Mechanism of electron multipacting with long bunched proton beam
— Theory and Simulation

L. Wang, BNL

In collaboration with J. Wei, M. Blaskiewicz, P. He, Y.Y Lee, D. Raparia, S.Y. Zhang (BNL), R. Macek (LANL), K. Ohmi (KEK), F. Zimmermann (CERN), A. Chao (SLAC)

The 13th ICFA Beam Dynamics Mini-Workshop
Beam Induced Pressure Rise in Rings
Brookhaven National Laboratory, Upton, NY
December 9 - 12, 2003

Content

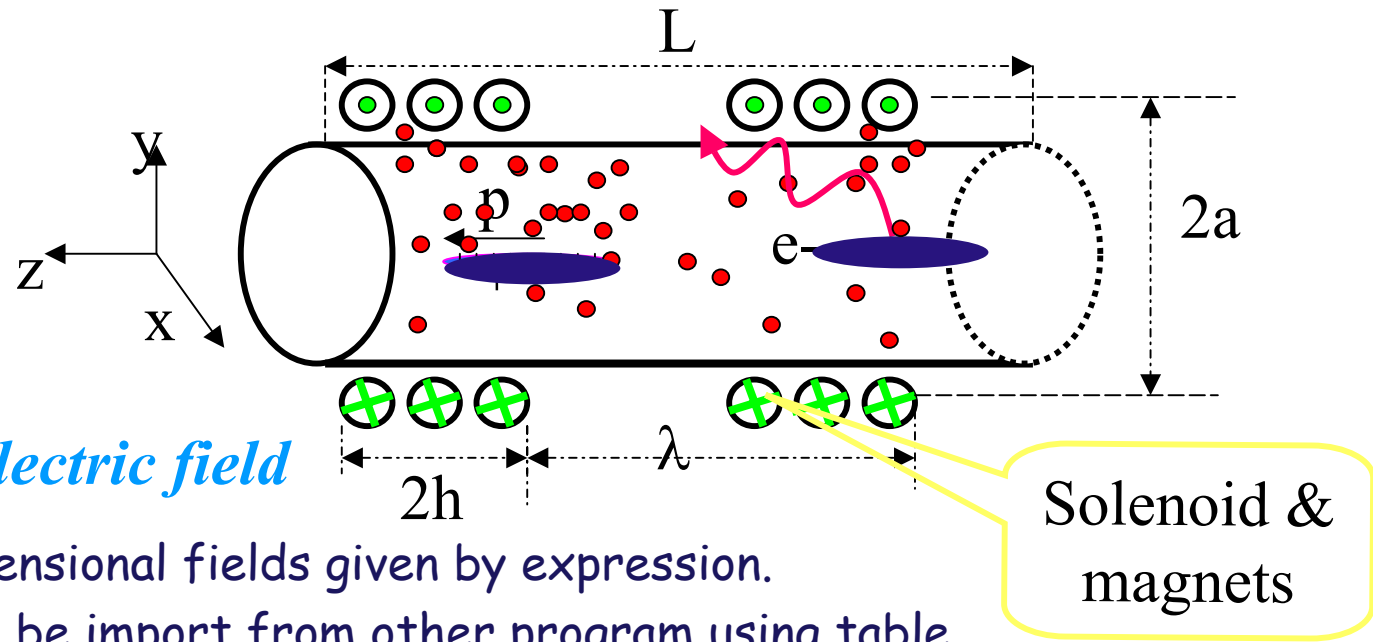
- **Motivation**
- Electron motion in beam space charge field & Mechanism of trailing edge multipacting
- Important factors related to Multipacting
  - Longitudinal beam profile
  - Transverse beam shape & size
  - Beam intensity
  - Chamber size
  - Beam in gap
  - Peak SEY and Energy at Peak SEY
  - ……
- Electron cloud in **dipole** magnets & Quadrupole & Sextupole magnets
- Electron cloud clearing with **solenoids** & clearing electrodes
- Summary

Motivation

- What is the mechanism of trailing edge multipacting?
- Which factors affect e-cloud multipacting? And How?
- How to Clear electron cloud?
- Why e-cloud is important in some accelerators, but not in others?

3D PIC Program--- CLOUDLAND

CLOUDLAND is a complete 3D PIC code for e-cloud initially developed for KEKB (*PRST-AB 124402*)



Program model

- PIC methods

Magnetic and electric field

- General 3-dimensional fields given by expression.
- Fields can also be import from other program using table

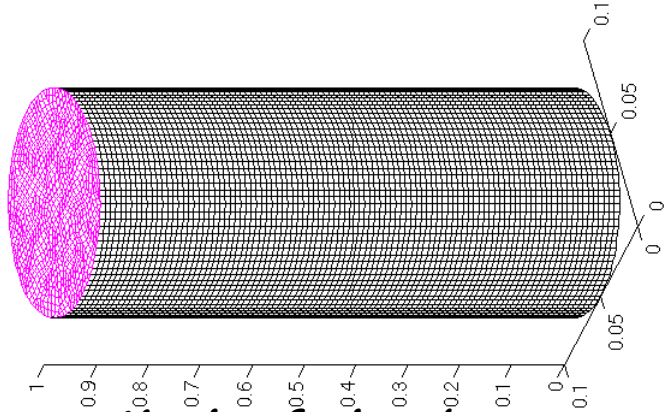
Beam potential

- Gaussian bunch in round chamber (image charge is included)
- PIC method for general geometry

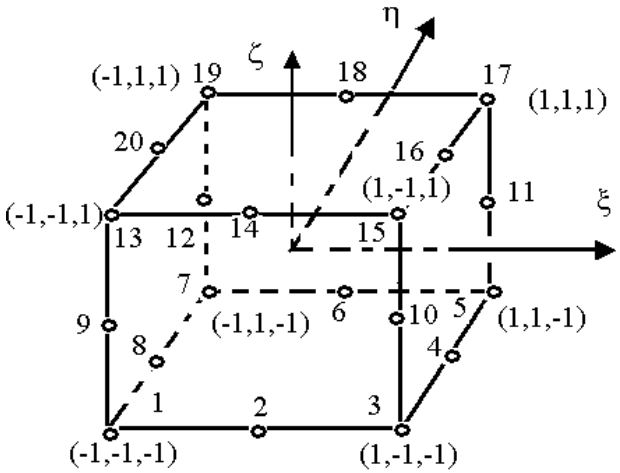
Secondary emission and reflective electron are included

Particle Mesh technique applied in the Space charge potential solver

- Three dimensional irregular mesh to better represent the general chamber geometry
- handle accuracy with high order elements.



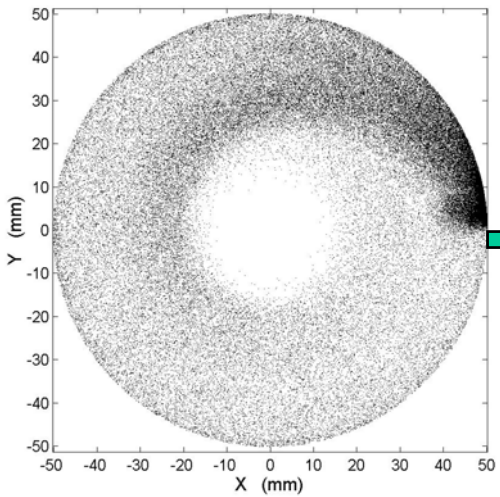
Mesh of chamber



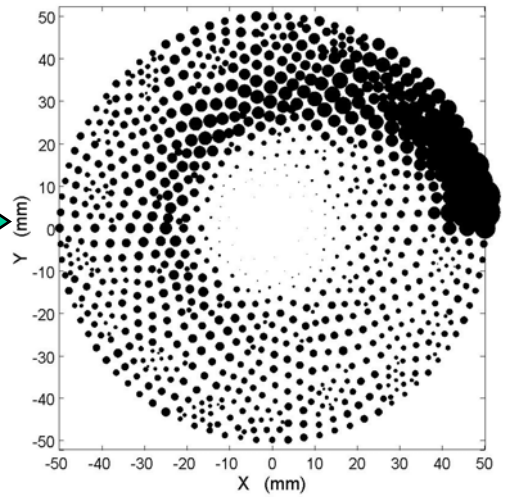
20 node element

Charge assignment

$$Q_i = N_i Q_0 \quad \sum_i N_i = 1$$



Real charge distribution

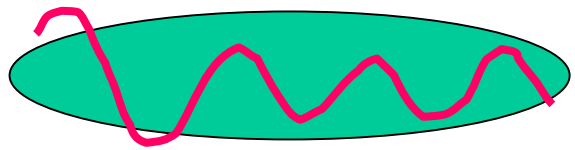


Meshed Charge distribution

Classification of electron multipacting

If an electron can oscillate many times under the bam force during the passage of one bunch, then the bunch is called long bunch

$$\frac{\hat{z}\overline{\omega}}{\pi\beta c} >> 1$$



Long bunch (Single bunch multipacting): PSR, SNS,JPAC, ISIS, ESS…

Short bunch (Multibunch multipacting): B-factories, PF, NLC damping ring, SPS, LHC, RHIC,…..

Source of electrons

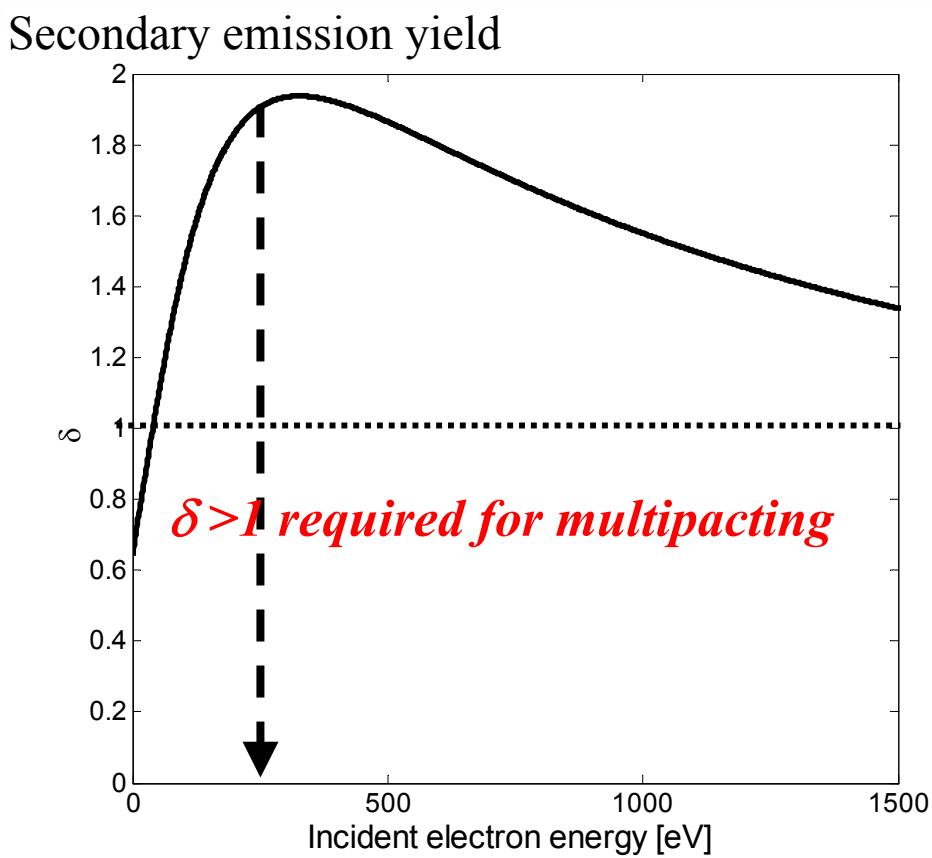
Primary electrons

- Photon-electrons (electron machine & LHC)
- **Beam loss** at the chamber surface (PSR, SNS, ISIS, ESS, JPARC)
- Residual gas ionization
- **Stripped & scattered electrons**

Table 1 Parameters for the SNS

Parameter	Description	SNS
E (GeV)	Beam energy	1.9
C (m)	Circumference	248
N_p	Bunch population	**2.05×10^{14}**
a_x,a_y (mm)	Transverse beam size	28, 28
τ_b (ns)	Bunch length	***700***
b (cm)	Beam pipe radius	10
P_l	Proton loss rate/turn	1.1×10^{-6}
Y	Assumed proton-electron yield	100

Secondary electrons



courtesy *P. He, M. Blaskiewicz*

Key parameters for Multipacting (Strong energy and SEY dependence)

- SEY depend on the material property of the chamber surface (peak SEY and energy at peak SEY)
- Beam-electron interaction dependence (beam pattern, bunch current, bunch shape, bunch length, chamber size…)

Beam space charge field

SNS beam transverse profile shape

- **Square shape** resulting from correlated painting during the injection.
- Inclusion of the space charge causes rapid diffusion in azimuthal direction and results in **round beam shape**
- **Electron Multipacting (energy at the wall surface) does not depend on transverse profile**

Space charge field of uniform cylinder beam

$$E_r(r,t) = \begin{cases} \dfrac{\lambda(t)}{4\pi\varepsilon_0}\dfrac{2}{r} & (\mathrm{r} > a) \\ \dfrac{\lambda(t)}{4\pi\varepsilon_0}\dfrac{2r}{a^2} & (\mathrm{r} < a) \end{cases} \qquad U(r,t) = \begin{cases} \dfrac{\lambda(t)}{4\pi\varepsilon_0}\left(1 + 2\ln\dfrac{r}{a}\right) & (\mathrm{r} > a) \\ \dfrac{\lambda(t)}{4\pi\varepsilon_0}\dfrac{r^2}{a^2} & (\mathrm{r} < a) \end{cases}$$

Nonlinear Hamiltonian of the radial motion

$$H = \frac{p^2}{2m} + eU(r,t)$$

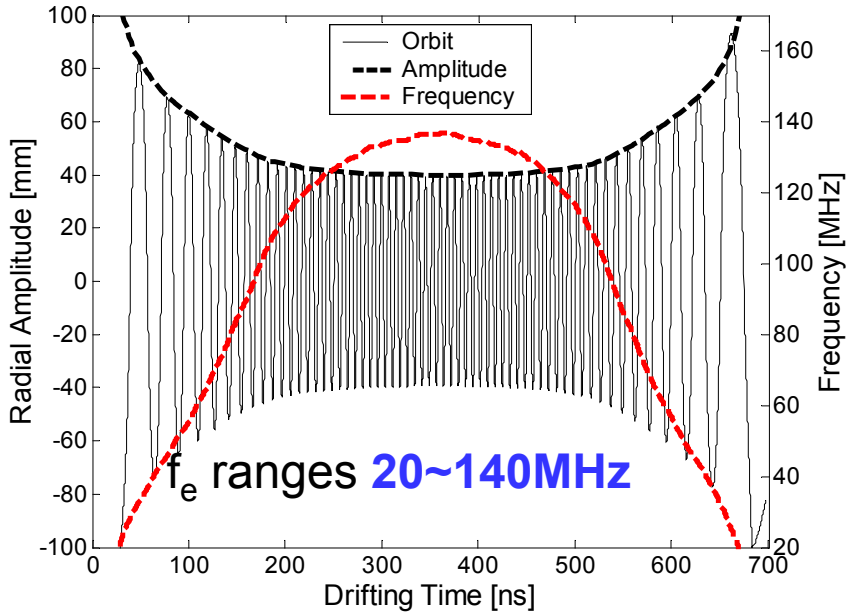
The longitudinal beam force is neglected

Nonlinear Oscillation Period and Adiabatic invariant

Nonlinear Oscillation Frequency

$$T = 4.0\int_0^{r_{amp}} \frac{dr}{\upsilon(r)} = 4.0\int_0^{r_{amp}} \frac{dr}{\sqrt{2\Phi e/m}}$$

$$T = \begin{cases} 4.0\sqrt{\dfrac{\pi\varepsilon_0 m}{\lambda e}}\left(\sqrt{2}a\arcsin\dfrac{1}{\sqrt{1+2\ln(r_{amp}/a)}} + \displaystyle\int_a^{r_{amp}}\dfrac{dr}{\sqrt{\ln(r_{amp}/r)}}\right) \\ 2\pi a\sqrt{\dfrac{2\pi\varepsilon_0 m}{\lambda e}} \qquad (r_{amp} \le a) \end{cases}$$



