Single Transverse-Spin Asymmetry in $p^\uparrow p \to \pi X$ and $e p^\uparrow \to \pi X$

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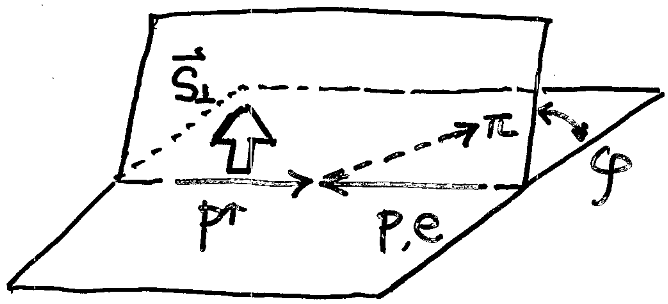
@SPIN 2002, BNL

- π Production with large transverse momentum



$$p^\uparrow(p.S_\perp) + \left\{ \begin{matrix} p \\ e \end{matrix} \right\}(p') \to \pi(\ell) + X$$

$$A_N \sim \epsilon_{\mu\nu\lambda\sigma} p^\mu p'^\nu \ell^\lambda S_\perp^\sigma \sim \sin\varphi$$

* Study A_N in the framework of Collinear factorization

* Twist-3 observable = Probe of quark-gluon correlation.

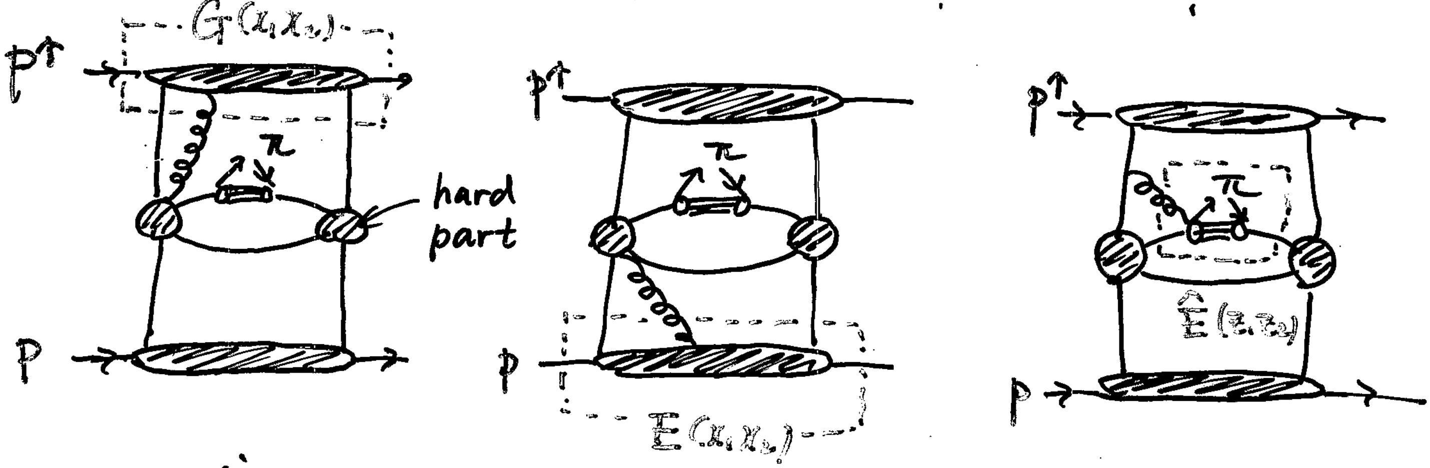
QCD factorization for Twist-3

$p^\uparrow + p \to \pi + X$

chiral-odd

	$p^\uparrow$		p		π			
$\Delta\sigma \sim$	$G(x_1 x_2)$	$\otimes$	$\begin{Bmatrix} f_1(x') \\ G(x') \end{Bmatrix}$	$\otimes$	$\hat{f}_1(z)$	$\otimes$	$\hat{\sigma}_1$	(i) (Qiu & Sterman)
$+$	$h_1(x)$	$\otimes$	$E(x_1' x_2')$	$\otimes$	$\hat{f}_1(z)$	$\otimes$	$\hat{\sigma}_2$	(ii) → tiny (Kanazawa & Y.K. P.L. 490B)
$+$	$h_1(x)$	$\otimes$	$\begin{Bmatrix} f_1(x') \\ G(x') \end{Bmatrix}$	$\otimes$	$\hat{E}(z_1 z_2)$	$\otimes$	$\hat{\sigma}_3$	(iii)

