma}{dE'd\Omega} = \Gamma_\nu \cdot \Big[\sigma_T + \epsilon\sigma_L + P_e P_t \Big(\sqrt{1-\epsilon^2}\,\boldsymbol{A}_1 \sigma_T \cos\psi + \sqrt{2\epsilon(1-\epsilon)}\,\boldsymbol{A}_2 \sigma_T \sin\psi\Big)\Big]$$

$$A_1 = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} \equiv \frac{\sigma_{TT'}}{\sigma_T}, \qquad A_2 = \frac{\sigma_{LT'}}{\sigma_T}, \qquad \frac{g_1}{F_1} = f_1^{\text{kin}} A_1 + f_2^{\text{kin}} A_2$$

Math in red due to longitudinal pol. component of photon



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Generalization of GDH sum

- Experimentalists favorite:

$$I^{\mathrm{GDH}}(Q^2) = -\int_0^\infty \frac{d\nu}{\nu} \cdot \left(\sigma_{3/2}(\nu, Q^2) - \sigma_{1/2}(\nu, Q^2)\right)$$

- Theorists favorite: first moment of g_1

$$\Gamma_1(Q^2) = \int_0^1 dx\ g_1(x, Q^2) = Q^2/2 \int_0^\infty \frac{d\nu}{\nu} G_1(\nu, Q^2)$$

  - $\boldsymbol{Q^2 = 0}$: GDH Sum Rule fixes slope
    $\Gamma_1/Q^2 = -\kappa^2/8m^2$, rel. precision 10^{-6}
  - $\boldsymbol{Q^2 \longrightarrow \infty}$: Bjorken Sum Rule $\longrightarrow$ isovector
    $\Gamma_1^p - \Gamma_1^n = g_A/6$, rel. precision $3 \cdot 10^{-3}$
  - $0 < Q^2 < \infty$: ***Study transition from hadronic to partonic degrees of freedom!***

Unified formalism

Virtual Compton forward scattering spin-amplitudes ($\vec{\gamma}^* N \to \vec{\gamma}^* N$):

$S_1(\nu, Q^2), S_2(\nu, Q^2)$, Low-Theorem: $S_1(0,0) = -\kappa^2/m^2$

Ji & Osborne: $$\bar{S}_1(0, Q^2) = 4 \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} G_1(\nu, Q^2)$$

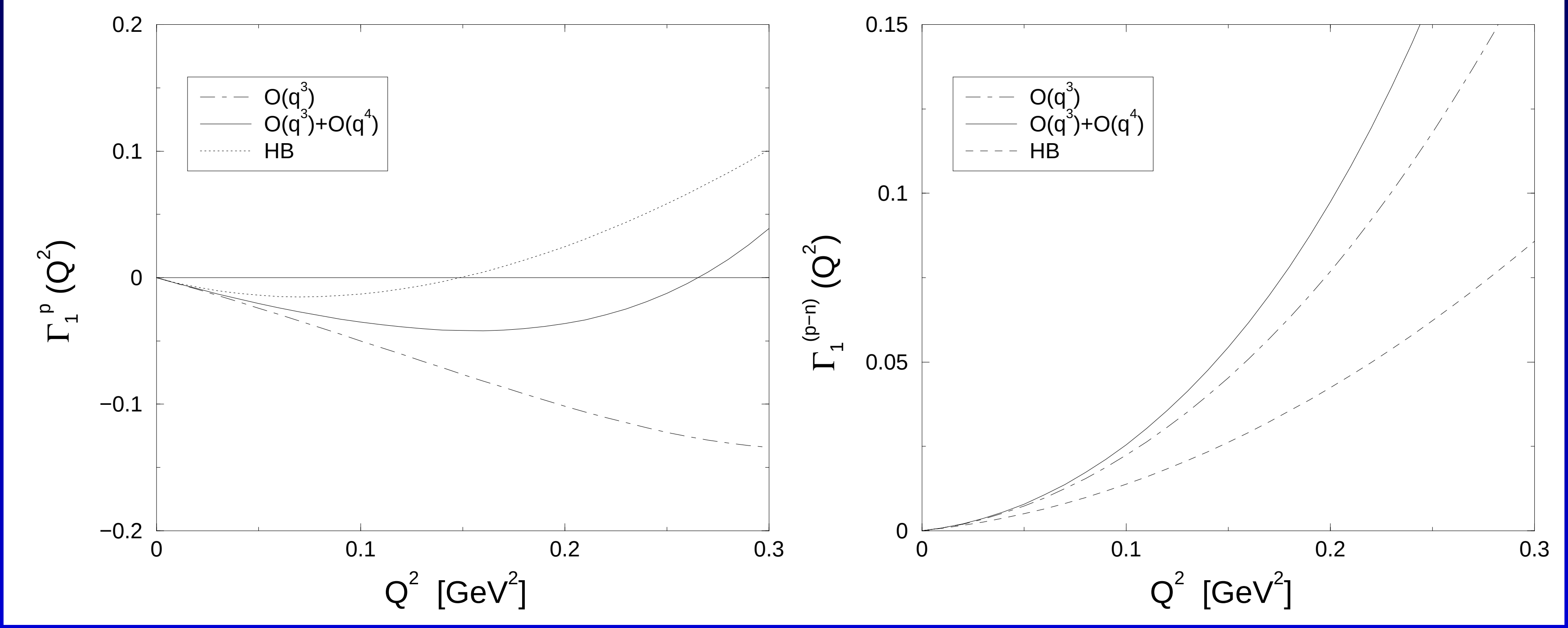
- Right hand side *measurable*
- Left hand side *calculable* ... but longitudinal polarization of photon complicates structure
  $\Rightarrow$ **no strict rule to sum!**
  - $Q^2 > 1$ GeV^2: QCD - OPE
  - $Q^2 > 0.1 - 1$ GeV^2: Lattice ... one day
  - $Q^2 < 0.1$ GeV^2: χPT