Oscillation amplitude and frequency

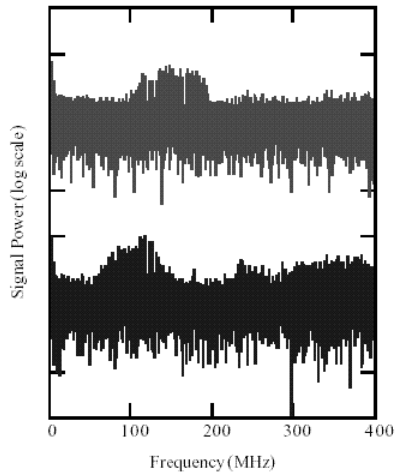
Adiabatic invariant

$$\frac{1}{\omega_e^2}\frac{d\omega_e}{dt} << 1$$ (if t>20ns and t<680ns for SNS)

$$J = \oint p\,dq$$

$$J = \begin{cases} \dfrac{\pi r_{amp}^2}{a}\sqrt{\dfrac{me\lambda}{2\pi\varepsilon_0}} \qquad (r_{amp} < a) \\ 4a\sqrt{\dfrac{me\lambda}{2\pi\varepsilon_0}}\left(\dfrac{\sqrt{2}}{2}x^{1/2} + \dfrac{1+2x}{2}arctg\dfrac{1}{\sqrt{2x}} + \dfrac{\sqrt{2}}{a}\displaystyle\int_a^{r_{amp}}\sqrt{\ln\dfrac{r_{amp}}{r}}dr\right) (r_{amp} > a) \end{cases}$$

$$x = ln(r_{amp}/a)$$



LANL PSR

beam spectrum

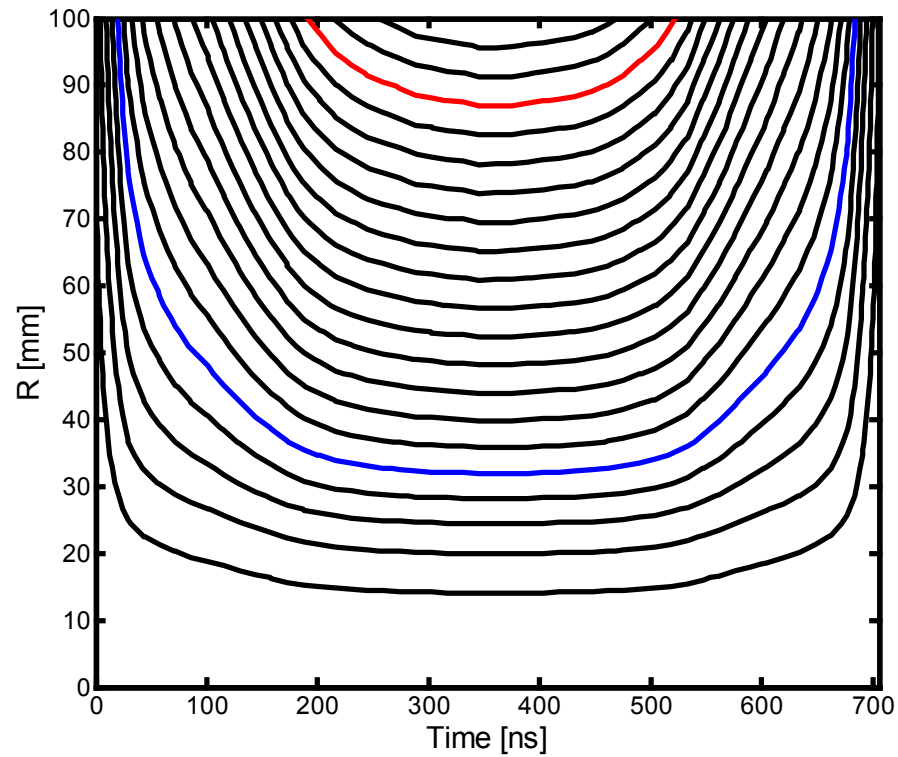
50~150MHz

courtesy

Robert J. Macek

Oscillation amplitude from adiabatic invariant

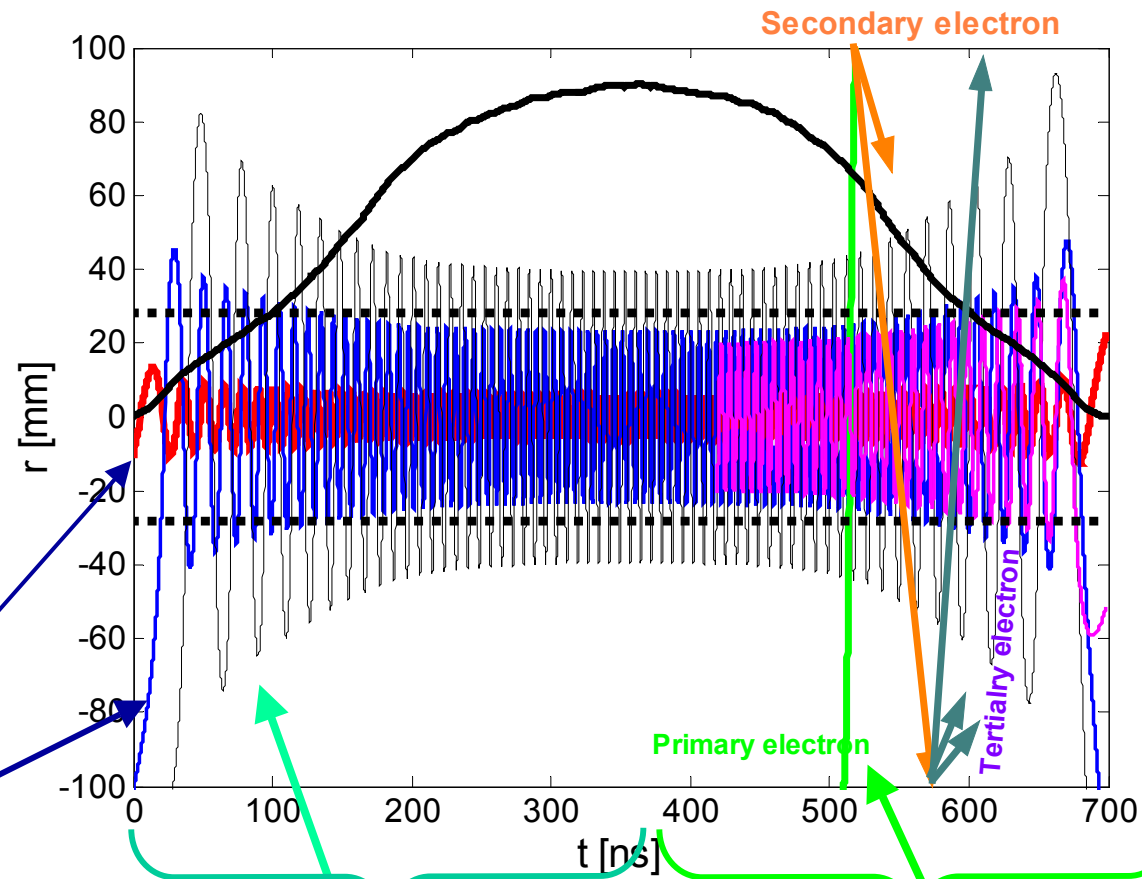
- Contour plot from adiabatic invariant can clearly descript the electron orbit
- All electron emitted (including gas ionization) before the bunch center or survived from last bunch gap can be trapped (inside beam for the survived electrons) during the bunch passage and are released at the bunch tail. The trapped electrons, most of them are the survived electrons from the last bunch gap, contribute to beam dynamics (instabilities)
- All electrons which emitted from the wall after bunch center will directly drift to the opposite of wall surface. The straight drifting electrons contribute to multipacting due to their short drifting time &high energy when they hit the wall surface.



Contour plot of the oscillation amplitude resulting from adiabatic invariant for SNS beam

Particle motion vs. instability & multipacting

Typical orbits of various electrons trapped by SNS beam, bold solid line shows the longitudinal beam profile shape and the dashed back lines show the beam size



Electrons by ionization (beam instabilities)

Electrons survived from the bunch gap(beam instabilities)

Electrons emitted before bunch center (trapped and lost after bunch center)

Electrons emitted after the bunch center(multipacting)

Energy Gain of straight drifting electron & Mechanism of trailing edge multipacting

Assuming the beam line density is a linear function of time during the short electron drifting time(~10ns)

$$\Delta\lambda \approx \frac{\partial\lambda}{\partial t}\Delta t = \frac{\partial\lambda}{\partial z} c\beta\Delta t$$

$$\Delta E = -\frac{1}{2}\sqrt{\frac{me}{2\pi\varepsilon_0}}\beta c\left(a(2\zeta-1)\arcsin\frac{1}{\sqrt{\zeta}} + a\sqrt{2\ln\frac{b}{a}} + \sqrt{2}\zeta\int_a^b \frac{dr}{\sqrt{\ln(b/r)}} - \frac{1}{\sqrt{2}}\int_a^b \frac{1+2\ln(r/a)}{\sqrt{\ln(b/r)}}dr\right)\frac{\partial\lambda}{\partial z}\frac{1}{\sqrt{\lambda}}$$

$$\zeta = 1 + 2\ln(b/a)$$

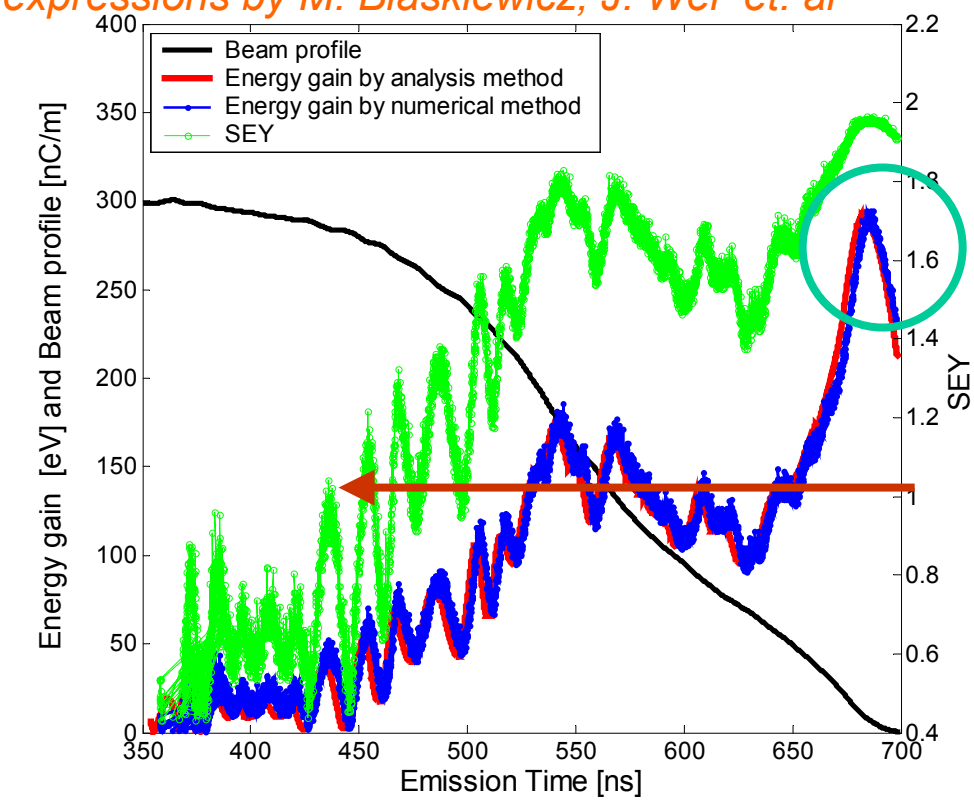
Also see other expressions by M. Blaskiewicz, J. Wei et. al

a: beam size, *b*, chamber radius, λ is beam line density

Longitudinal beam profile factor

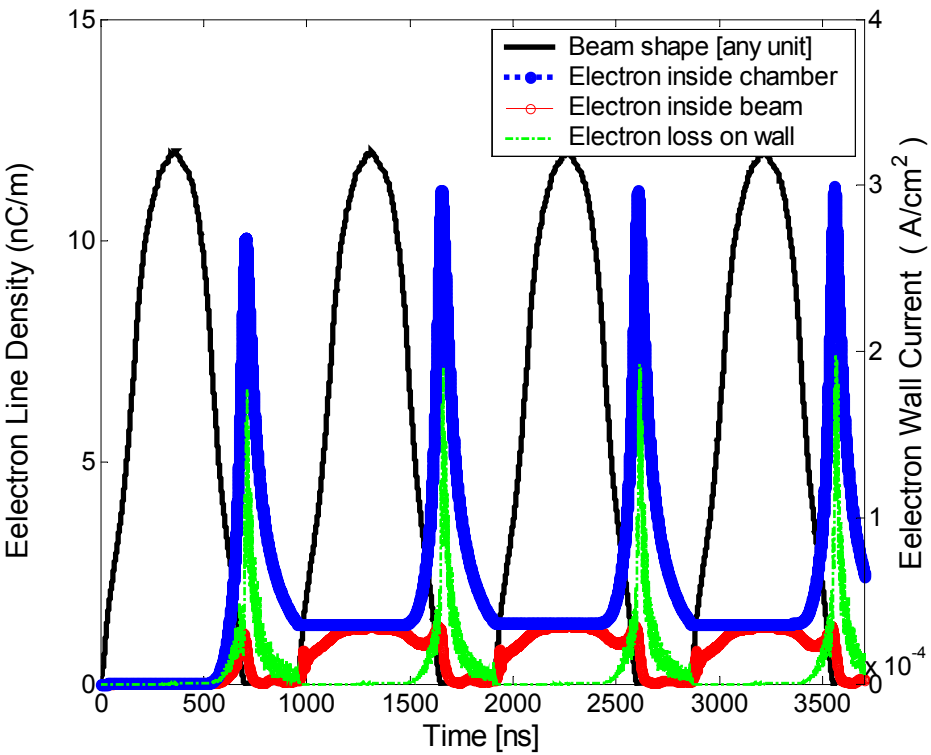
$$Factor_{profile} = -\frac{\partial\lambda}{\partial z}\frac{1}{\sqrt{\lambda}}$$

- Good agreement with numerical method
- Calculated SEY can be used to predict the multipacting directly
- Adiabatic motion and Energy gain can explain the mechanism of "trailing edge multipactor"

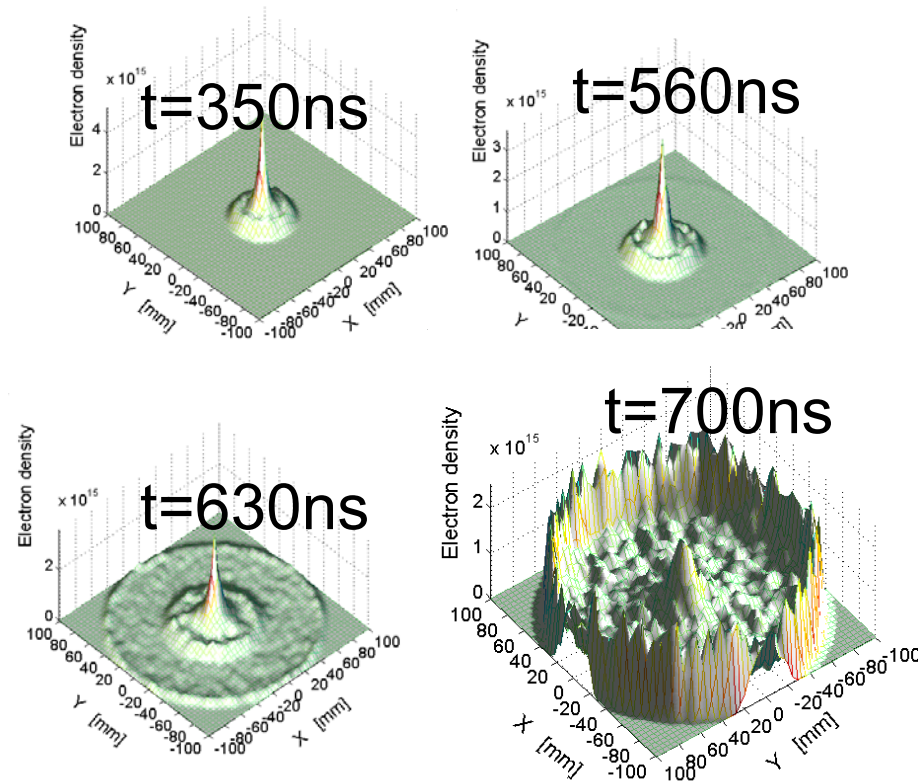


E-cloud in drift region

- **Single bunch multipacting**
- **Trailing Edge Multipacting**
- **All surviving electron from the last gap are trapped inside beam during the bunch passage (Contributing to beam instabilities)**
- **Bunch gap is important for *beam dynamics***