Transversity



(i) (ii) (iii)

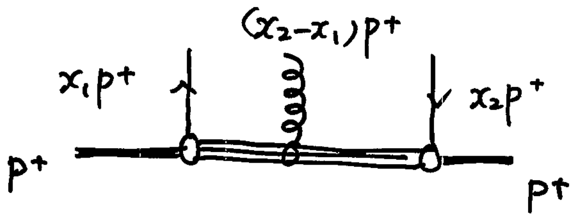
$e + p^\uparrow \to \pi + X$

	$p^\uparrow$		π			
$\Delta\sigma \sim$	$G(x_1 x_2)$	$\otimes$	$\hat{f}_1(z)$	$\otimes$	$\hat{\sigma}_1'$	(i)$'$
$+$	$h_1(x)$	$\otimes$	$\hat{E}(z_1 z_2)$	$\otimes$	$\hat{\sigma}_2'$	(iii)$'$

$\perp$ Pol. Twist-3 distribution function

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS_\perp | \psi(0)\, g F^{\alpha\beta}(\mu n) n_\beta\, \bar{\psi}(\lambda n) | pS_\perp \rangle$$

$[0\ \mu n]$ $\quad$ $[\mu n\ \lambda n]$



$$\left(\begin{array}{l} p^+ = p^+ \\ p^2 = n^2 = 0, \quad p\cdot n = 1 \end{array} \right)$$

$$= \frac{M_N}{4} \not{p}\, \epsilon^{\alpha p n S_\perp}\, \underline{G_F(x_1 x_2)} + i \frac{M_N}{4} \gamma_5 \not{p}\, S_\perp^\alpha\, \underline{G_F^5(x_1 x_2)} + \cdots$$

$\cdot\ F^{\alpha\beta} n_\beta \to D^\alpha = \partial^\alpha - igA^\alpha$ defines $\underline{G_D(x_1 x_2)}$, $\underline{G_D^5(x_1 x_2)}$.

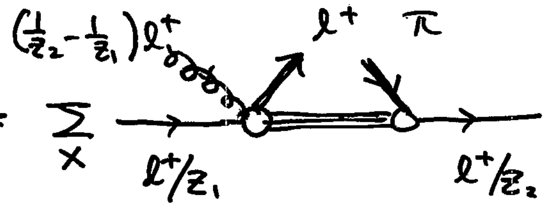
Hermiticity & T-inv. gives:

$$\left\{ \begin{array}{l} G_F(x_1 x_2) = G_F(x_2 x_1) \\ G_F^5(x_1 x_2) = -G_F^5(x_2 x_1) \\ G_D(x_1 x_2) = -G_D(x_2 x_1) \\ G_D^5(x_1 x_2) = G_D^5(x_2 x_1) \end{array} \right.$$

real functions

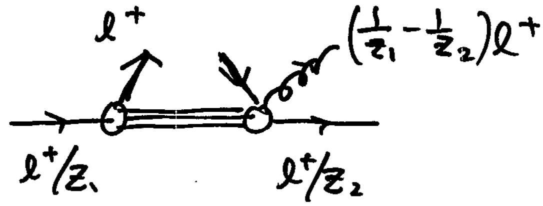
Unpolarized Twist-3 fragmentation function for π

$$\frac{1}{N_c}\sum_X \int\frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi}\, e^{-i\lambda/z_1} e^{-i\mu\left(\frac{1}{z_2}-\frac{1}{z_1}\right)} \langle 0|\,\psi(0)\,|\pi(\ell)X\rangle\langle\pi(\ell)X|\, \underline{g F^{\alpha\beta}}(\mu w) n_\beta\, \bar{\psi}(\lambda w)\,|0\rangle$$