$\bar{S}_i = S_i - S_i^{\mathrm{el}}$; ν_0 : inelastic threshold

Status: ChPT

Bernard, Hemmert, Meissner: hep-ph/0203167
Comparison of 4th to 3rd order calculation and
"heavy baryon approach" by Ji, Kao, Osborn

Proton Isovector



About 30 – 50% uncertainty at $Q^2 = 0.1$ GeV^2

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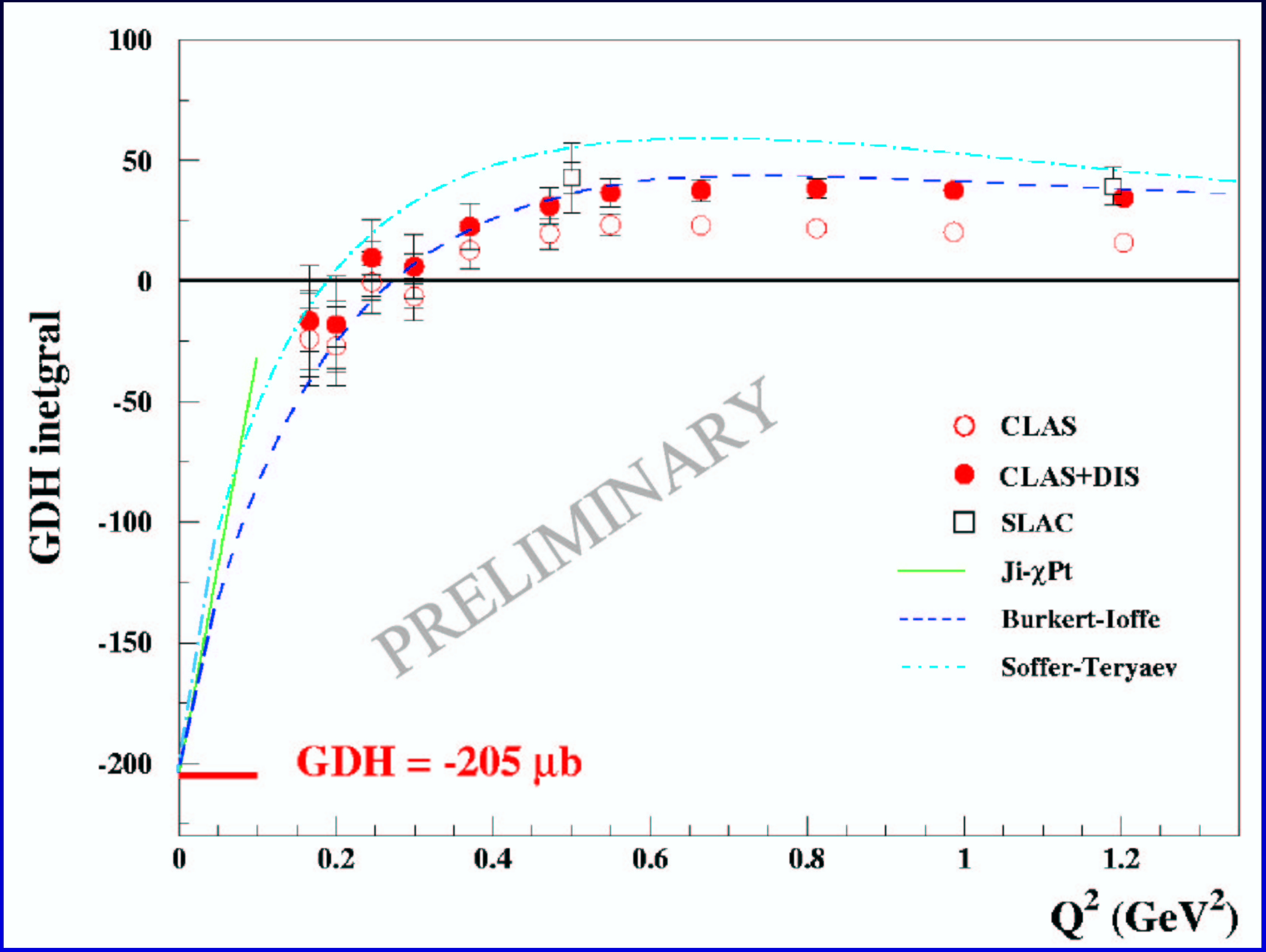
GDH sum: proton