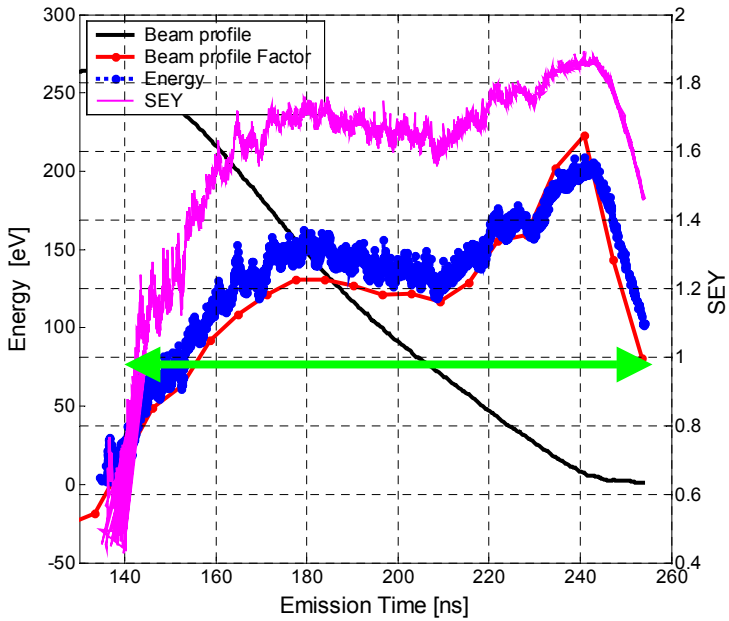


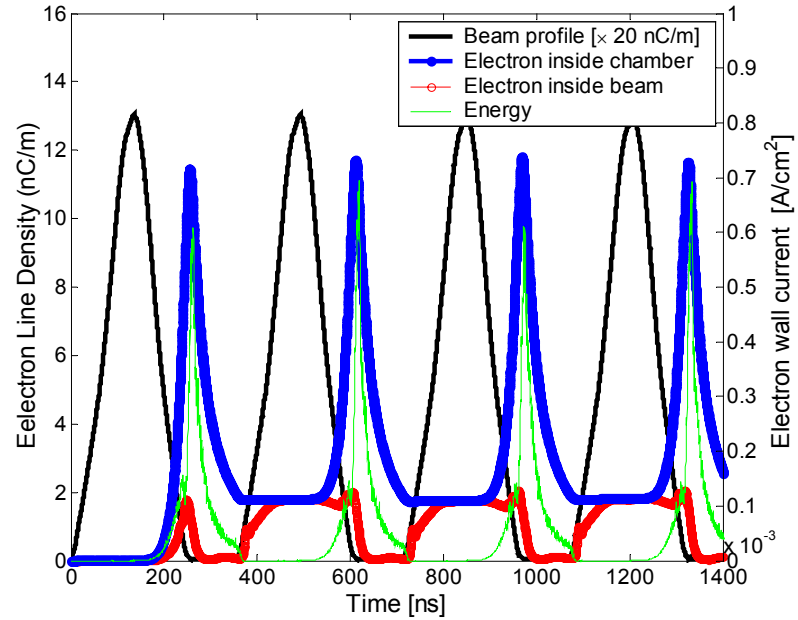
E-cloud build-up in SNS drift region



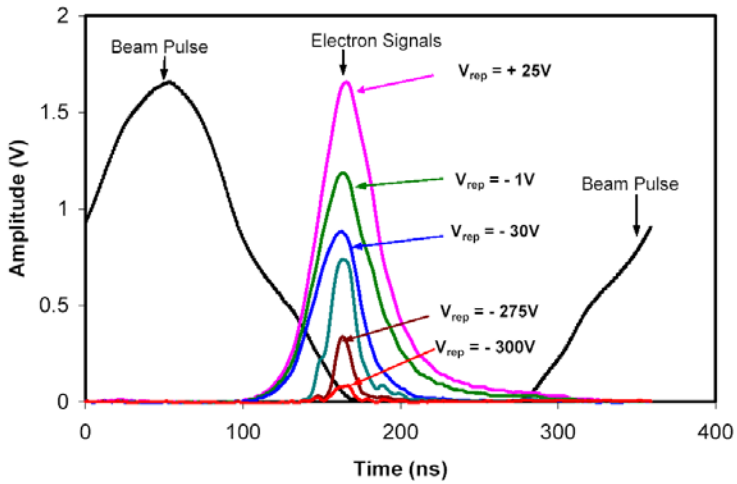
E-cloud distribution in different time

Ecloud in PSR





Electron signals from RFA in straight section 4

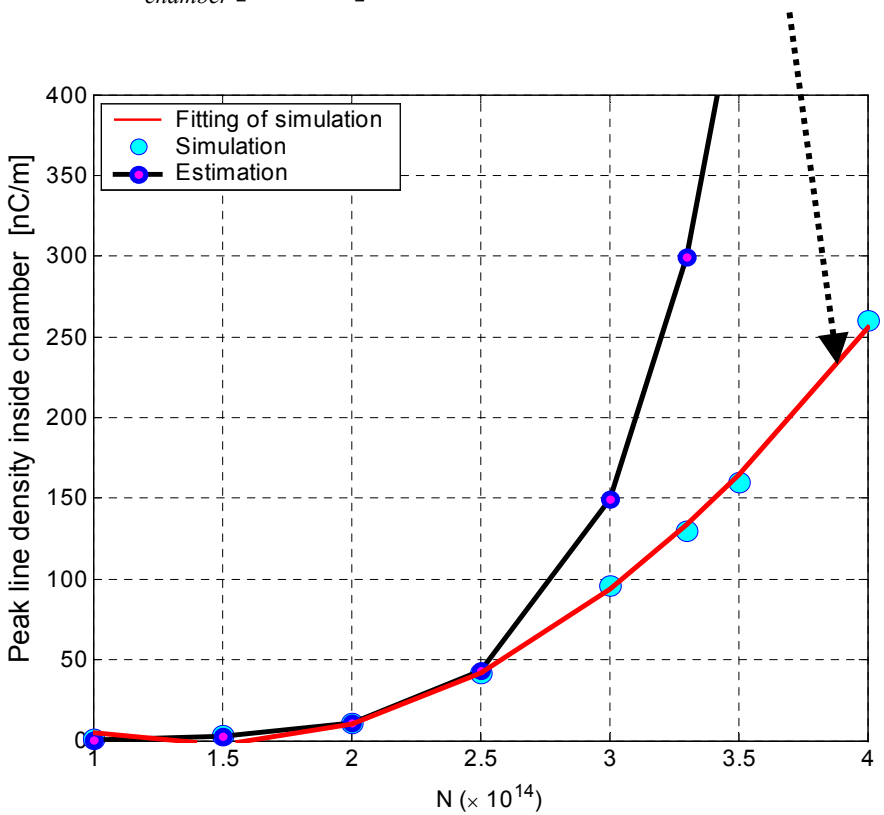


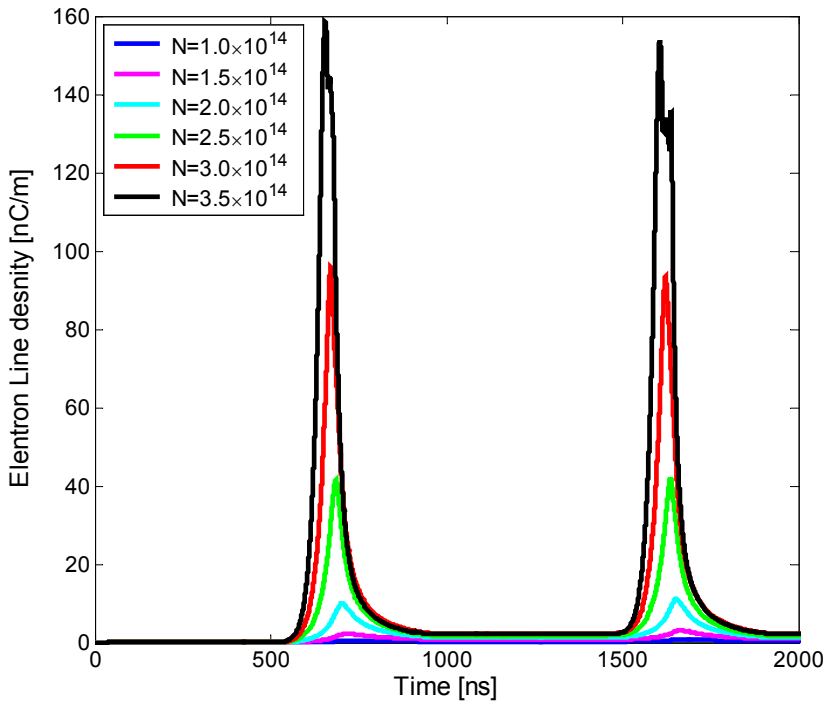
courtesy **Robert J. Macek**

Beam intensity effects (I)

- **High beam intensity causes high electron energy gain** $\Delta E \propto \sqrt{\lambda}$
- **High beam intensity increases multipacting frequency** $f_{multipacting} \propto \sqrt{\lambda}$
- **Space charge slows the growth of electron density inside chamber when strong multipacting case happens**

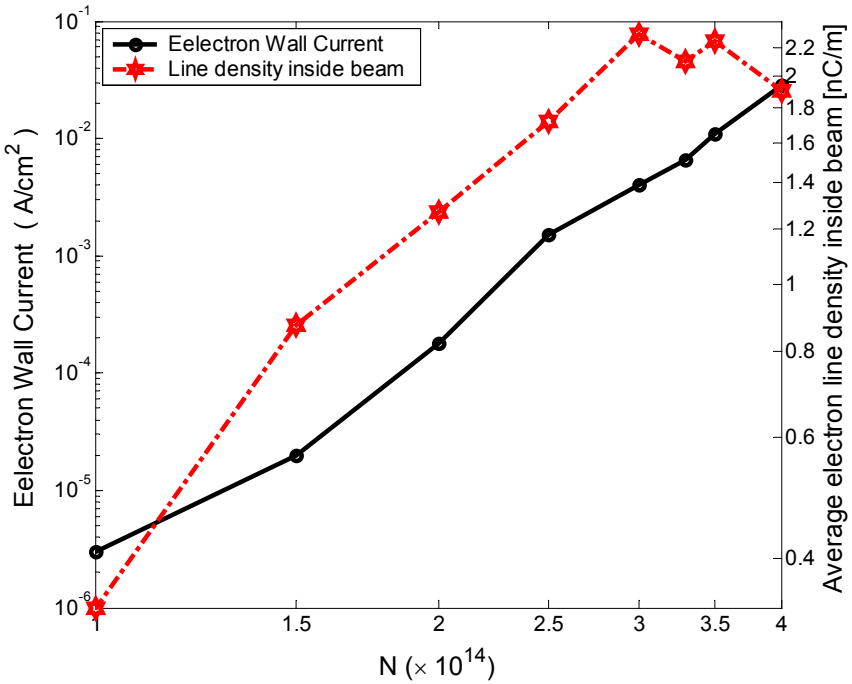
$$\lambda_{chamber}[nC/m] = 78 - 112 \times 1.0^{-14} N + 39 \times 1.0^{-28} N^2$$



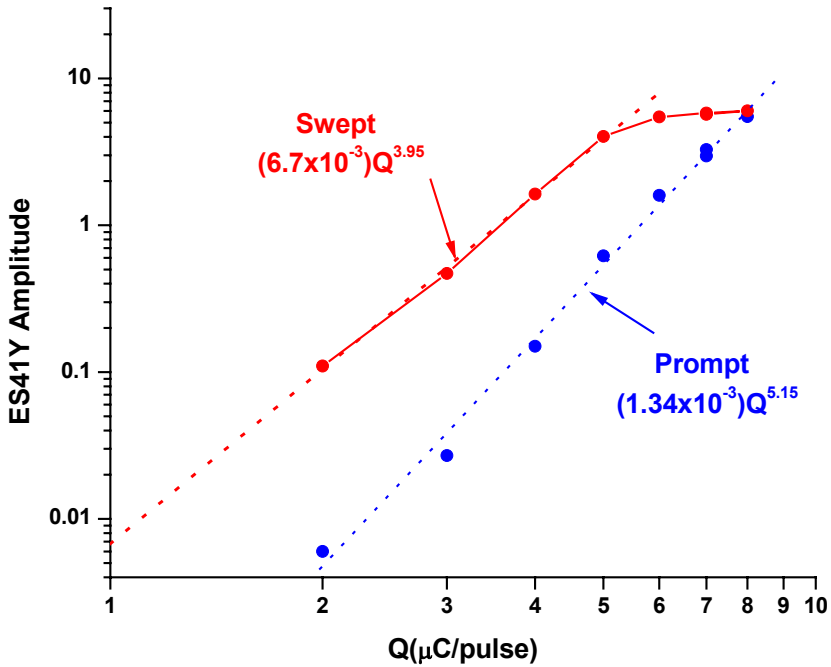


Beam intensity effects (II)

- Space charge makes the electron density inside beam saturated when strong multipacting case happens

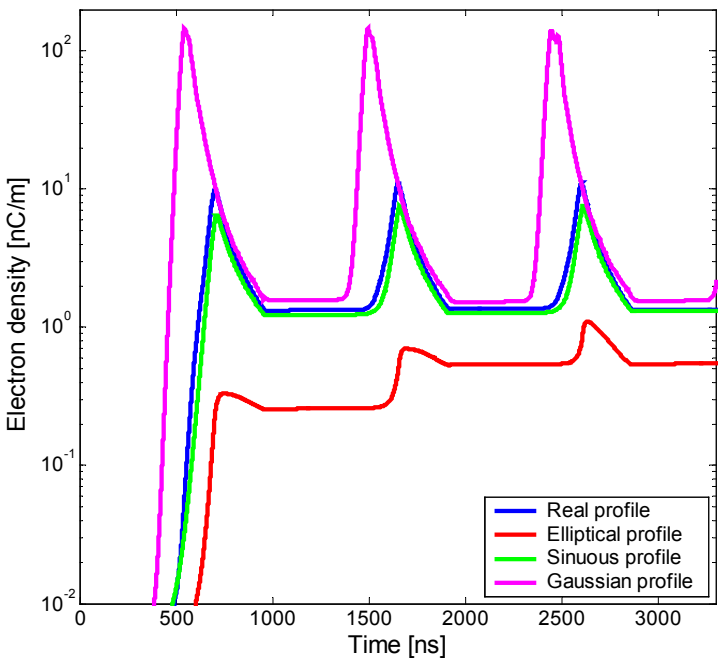


SNS, simulation



LANL PSR, Experiment, R. Macek

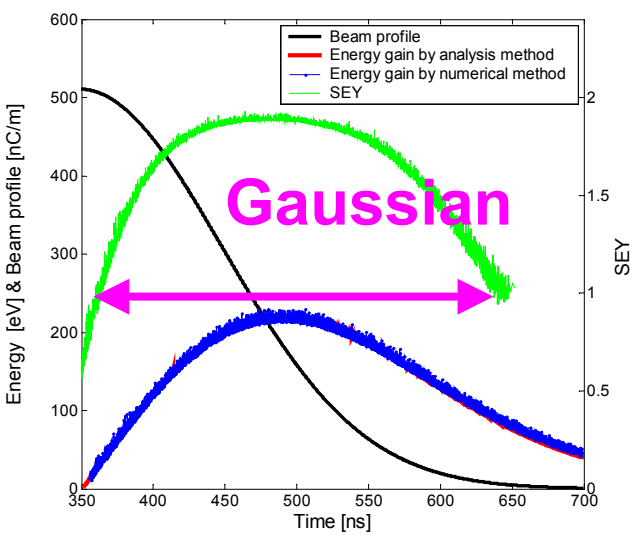
Important Factors Related to Electron multipacting(1)