$= \sum_X$ 

$$\left(\begin{array}{c} \ell^2 = w^2 = 0 \\ \ell\cdot w = 1 \end{array}\right)$$

$$= \frac{M_N}{2}\gamma_5 \not{\ell}\, \gamma_\nu\, \epsilon^{\nu\alpha\sigma\tau} w_\sigma \ell_\tau\, \underline{\underline{\hat{E}_F(z_1,z_2)}}/z_2 \quad + (\text{other twist-3}) + \cdots$$

- From $\sum_X$ 

$\Rightarrow$ $\underline{\hat{E}_{FR}(z_1,z_2)}$

- $F^{\alpha\beta} n_\beta \to D^\alpha = \partial^\alpha - igA^\alpha$ defines : $\hat{E}_D(z_1,z_2)$ $\hat{E}_{DR}(z_1,z_2)$.

- Hermiticity & (naive) T-inv. gives

$$\begin{cases} \hat{E}_F(z_1 z_2) = \hat{E}_{FR}(z_2 z_1) \\ \hat{E}_D(z_1 z_2) = -\hat{E}_{DR}(z_2 z_1) \end{cases}$$

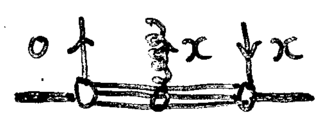
real functions

chiral-odd

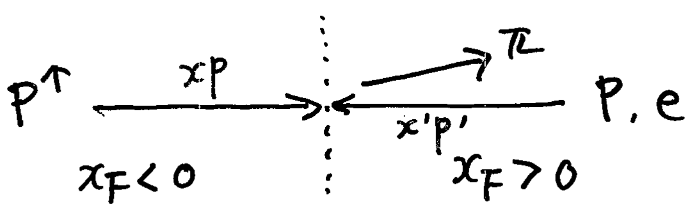
Analysis of twist-3 cross section (cf. Qiu & Sterman '99)

- $G_F(x,x)$ and $\hat{E}_F(z,z)$ appear as soft-gluon-pole (i.e. $x_1 = x_2$, $z_1 = z_2$), and their derivatives $\frac{d}{dx}G_F(x,x)$ and $\frac{d}{dz}\hat{E}_F(z,z)$ also appear.

- $G_D(x_1\, x_2)$ and $\hat{E}_D(z_1\, z_2)$ appear as soft-fermion-pole (i.e. $x_i = 0$ or $z_i = 0$), which is physically expected to be suppressed. $\longrightarrow$ ignore!



- Focuss on soft-gluon-pole :
  At $x_F \to 1$, main contribution is from the region with $x \to 1$, $x' \to 0$, and $z \to 1$.



Here
$$\left| \frac{d}{dx} G_F(x,x) \right| \gg \left| G_F(x,x) \right| \quad \because)\ G_F(x,x) \sim (1-x)^\beta \quad (\beta > 0)$$

$$\left| \frac{d}{dz} \hat{E}_F(z,z) \right| \gg \left| \hat{E}_F(z,z) \right| \quad \because)\ \hat{E}_F(z,z) \sim (1-z)^{\beta'} \quad (\beta' > 0)$$

★ In $p^\uparrow p \to \pi X$, keep only $\frac{d}{dx}G_F(x,x)$ and $\frac{d}{dz}\hat{E}_F(z,z)$ for valence component. (valence-quark soft-gluon approximation.)

☆ In $ep^\uparrow \to \pi X$, include all soft-gluon-poles.