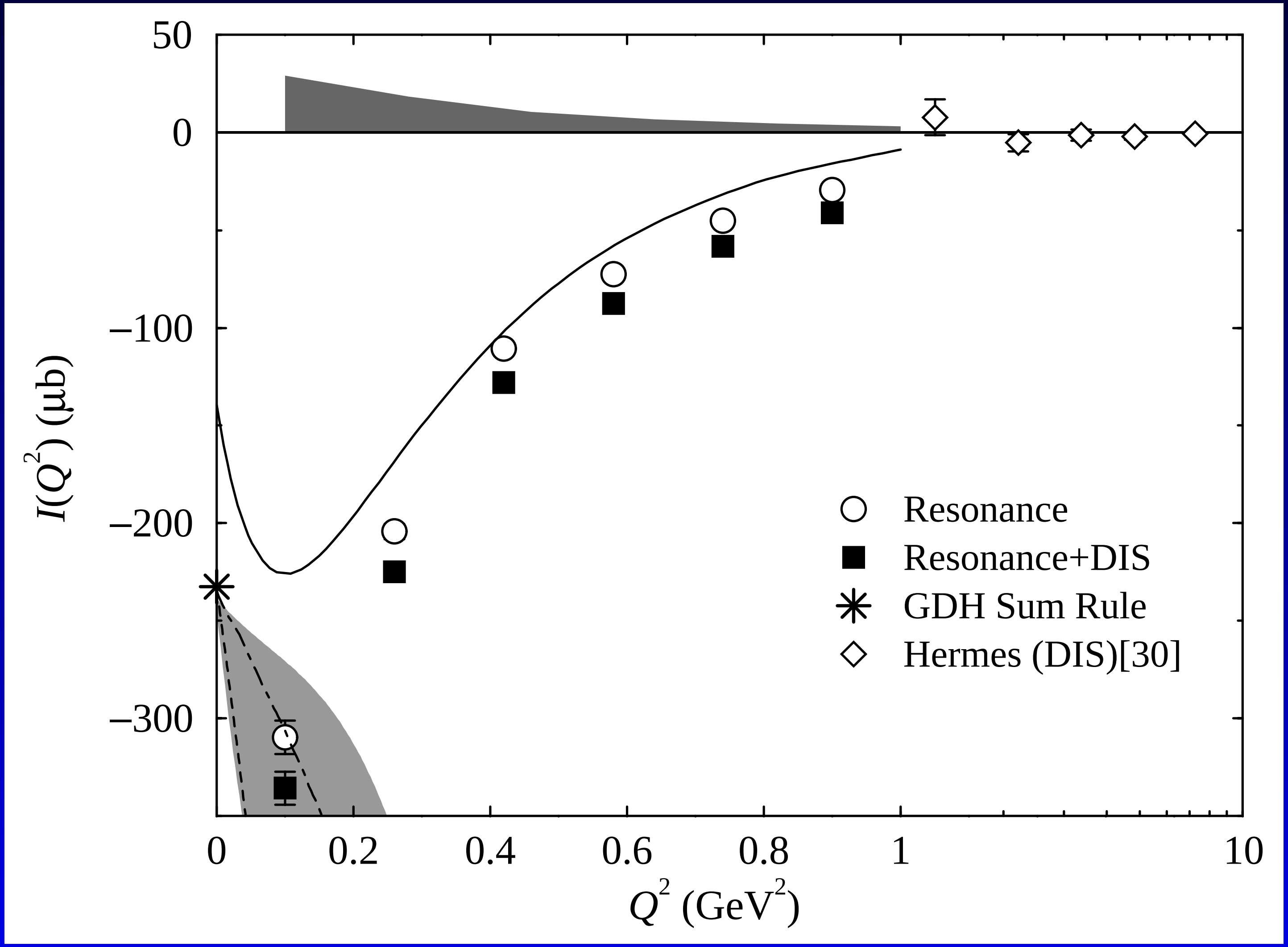
JLAB: Hall B/CLAS



→ more this afternoon: G. Dodge, K. Dharmawardane

GDH sum: Neutron

JLAB: Hall A (nucl-ex/0205020)