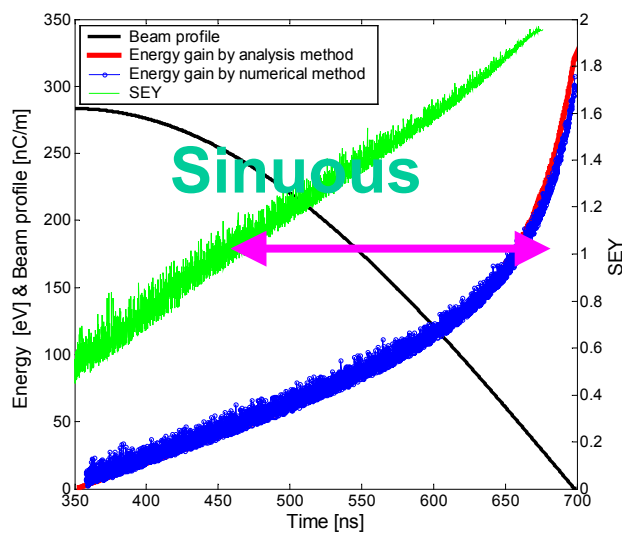


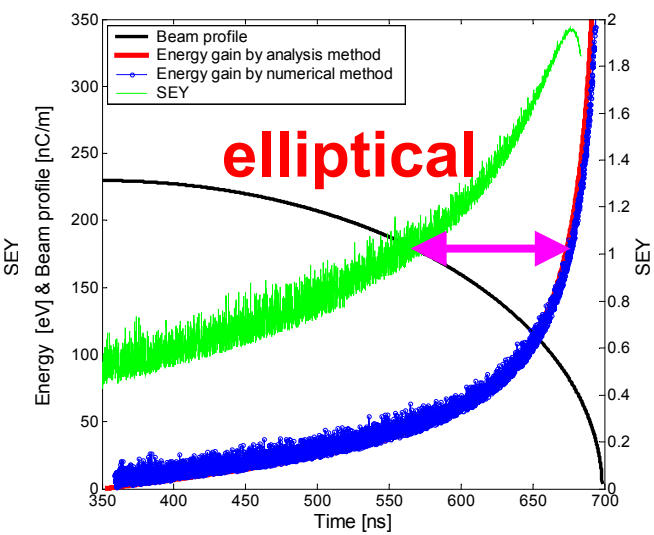
Longitudinal Beam Profile

For assumed Gaussian, sinuous & elliptical beam profile:

- Gaussian profile excites the strongest multipacting due to long bunch tail
- Elliptical profile has the weakest multipacting
- Electron cloud of the real profile is close to that of the sinuous profile



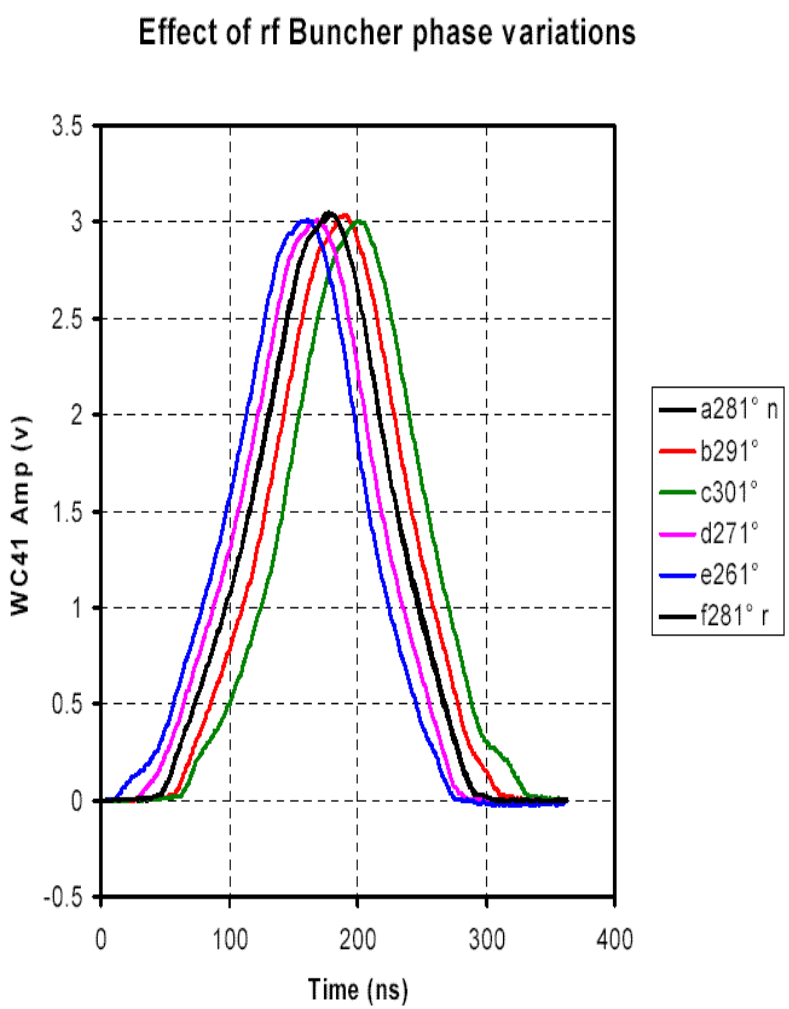


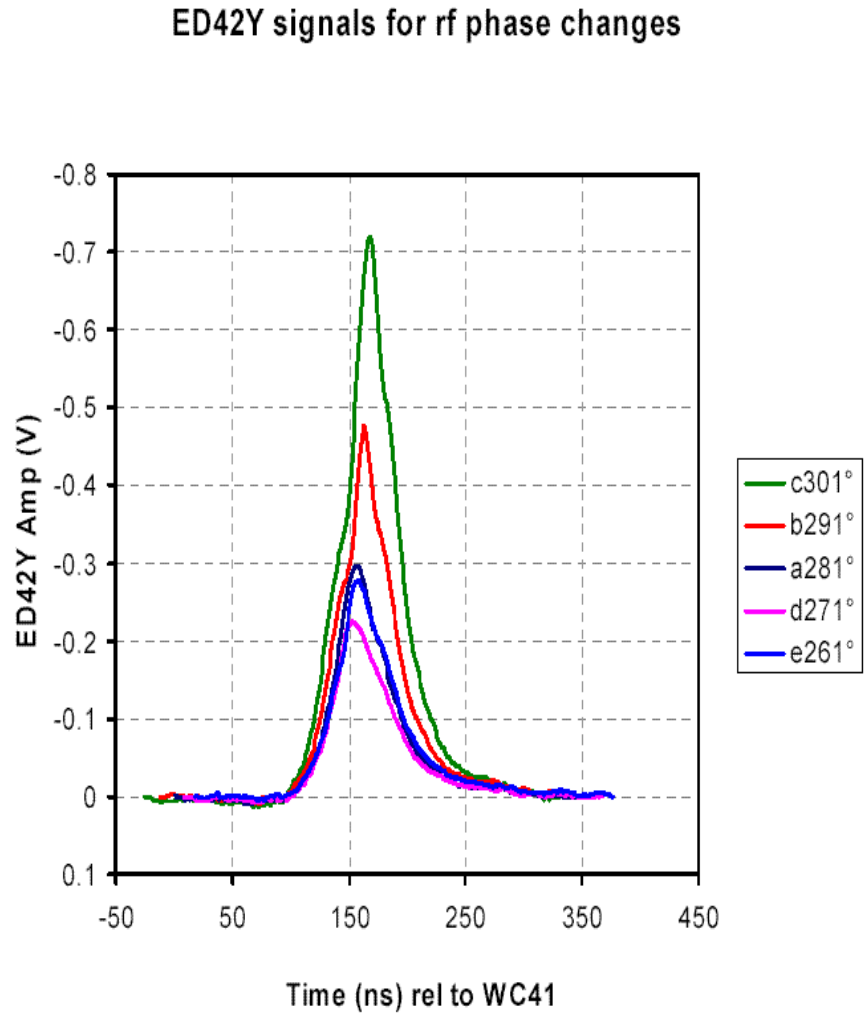


Longitudinal beam profile effects (cont.)

Electron signals vs. bunch shape

(LANL PSR experiment, courtesy **Robert J. Macek, ecloud'02)**



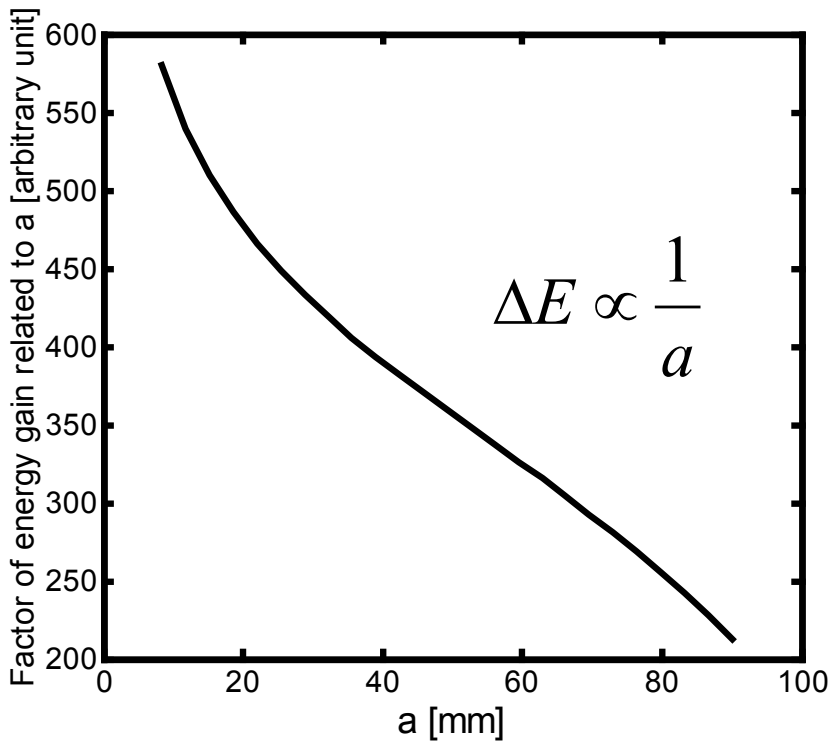


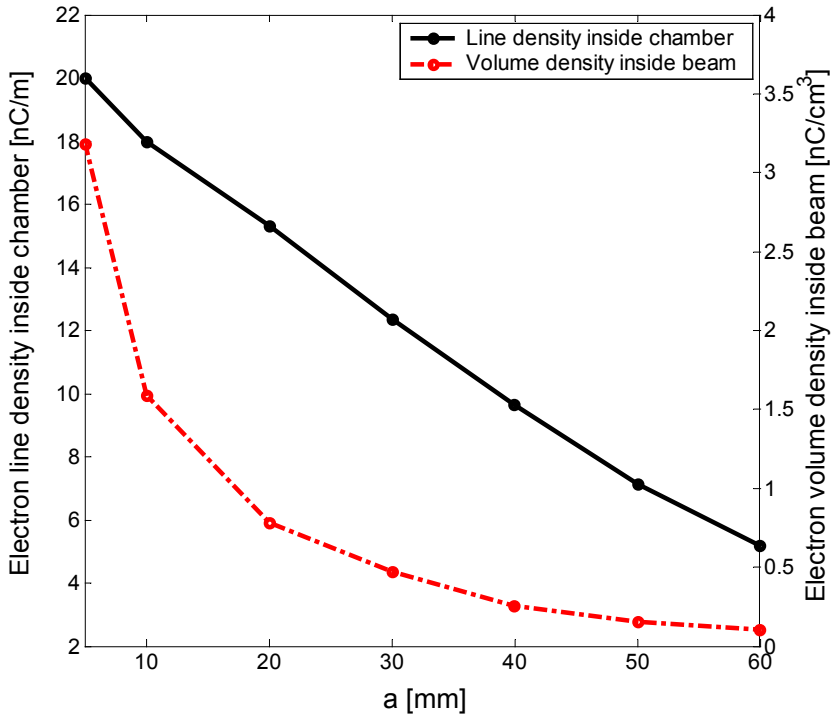
Transverse Beam Size Effects

- A smaller beam size contributes to stronger space charge field and hence larger electron energy gain and stronger multipacting.
- Instabilities is sensitive to *a*. Small *a*, strong instability

$$\lambda_{chamber}[nC/m] = 21 - 0.27a[\mathrm{mm}]$$

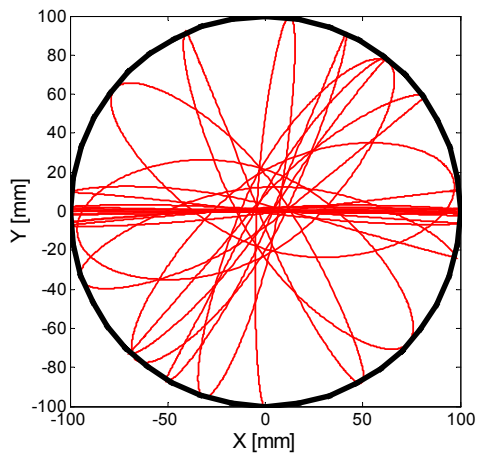
$$\rho_{cen}[nC/cm^3] = 4.9e^{-0.1a[mm]}$$