$p^{\uparrow}p \to \pi(\ell)X$

(i) $\frac{d}{dx} G_F(x\,x) \otimes \left\{ \begin{matrix} f_1(x') \\ G(x') \end{matrix} \right\} \otimes \hat{f}_1(z) \otimes \hat{\sigma}_1$ (chiral-even) (Qiu & Sterman '99)

- $\underline{G_F^a(x\,x) \equiv K_a f_1^a(x)}$

  ↑ same Dirac structure f_1^a: GRV, LO

  $\underline{K_u = -K_d = 0.07}$ (E704 $A_N^{\pi^+}$ and $A_N^{\pi^-}$ have opposite sig

(ii) $h_1(x) \otimes \left\{ \begin{matrix} f_1(x') \\ G(x') \end{matrix} \right\} \otimes \frac{d}{dz} \hat{E}_F(z\,z) \otimes \hat{\sigma}_3$ (chiral-odd)

- $\underline{h_1^a(x) \equiv g_1^a(x)}$: GRV, LO
- $\underline{\hat{E}_F^a(z\,z) \equiv \hat{K}^a \hat{f}_1^a(z)}$ $\hat{f}_1$ for π : BKK'95

  $\underline{\hat{K}_u = -0.11,\ \hat{K}_d = -0.19}$ to reproduce E704 A_N

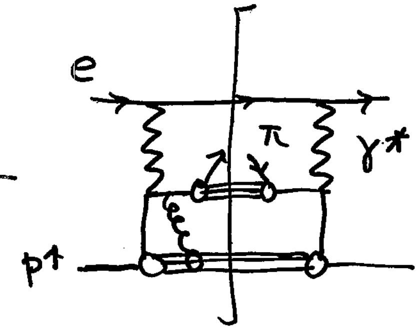
  (Recall g_1^u and g_1^d have opposite signs.)
- N.B. In general $\hat{E}_F(z_1 z_2) \neq \hat{E}_F(z_2 z_1)$

  But assume $\left.\frac{\partial}{\partial z_1} \hat{E}_F(z_1 z)\right|_{z_1=z} = \left.\frac{\partial}{\partial z_2} \hat{E}_F(z\,z_2)\right|_{z_2=z}$

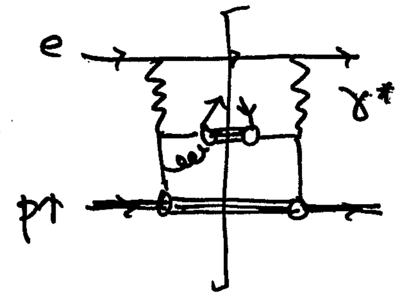
(i) (iii) $\overset{\text{large } x_F}{\sim}$ $\underbrace{\frac{K\pi M \ell_T}{(-U)} \left[1 + O\left(\frac{U}{T}\right)\right]}_{\sim M\left(O\left(\frac{\ell_T}{S}\right) + O\left(\frac{1}{\ell_T}\right)\right)} \frac{1}{1-x_F} \sin\varphi$

$ep^{\uparrow} \to \pi(\ell)\, X$

$$\left\{ \begin{matrix} G_F(x\,x) \\ \frac{d}{dx} G_F(x\,x) \end{matrix} \right\} \otimes \hat{f}_1(z) \otimes \hat{\sigma}_1' \quad \leftarrow$$



$$+ \quad h_1(x) \otimes \left\{ \begin{matrix} \hat{E}_F(z\,z) \\ \frac{d}{dz} \hat{E}_F(z\,z) \end{matrix} \right\} \otimes \hat{\sigma}_2' \quad \leftarrow$$



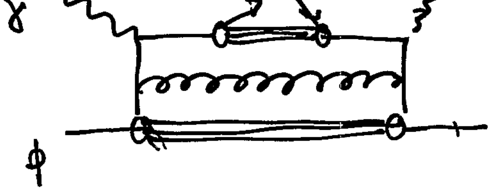
x_F: large

$$\frac{K\pi M \ell_T}{(-T)} \frac{1}{1-x_F} \sin\varphi \sim \frac{K\pi M}{\ell_T} \frac{1}{1-x_F} \sin\varphi$$

only $\frac{M}{\ell_T}$ term !