→ more this afternoon: X. Zheng

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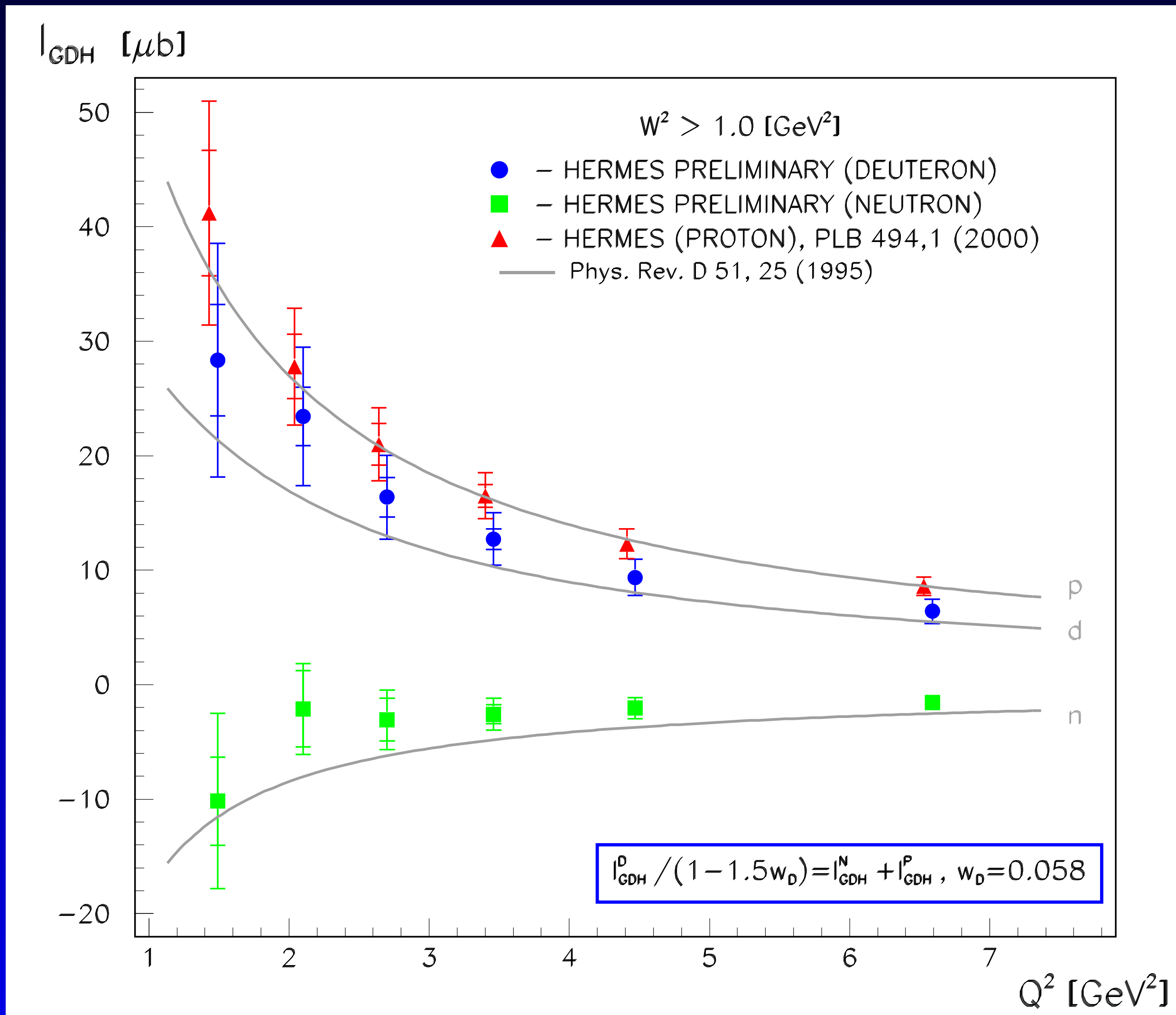
GDH sum: QCD region

DESY: HERMES



Consistent with $1/Q^2$ behavior

more this afternoon: W.-D. Nowak

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Summary

- 1st time verification of GDH Sum Rule for proton performed at ELSA and MAMI at 5% level.
- Negative $\Delta\sigma$ at $E_\gamma > 2.5$ GeV improves agreement with SR $\longrightarrow$ to be confirmed with high statistics & neutron data.
- Response of resonances to polarization (except Δ) differs largely from predictions.
- Regge extrapolation from DIS to $Q^2 = 0$ does not work.
- Indication for high asymmetry for neutron for $E_\gamma > 1$ GeV.
- Several experiments under way to verify results from ELSA/MAMI; plans for extension up to 40 GeV.
- Formalism for GDH extension to finite Q^2 established.
- Yet no strict predictions available at finite Q^2.