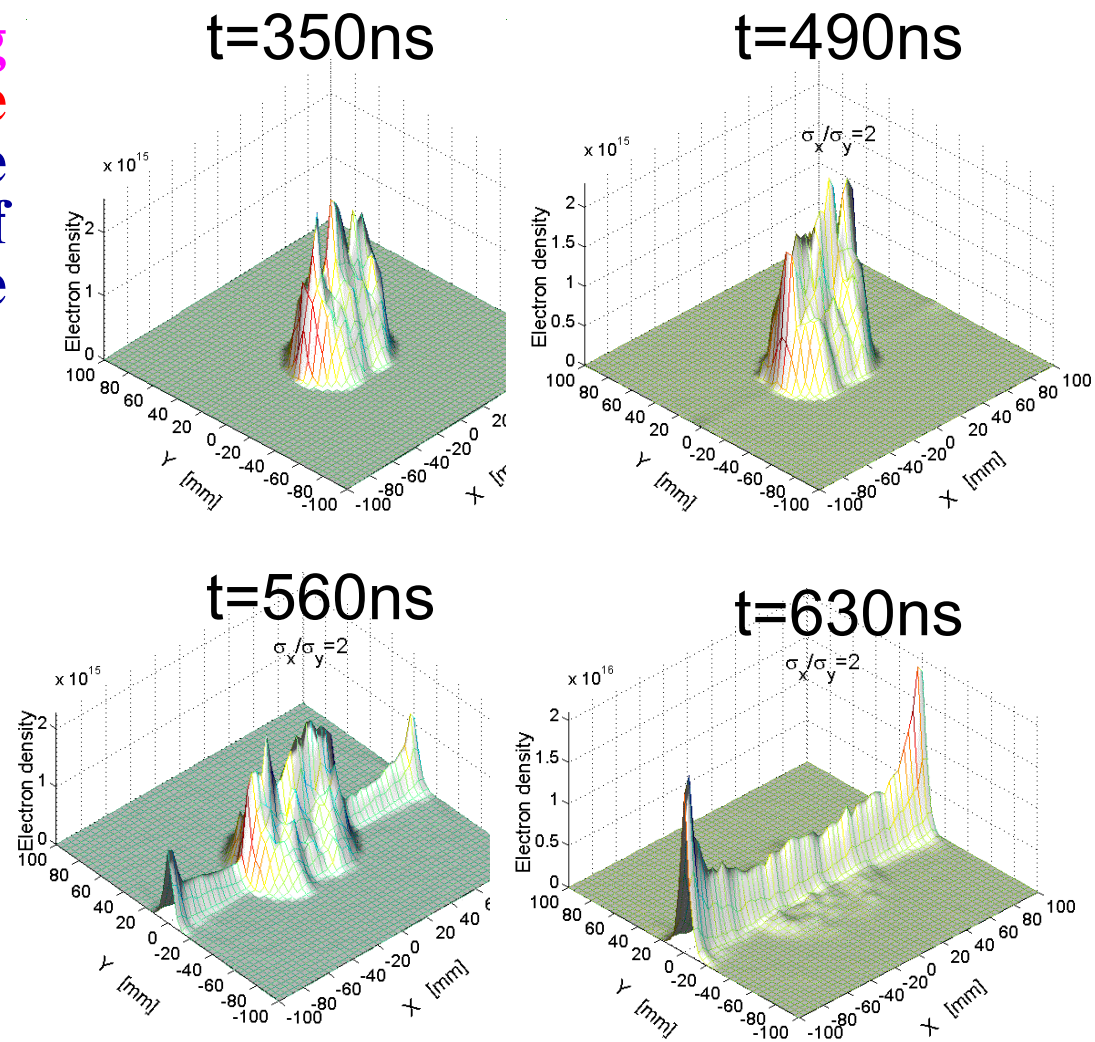




Flat beam effect on EC distribution

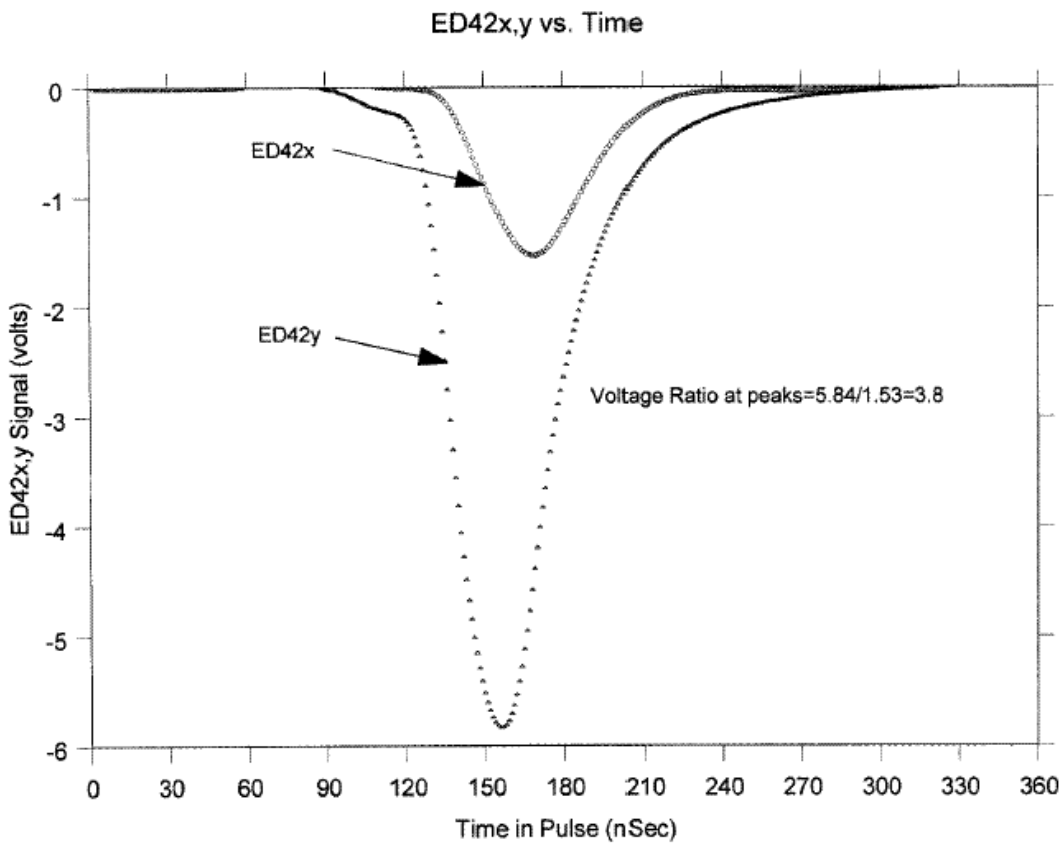
- Flat beam σ_x: σ_y=2:1
- Stronger multipacting in larger beam size direction due to the "polarization effect" of strong beam space charge force





PSR experimental study----flat beam

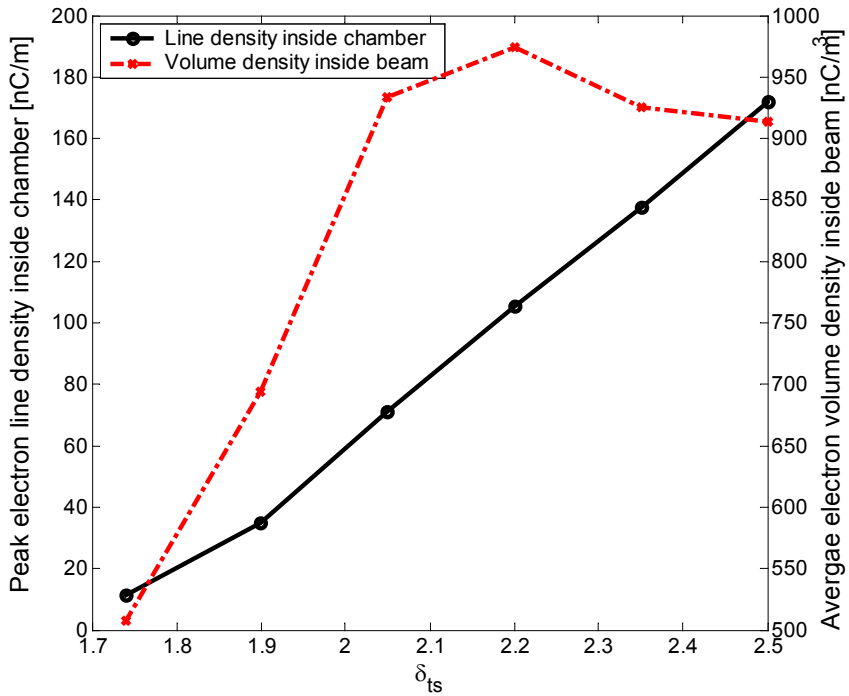
- Qualitatively agrees with LANL PSR observation
- Instability & detector

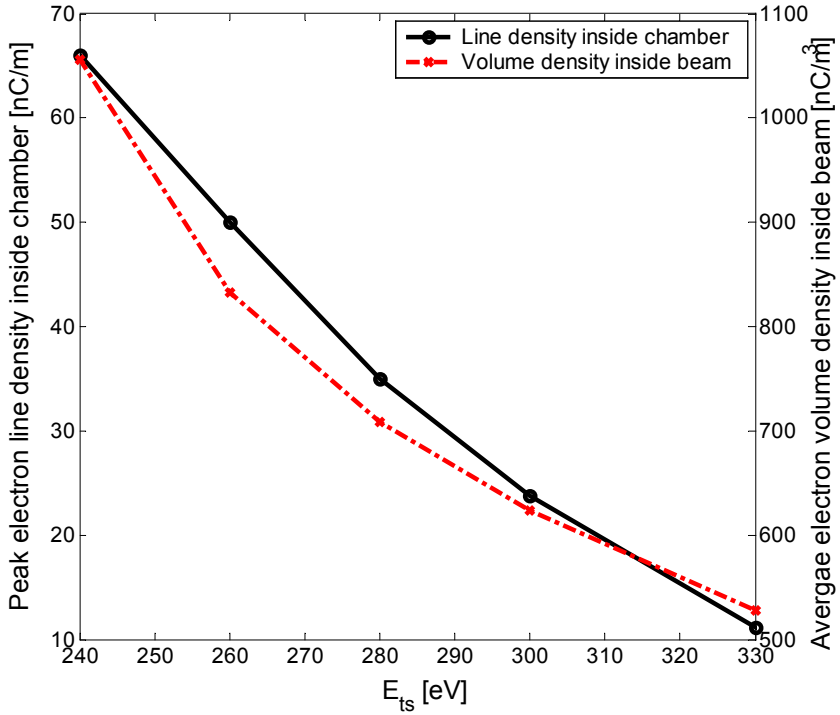


A. Browman Two-stream-2000

Peak SEY and Energy at Peak SEY

- **E-cloud density inside chamber is a linear function of peak SEY and energy at peak SEY.**
- **E-cloud density inside chamber (and hence Instability growth rate) increases linear with peak SEY and finally saturates at some level.**
- **E-cloud density inside chamber (and hence Instability growth rate) decreases with the energy at peak SEY.**





Electron motion in dipole magnets

- **Dipole magnetic field B**=(0,By,0)
- Beam electric field **E**=(Ex,Ey,0)

$$\boxed{\frac{d\upsilon_y}{dt} = eE_y / m}$$

$$\upsilon_x = \boxed{\upsilon_{x0}\cos\omega t + \upsilon_{z0}\sin\omega t} + \boxed{\frac{E_{x0}}{B}\sin\omega t} + \frac{1}{\omega B}\frac{dE_x}{dt} - \frac{1}{\omega B}\left(\frac{dE_x}{dt}\right)_0 \cos\omega t$$

$$-\frac{1}{\omega B}\cos\omega t\int_0^t \frac{d^2E_x}{dt^2}\cos\omega t dt - \frac{1}{\omega B}\sin\omega t\int_0^t \frac{d^2E_x}{dt^2}\sin\omega t dt$$

$$\upsilon_z = \boxed{\upsilon_{z0}\cos\omega t - \upsilon_{x0}\sin\omega t} + \boxed{\frac{E_{x0}}{B}\cos\omega t} - \boxed{\frac{E_x}{B}} - \frac{1}{B}\cos\omega t\int_0^t \frac{dE_x}{dt}\cos\omega t dt + \frac{1}{B}\sin\omega t\int_0^t \frac{dE_x}{dt}\sin\omega t dt$$

- In strong dipole magnet(B~1T for SNS) $\left|\frac{1}{\omega E_x}\frac{dE_x}{dt}\right| << 1$
- Electron motion ≅ **gyration motion** + **translation** (cross-field drifting) + movement along B-field lines
- The kinetic energy of gyration motion and cross-field drifting is **smaller** comparing with the kinetic energy in B-field direction.

Electron energy gain in strong dipole magnet

$$\Delta E(X) = -c\beta\sqrt{\frac{me}{2\pi\varepsilon_0}}\frac{\partial\lambda}{\partial z}\frac{1}{\sqrt{\lambda}}\left(1-\frac{X^2}{a^2}+\ln\frac{b^2}{a^2}\right)\left(aG+\int_{\sqrt{a^2-X^2}}^{\sqrt{b^2-X^2}}\left(\ln\frac{b^2}{X^2+y^2}\right)^{-1/2}dy\right)$$

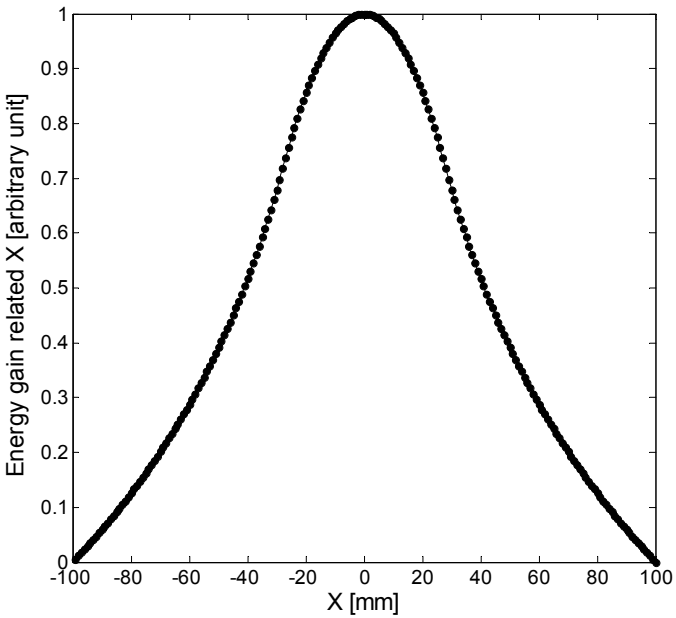
$$+\frac{1}{2}c\beta\sqrt{\frac{me}{2\pi\varepsilon_0}}\frac{\partial\lambda}{\partial z}\frac{1}{\sqrt{\lambda}}\int_0^{\sqrt{a^2-X^2}}\frac{y^2}{a^2}\left[\ln\left(1-\frac{X^2}{a^2}+\ln\frac{b^2}{a^2}+\frac{y^2}{a^2}\right)\right]^{-1/2}dy$$

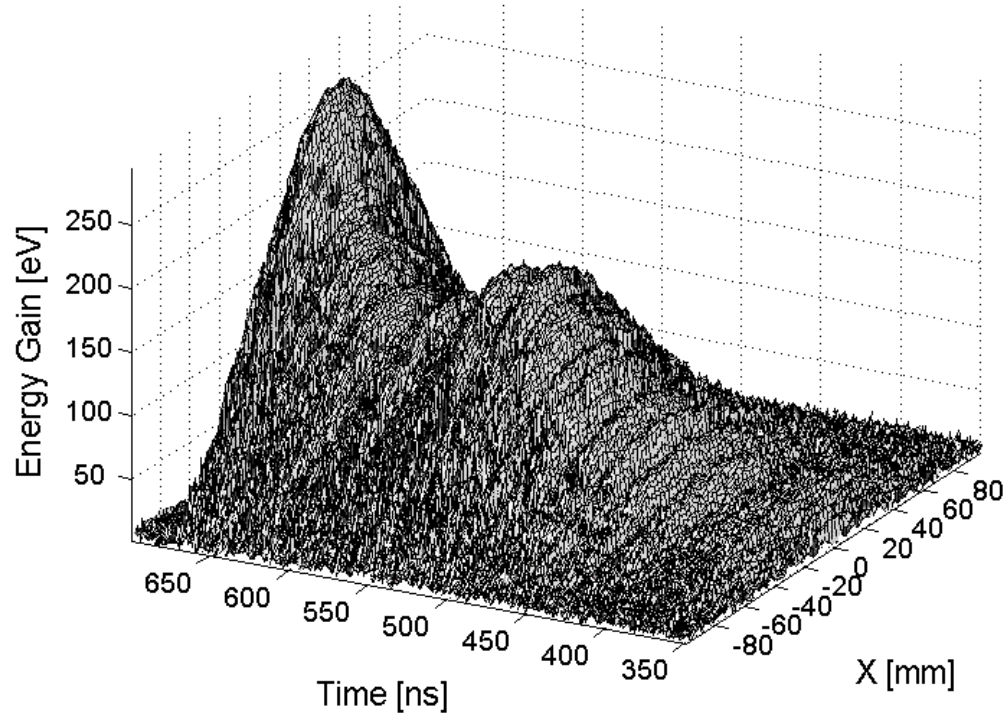
$$+\frac{1}{2}c\beta\sqrt{\frac{me}{2\pi\varepsilon_0}}\frac{\partial\lambda}{\partial z}\frac{1}{\sqrt{\lambda}}\int_{\sqrt{a^2-X^2}}^{\sqrt{b^2-X^2}}\left(1-\frac{X^2}{a^2}+\ln\frac{X^2+y^2}{a^2}\right)\left[\ln\left(\frac{b^2}{X^2+y^2}\right)\right]^{-1/2}dy \qquad (|X|<a)$$

$$\Delta E(X) = -c\beta\sqrt{\frac{me}{2\pi\varepsilon_0}}\frac{\partial\lambda}{\partial z}\frac{1}{\sqrt{\lambda}}\left[\frac{b^2}{X^2}\int_0^{\sqrt{b^2-X^2}}\left(\ln\frac{b^2}{X^2+y^2}\right)^{-1/2}dy-\frac{1}{2}\int_0^{\sqrt{b^2-X^2}}\frac{X^2+y^2}{X^2}\left(\ln\frac{b^2}{X^2+y^2}\right)^{-1/2}dy\right]$$

$$G = \arcsin\left[\frac{\sqrt{a^2-X^2}}{a}\left(1-\frac{X^2}{a^2}+\ln\frac{b^2}{X^2+a^2}\right)^{-1/2}\right] \qquad (|X|>a)$$

