Spin asymmetry for proton deuteron collisions at forward angles

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Summary

Polarized $p\,d \to p\,d$ collisions

- 12 helicity amplitudes from P and T
- deuteron form factors
- proton spin asymmetry
- one photon exchange amplitudes
- hadronic spin dependence
- maximum analyzing power
- conclusions

Introduction

Proton carbon elastic scattering in the Coulomb interference region has been proposed by the E950 Collaboration as a polarimeter for RHIC.

Another example of a fermion boson collision process with interesting spin dependence is forward proton deuteron elastic scattering.

The spin dependent dynamics of polarized protons and deuterons colliding at small angles may be useful for polarimetry, where again the phases of hadronic amplitudes play a critical rôle.

Spin 1/2 – 1 amplitudes

The 36 helicity amplitudes reduce to 12 under time reversal and parity invariance. Of the hadronic amplitudes (em have $1/t$),

$$H_i(\lambda'_p, \lambda'_d \,|\, \lambda_p, \lambda_d), \quad (i = 1-12)$$

with $\lambda_p \in \{+, -\}$ and $\lambda_d \in \{+, 0, -\}$,

- four can be non-zero at $t = 0$

$$H_1(+\,+\,|\,+\,+) \quad H_2(+\,+\,|\,-\;0)$$

$$H_3(+\,0\,|\,+\;0) \quad H_4(+\,-\,|\,+\,-)$$

and relate to spin-spin total cross sections

- five have $\sqrt{-t}$ single flip dependence,

$$H_5(+\,+\,|\,+\;0) \quad H_6(+\,+\,|\,-\,+)$$

$$H_7(+\,+\,|\,-\,-) \quad H_8(+\,0\,|\,+\,-)$$
$$H_9(+\,0\,|\,-\;0)$$

- two have $(-t)$ double flip dependence

$$H_{10}(+\,+\,|\,+\,-) \quad H_{11}(+\,0\,|\,-\,+)$$

- $H_{12}(+\,-\,|\,-\,+)$ has $-t\sqrt{-t}$ dependence.

In the reduction from 36 to 12 amplitudes, the multiplicity of H_1, H_3, H_4, H_7, H_9, and H_{12} is two while the multiplicity of the other six amplitudes of the 12 is four.

Deuteron form factors

The electric, magnetic, and quadrupole form factors of the deuteron are normalized according to

$$
\begin{aligned}
F_1^d(0) &= 1 \\
F_2^d(0) &= Q + \mu_d - 1 \\
G_1^d(0) &= \mu_d
\end{aligned}
$$

where for the deuteron of mass M,

Q is the quadrupole moment (units e/M^2),

μ_d is the magnetic dipole moment.

Analyzing power

The asymmetry for polarized protons at CN interference involves amplitudes, particularly the following significant amplitudes

$$A_N = \frac{2\,\mathrm{Im}\left[H_6{}^*\,(H_1 + H_4) + \cdots\right]}{|H_1|^2 + |H_2|^2 + |H_3|^2 + |H_4|^2 + \cdots}$$

Each amplitude includes an electromagnetic element (and Coulomb phase δ)

$$H_j + e^{i\delta}\,H_j^{\mathrm{em}}$$

related to spin 1/2 and 1 currents, Waldenstrøm, Nuovo Cimento 3A, 491, (1971).

Coulomb amplitudes

One photon exchange helicity amplitudes (B. Corbett, 1984) have the approximate form, and here $H_2^{\mathrm{em}}(+-\,|\,-0) \approx 0$ for em,

$$H_i^{\mathrm{em}}(+j|+j) = \frac{\alpha s}{t} F_1(t)\, F_1^{\mathrm{d}}(t)$$

in the nonflip case labelled by $i = 1,\, 3,\, 4$ for deuteron helicities $j \in \{+,\, 0,\, -\}$.
The proton single spin flip em amplitudes $H_7^{\mathrm{em}}(++\,|\,--) \approx 0$ and $H_9^{\mathrm{em}}(+0\,|\,-0) \approx 0$ but

$$H_6^{\mathrm{em}}(++\,|\,-+) = \frac{\alpha s}{\sqrt{-t}} \frac{\kappa_p}{2m} F_2(t)\, F_1^{\mathrm{d}}(t)$$

where the anomalous magnetic moment of the proton is $\kappa_p = \mu_p - 1$.

The deuteron single flip em amplitudes are

$$H_5^{\rm em}(+\,+\,|\,+\,0) = \frac{\alpha s}{\sqrt{-2t}}\,F_1(t)\,\frac{1}{2M}\,G_1^{\rm d}(t)$$

$$H_8^{\rm em}(+0\,|\,+\,-) = \frac{\alpha s}{\sqrt{-2t}}\,F_1(t)\,\frac{1}{2M}\,G_1^{\rm d}(t)$$

where $G_1^{\rm d}(0)$ is related to $\mu_{\rm d} = 0.8574$, the deuteron dipole magnetic moment.

The double spin flip amplitudes,
$H_{10}^{\rm em}(+\,+\,|\,+\,-)$ and $H_{11}^{\rm em}(+0\,|\,-\,+)$,
play no important rôle in the interference region.

Asymmetry maximum

The Coulomb phase shift δ is about 2% in the interference region for pd collisions. The analyzing power for polarized protons

$$\frac{3}{2}\frac{m\,A_N}{\sqrt{-t}}\frac{16\pi}{\sigma_{\rm tot}^2}\frac{d\sigma}{dt}e^{-bt} = (\kappa_p - 2\,{\rm Im}\,r)\frac{t_c}{t} - 2\,{\rm Re}\,r + 2\rho\,{\rm Im}\,r$$

The unpolarized differential cross section is

$$\frac{16\pi}{\sigma_{\rm tot}^2}\frac{d\sigma}{dt}e^{-bt} = \left(\frac{t_c}{t}\right)^2 - 2(\rho+\delta)\frac{t_c}{t} + 1 + \rho^2$$

with ratio $\rho = {\rm Re}\,H_+/{\rm Im}\,H_+$ referring to the hadronic non-flip amplitude H_+ (an appropriate average of H_1, H_3, and H_4) and a kinematically scaled hadronic ratio of the proton helicity flip amplitude is defined as

$$r = -\frac{m}{\sqrt{-t}} \times \frac{H_6}{\operatorname{Im} H_+}.$$

A negative value of $\rho(s)$ would reduce the asymmetry maximum in pd elastic scattering. A detailed study of the pd differential cross section at interference could provide values of ρ at particular energies. Helicity dependent terms could then be isolated and their energy and momentum transfer dependence studied to facilitate the understanding of proton deuteron spin structure.

Conclusions

- Electromagnetic amplitudes for $\mathrm{p\,d} \to \mathrm{p\,d}$
- Maximum analyzing power at interference
- Polarimetry for proton & deuteron beams
- Dispersive spin independent amplitudes and tests of causality via analyticity
- The rôle of the Coulomb phase
- Understanding high energy spin dynamics