Electron energy gain in strong dipole magnet

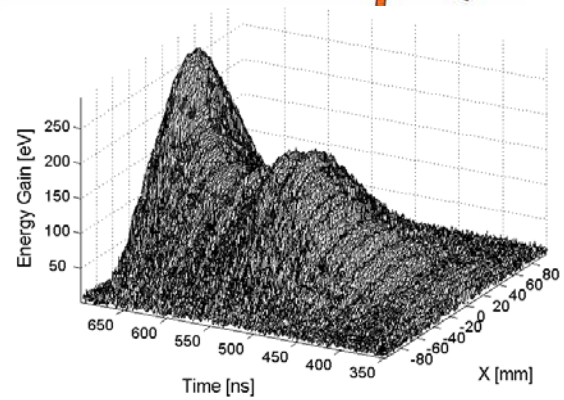


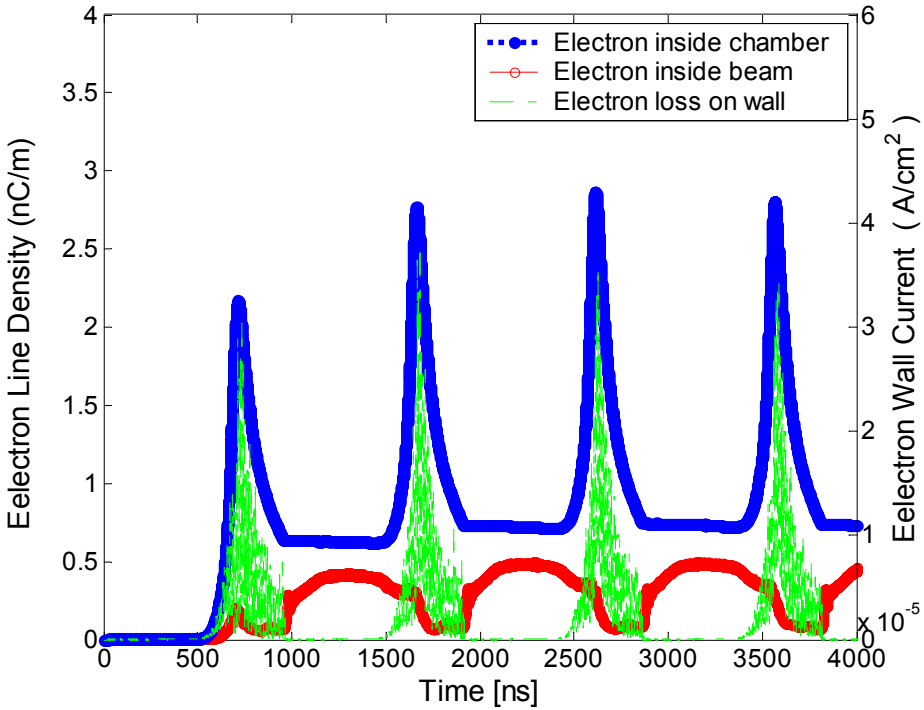


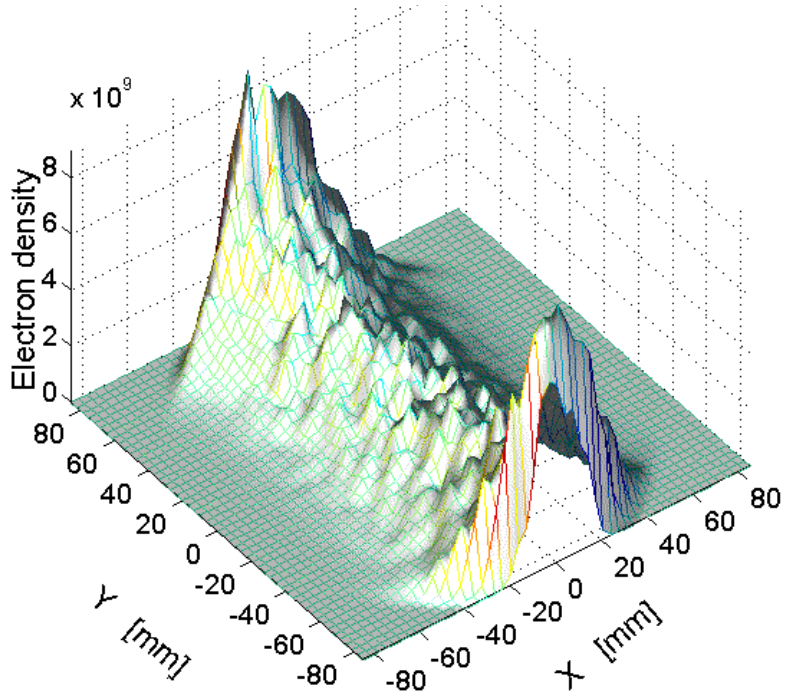
Energy gain at the wall surface for different X-coordinates. Left plot shows the electron energy gain as a function of horizontal coordinate. It is normalized by the peak energy gain at the chamber center X=0. Right plot shows the energy gain of direct drifting electrons in SNS dipole magnets with By=7935 Gauss.

Multipacting in Dipole magnets (By=0.79T)

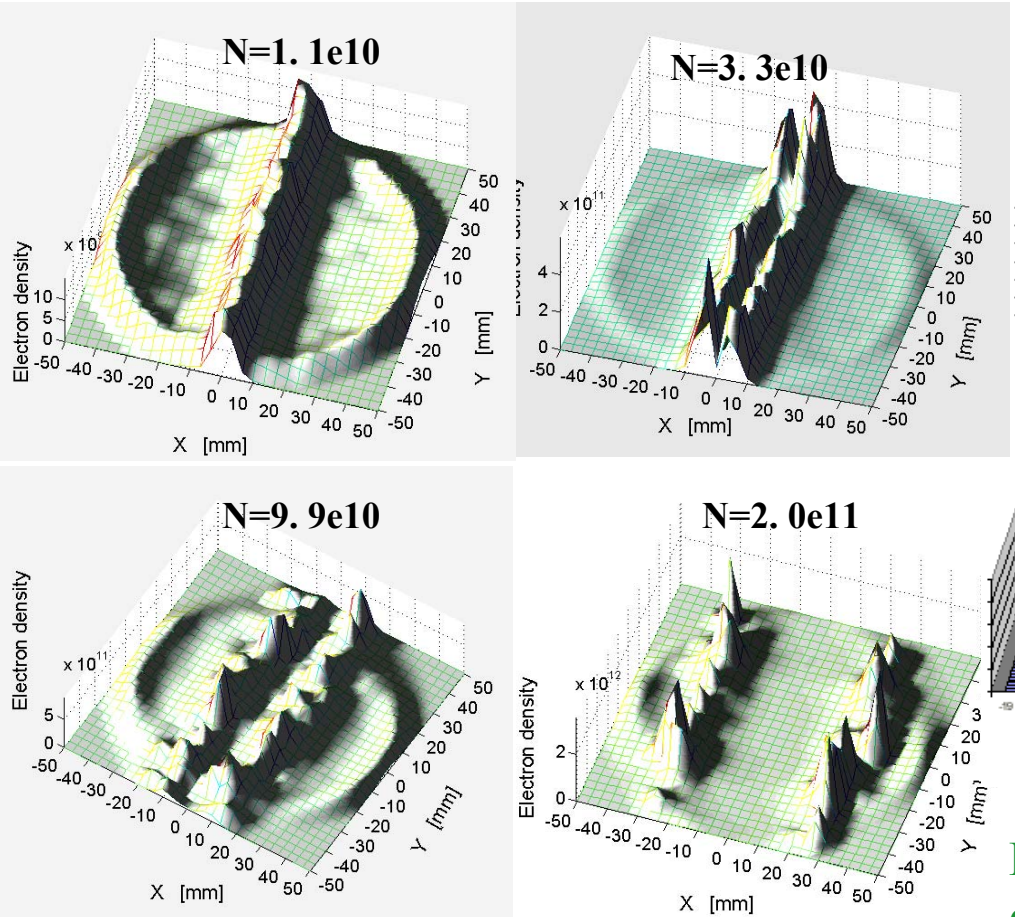
- Multipacting happen at the horizontal chamber center (**1 strip**, agree with estimation)
- E-cloud density is about 2 times smaller than the drift region



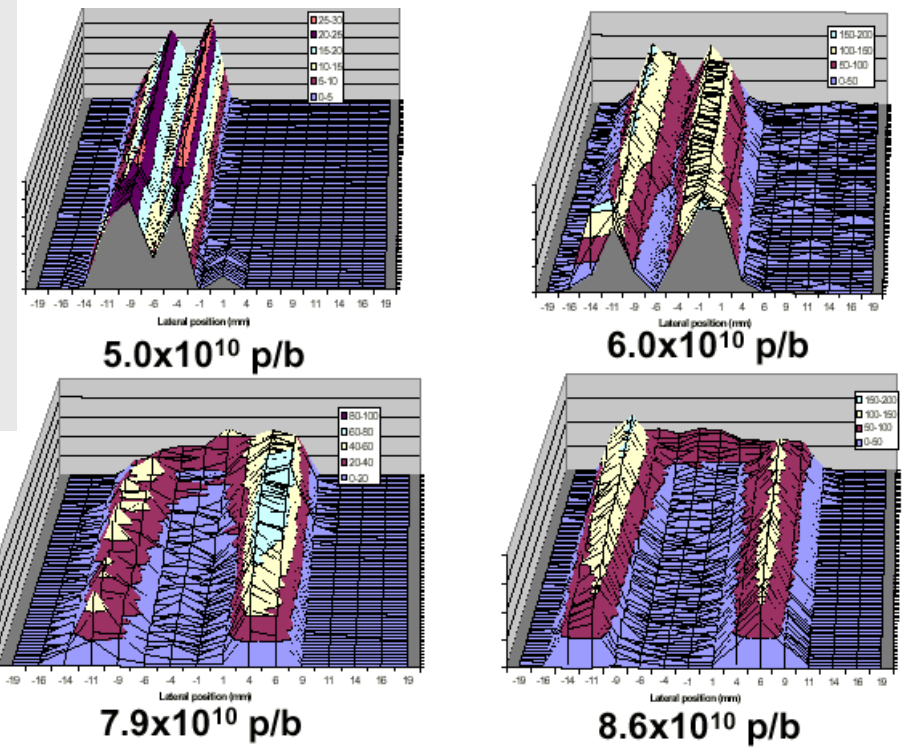




Bunch current effects on Multipacting in *dipole* for *short bunch*---strip position and lost charge density

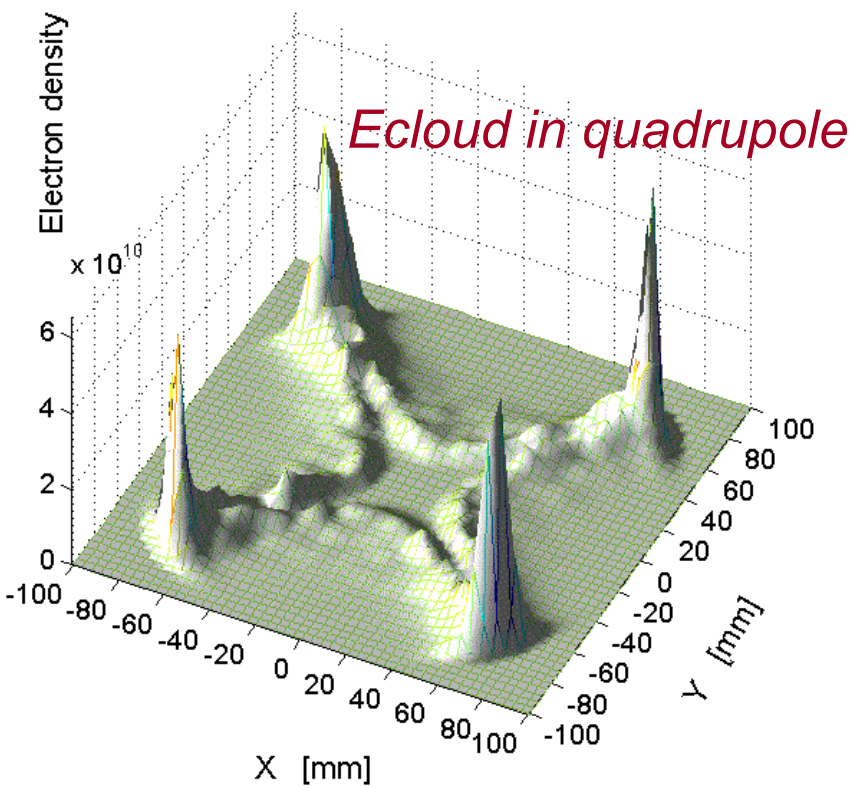


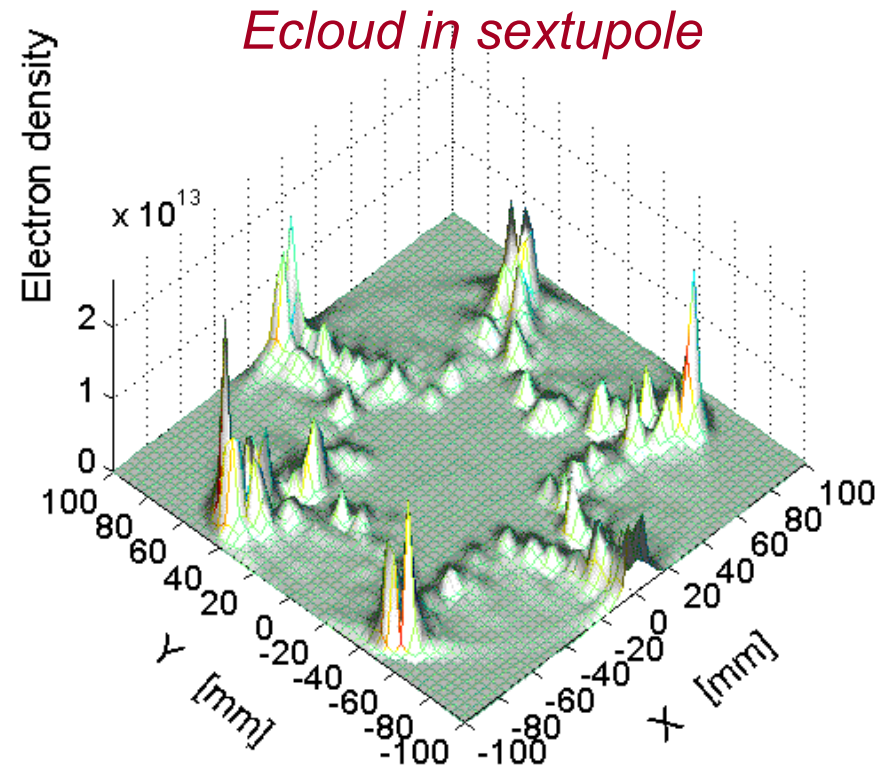
KEKB LER,simulation



Effect of a dipole field: 2 strips at different bunch intensities, CERN SPS experimental results, J. M.Jimenez, ECloud'02, CERN, 2002

E-cloud in Quadrupole and Sextupole magnets





- **Weak multipacting happens only near the middle of the pole surface**
- **No Trapping for long bunch! (trap was suspected in PSR, PSR-94-005)**

Trapping mechanism II– Beam field effect *(PRE 66, 036502, 2002)*

Invariation value of motion

$$W = \frac{m\upsilon^2}{2} = \frac{m\upsilon_{\|}^2}{2} + \frac{m\upsilon_{\perp}^2}{2} = cons\tan t$$

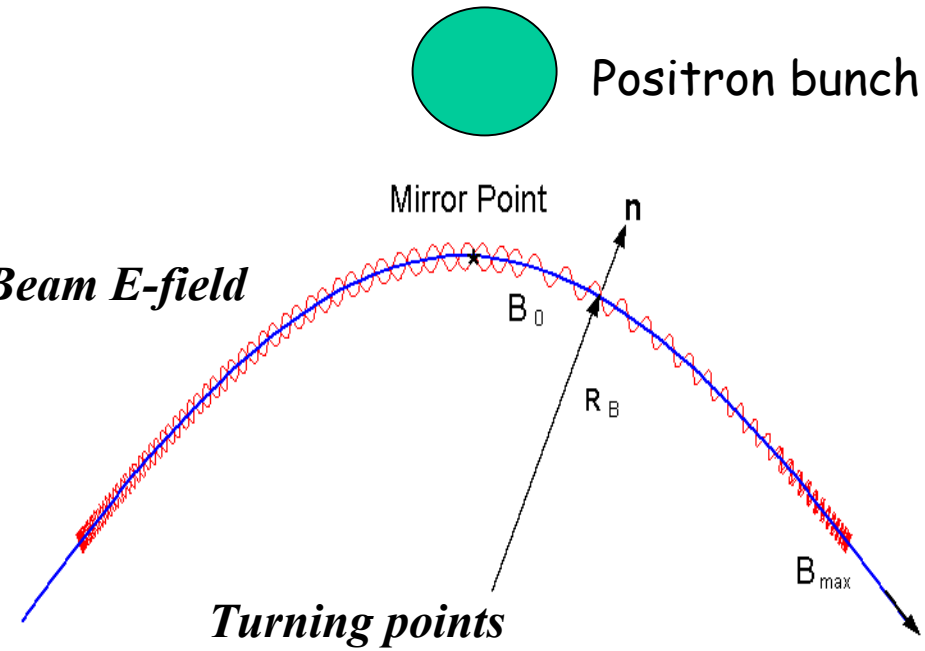
$$\frac{1}{2}m\upsilon_{\|}^2 + \mu_m B = const$$

→ Reflective Points: $\upsilon_{\|}=0$

Trapping condition

$$\Gamma_{trap} > 1$$

$$\Gamma_{trap} = \frac{F_\upsilon}{F_B} = \frac{\upsilon_{\perp 0}^2}{\upsilon_{\perp 0}^2 + \upsilon_{\|0}^2}\frac{B_{\max}}{B_0}$$



mirror field trapping

$$\Gamma_{trap} = \left.\frac{\upsilon_{\perp 0}^2}{\upsilon_{\perp 0}^2 + \upsilon_{\|0}^2}\right|_{\substack{at\ the\ emission \\ point}} = cons\tan t \le 1$$

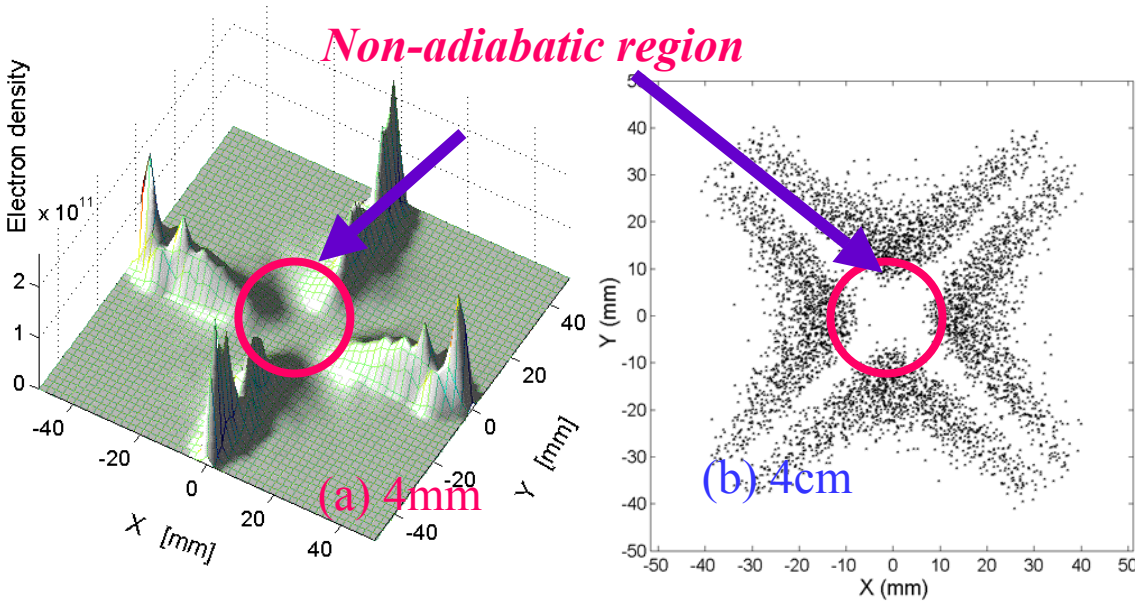
Trap factor is constant if no other force (except B force) disturbs the electron and smaller than 1.0, no trapping

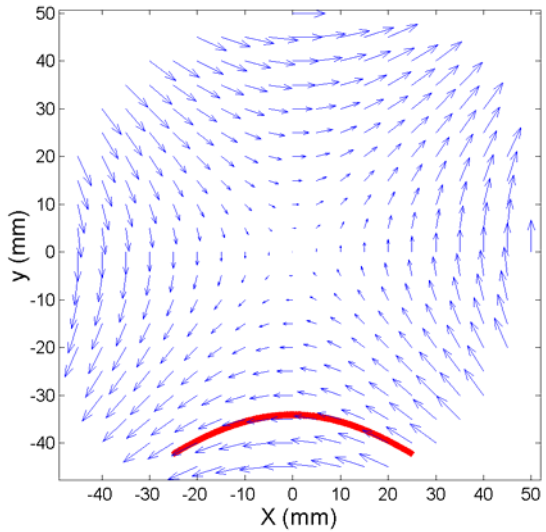
Trapping mechanism II --- Beam potential effect

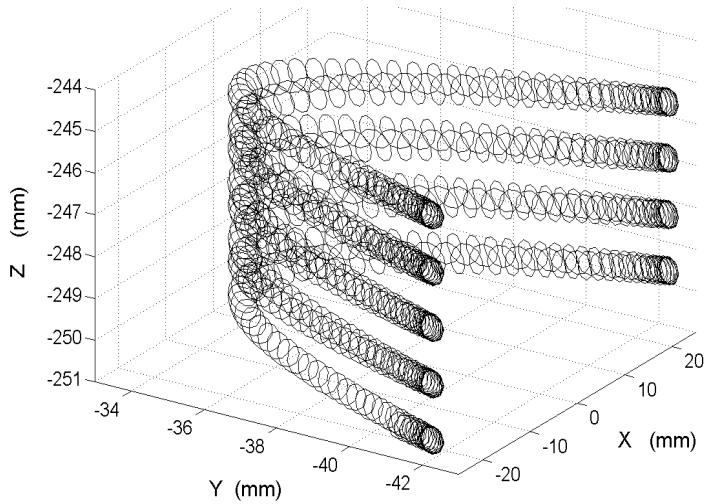
Trap requirement for positron bunch

Bunch length should be shorter than period of gyration motion

$$\sigma_l < \frac{2\pi cm}{e}\frac{1}{B} \Rightarrow \sigma_l(mm) < 10.7/B(T)$$



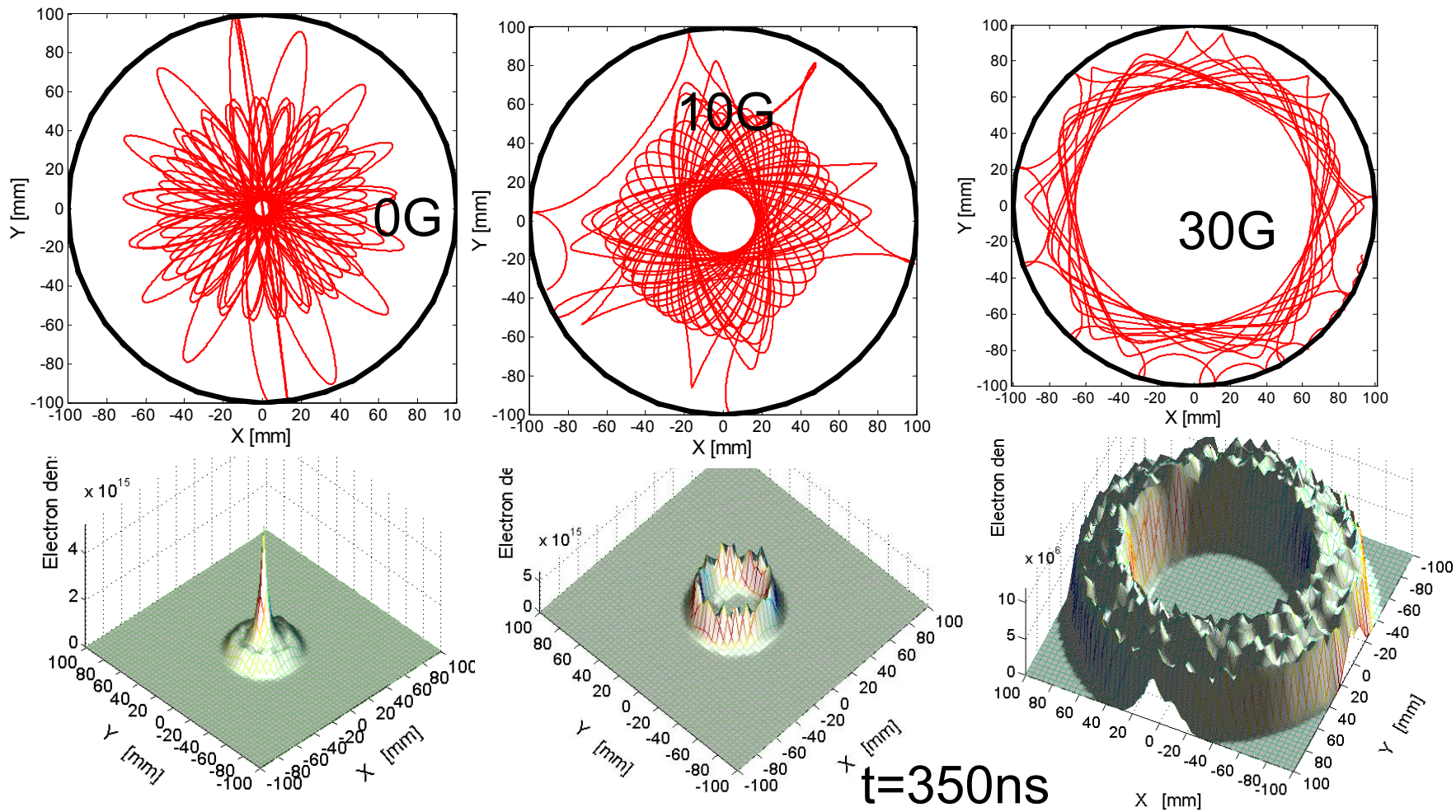




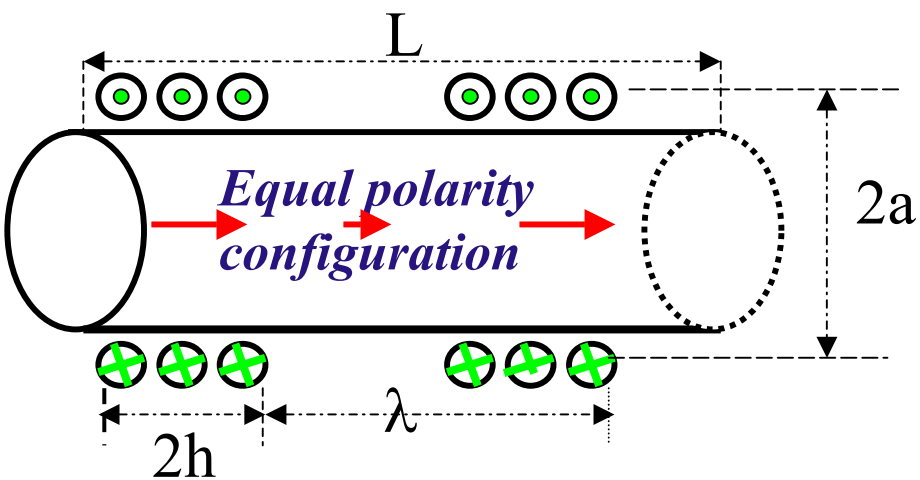
Trapped photoelectron distribution in quadrupole magnet with field gradient 10.3 T/m during the train gap for different bunch length

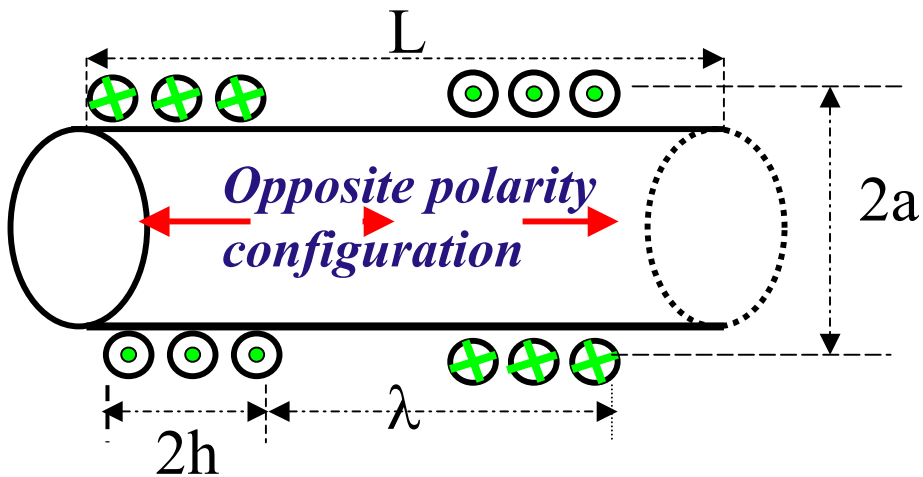
Uniform Solenoid effects

- 30G Solenoid field can reduce the e-cloud density with a factor 2000 !
- Zero density within beam



Solenoid---Configuration





$$B_r = B_0 \frac{2ka}{\pi} \sum_{n=1}^{\infty} \sin nhk K_1(nka) I_1(nkr) \sin nkz$$

$$B_z = B_0 \left(\frac{2h}{\lambda} + \frac{2ka}{\pi} \sum_{n=1}^{\infty} \sin nhk K_1(nka) I_0(nkr) \cos nkz \right)$$

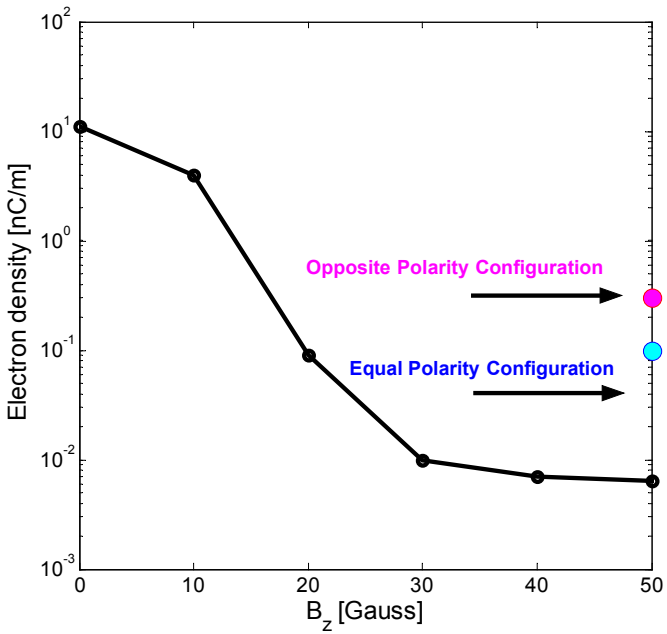
$$B_r = B_0 \frac{4ka}{\pi} \sum_{n=1,3,5}^{\infty} \sin nhk K_1(nka) I_1(nkr) \sin nkz$$

$$B_z = B_0 \frac{4ka}{\pi} \sum_{n=1,3,5}^{\infty} \sin nhk K_1(nka) I_0(nkr) \cos nkz$$

By E. Perevedentsev

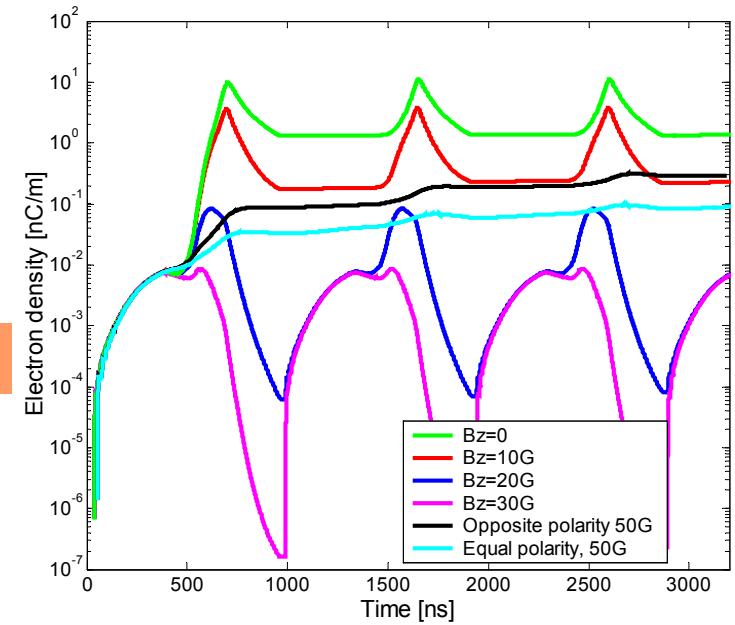
B_0=50 Gauss, h=0.4 m, a=120 mm, λ =1 m and 2 m

Solenoid configuration effects

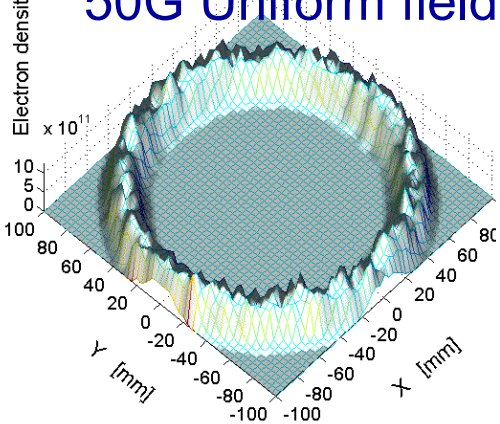


Factor 36

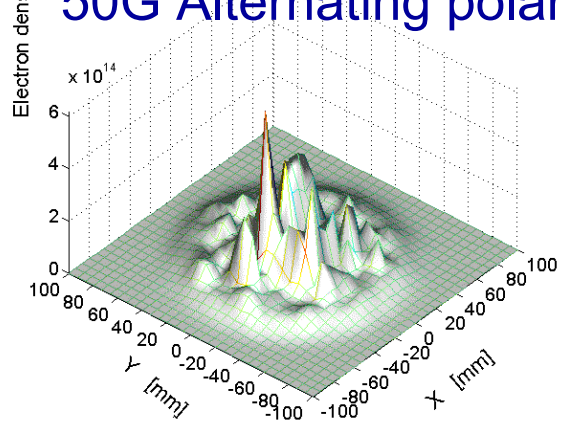
Factor 110



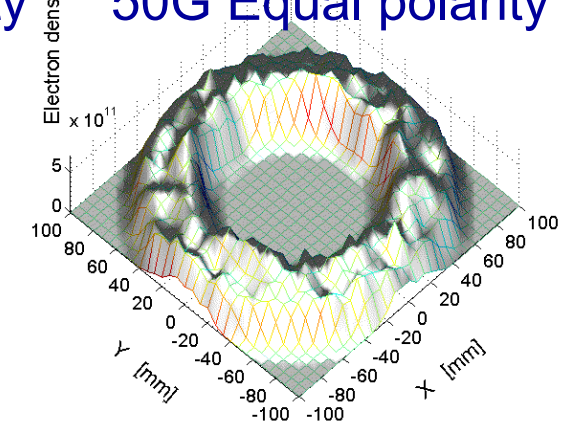
50G Uniform field



50G Alternating polarity

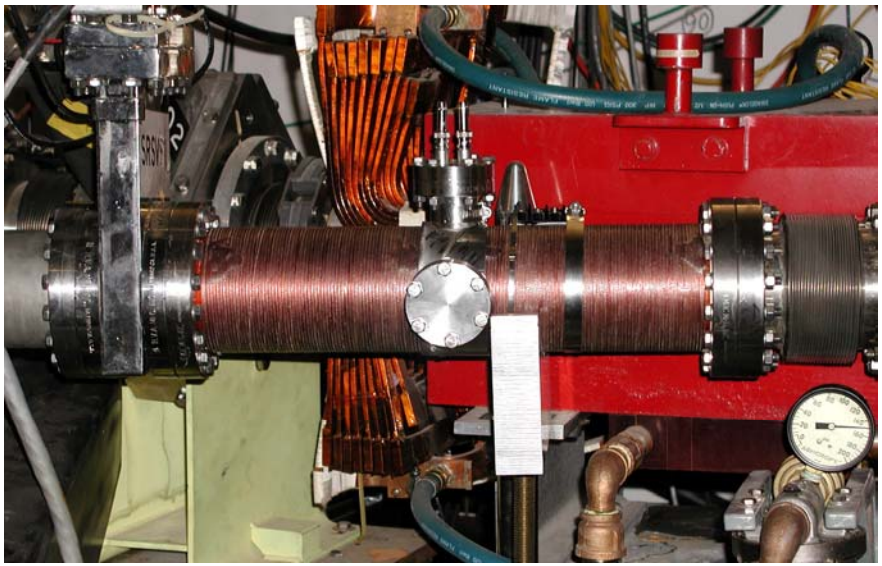


50G Equal polarity

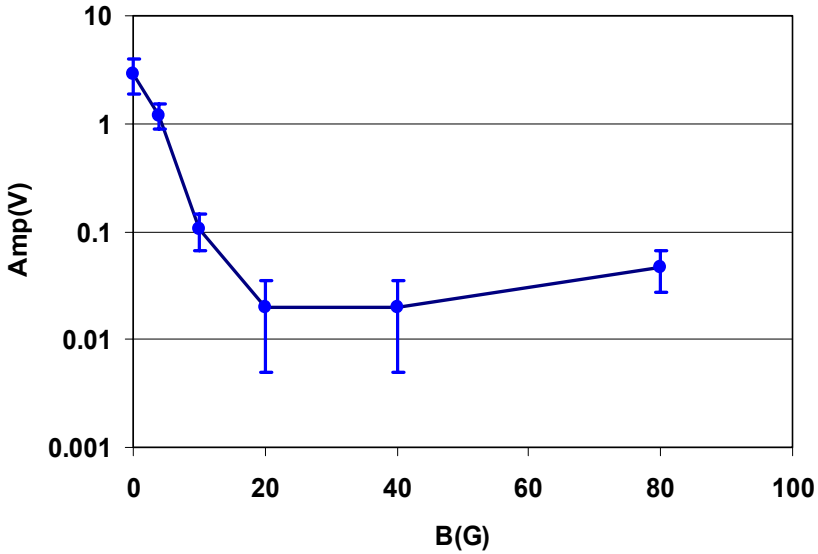


Solenoid effect ------PSR experiment

- **20G Solenoid field can reduce the e-cloud signal with a factor > 50 !**

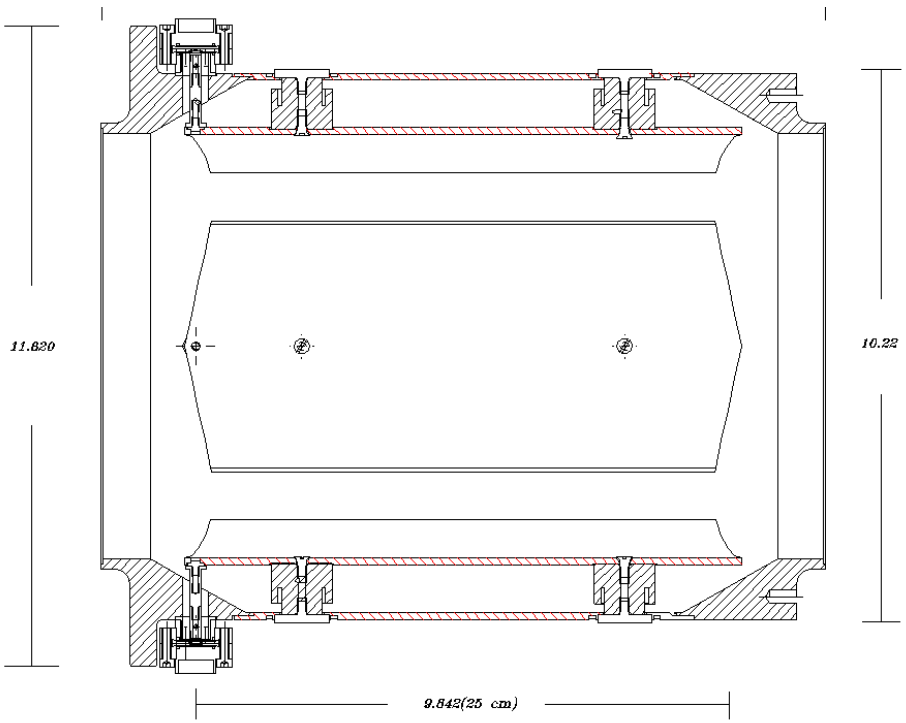


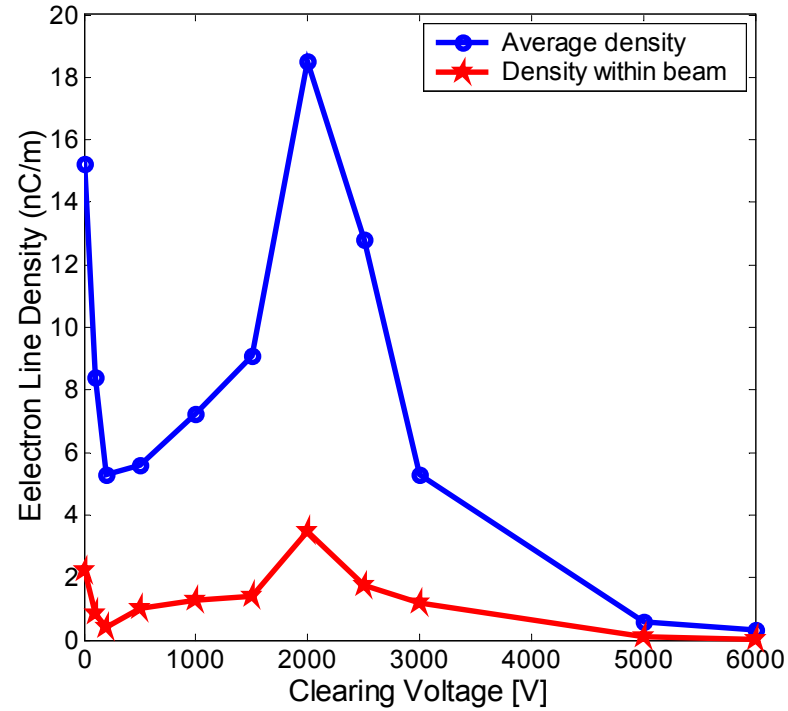
Picture of RFA (ED92Y) in a short solenoid in section 9



Effect of weak solenoid on prompt electron peak (ED92Y)

Electrode clearing effect vs. Clearing voltage (SNS Tech note 128, to be published)

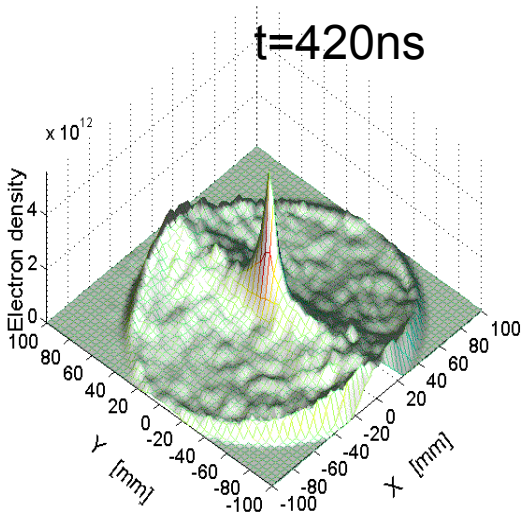


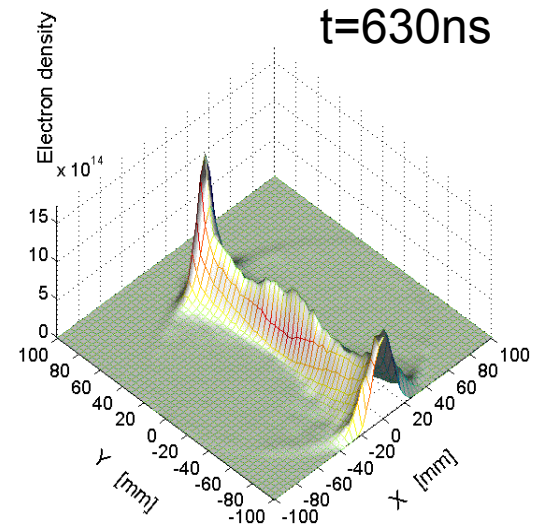


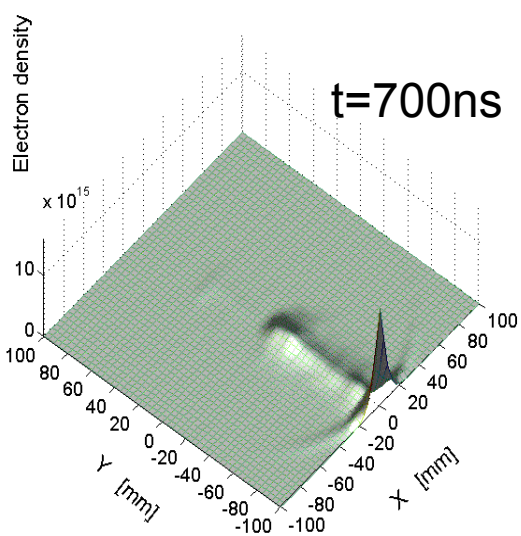
e-cloud density vs. clearing fields

- **Weak field(~200V) is very helpful**
- **Strong multipacting at 12kV, which could be stronger than zero field case**

E-cloud distribution at different time for 500V clearing field

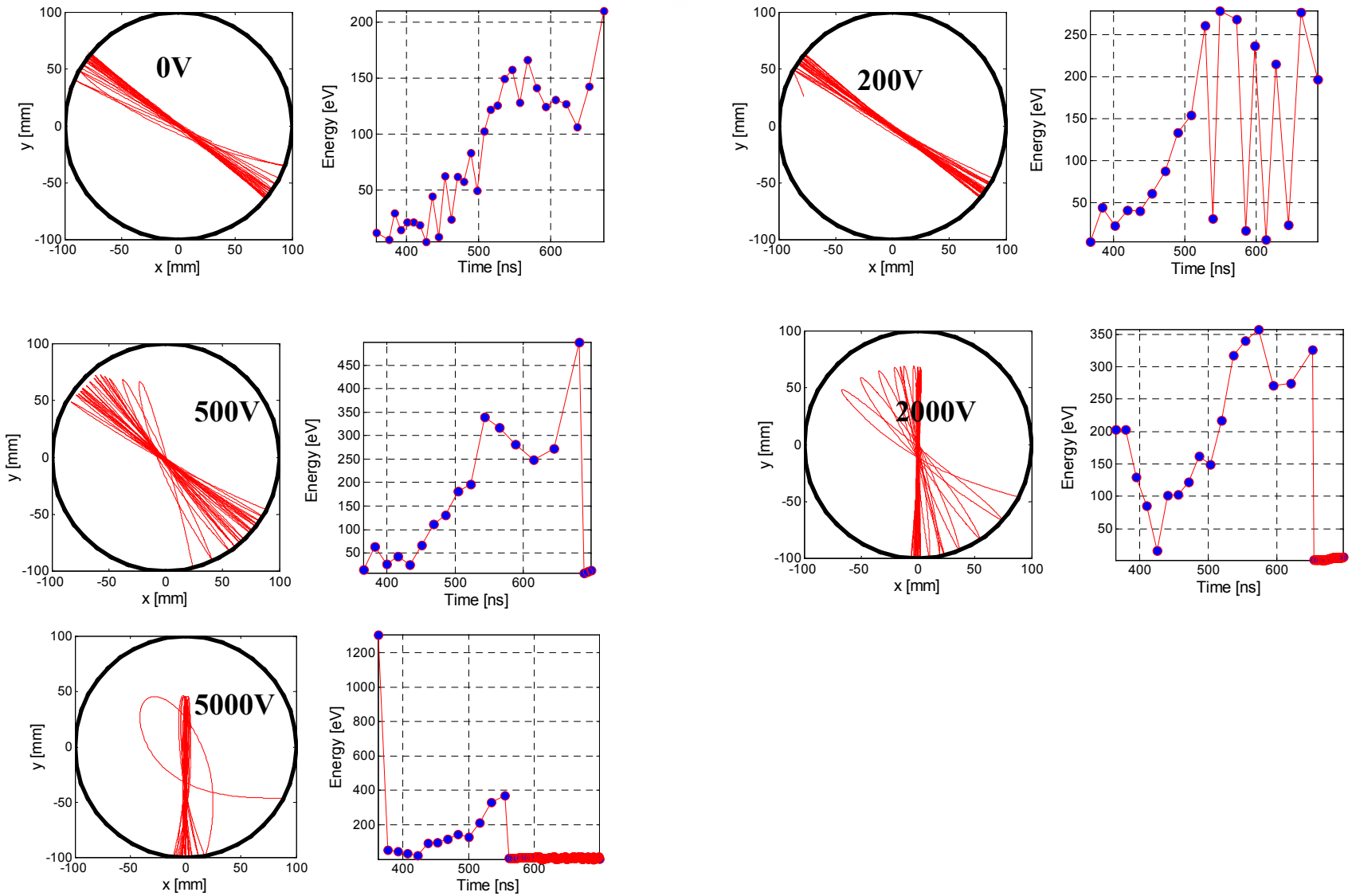






- **Clearing field can cause the particle polarized toward the clearing field direction. As a result, it causes strong multipacting near the positive electrode.**

Mechanism of strong multipacting due to clearing field



(BNL/SNS TECHNICAL NOTE 128, 2003)

Summary & outlook

- *Electron motion under beam space charge field is investigated. (Adiabatic invariant, Nonlinear oscillation frequency, electron energy gain). Mechanism of trailing edge multipacting is clearly explained*
- *Many factors related to the multipacting has been investigated one by one using 3D code. The results qualitatively agree with the our analysis and experiment studies. Beam intensity, Longitudinal beam profile, transverse beam size, beam in gap are important.*
- *Good agreement is achieved between analysis and simulation for the multipacting in drift region and dipole magnet*
- *Mechanism of the clearing of e-cloud with clearing electrode and solenoid is investigated*