N.B. Experimentaly, must select $Q^2 \geq \frac{S}{2}\left[1 - \frac{x_F}{\sqrt{x_F^2 + x_T^2}}\right]$ in $ep^{\uparrow} \to \pi X$. $\left(x_T = \frac{2\ell_T}{\sqrt{S}} \right)$

For $Q^2 = 0$



$p^\uparrow p \to \pi(\ell) X \;:\; \hat{h}_1(x) \otimes \begin{Bmatrix} f_1(x) \\ G(x') \end{Bmatrix} \otimes \frac{d}{dz}\hat{E}_F(z,z) \otimes \hat{\sigma}$ piece

$$E_\pi \frac{d^3\Delta\sigma}{d\ell^3} = \frac{\pi M \alpha_s^2}{S} \underline{\epsilon^{\alpha \ell w S_\perp}} \sum_a \int_{z_{min}}^1 \frac{dz}{z^2} \int_{x_{min}}^1 \frac{dx}{x} \frac{1}{xS + U/z} \int_0^1 \frac{dx'}{x'}$$

$$\times \delta\left(x' + \frac{xT/z}{xS + U/z}\right)$$

$$\times \Big\{ 2 \sum_b \underline{h_1^a(x)} f_1^b(x') \underline{\left(-z^2 \frac{\partial}{\partial z_1} \hat{E}_F^a(z_1, z)\right)}\Big|_{z_1 = z} \left(\frac{-2p_\alpha}{T} \hat{\sigma}^{\mathrm{I}}_{ab\to a} + \frac{-2p'_\alpha}{U} \hat{\sigma}^{\mathrm{II}}_{ab\to a} \right)$$

$$+ 2 \sum_b \underline{h_1^a(x)} f_1^b(x') \underline{\left(-z^2 \frac{d}{dz} \hat{E}_F^a(z,z)\right)} \frac{x p_\alpha + x' p'_\alpha}{|xT + x'U|} \left(\hat{\sigma}^{\mathrm{I}}_{ab\to a} + \hat{\sigma}^{\mathrm{II}}_{ab\to a} \right)$$

$$+ 2 \underline{h_1^a(x)} G(x') \underline{\left(-z^2 \frac{\partial}{\partial z_1} \hat{E}_F^a(z_1, z)\right)}\Big|_{z_1 = z} \left(\frac{-2p_\alpha}{T} \hat{\sigma}^{\mathrm{I}}_{ag\to a} + \frac{-2p'_\alpha}{U} \hat{\sigma}^{\mathrm{II}}_{ag\to a} \right)$$

$$+ 2 \underline{h_1^a(x)} G(x') \underline{\left(-z^2 \frac{d}{dz} \hat{E}_F^a(z,z)\right)} \frac{x p_\alpha + x' p'_\alpha}{|xT + x'U|} \left(\hat{\sigma}^{\mathrm{I}}_{ag\to a} + \hat{\sigma}^{\mathrm{II}}_{ag\to a} \right) \Big\}$$

$$z_{min} = \frac{-T-U}{S} = \sqrt{x_F^2 + x_T^2}, \quad x_{min} = \frac{-U/z}{S + T/z}$$

$\left(\hat{\sigma}^{\mathrm{I}}, \hat{\sigma}^{\mathrm{II}} : \text{partonic cross section} \right)$

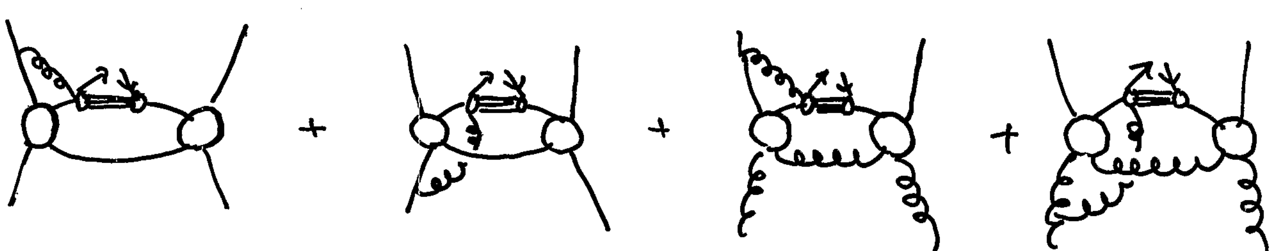
$$\epsilon^{\alpha \ell w S_\perp} \equiv \epsilon^{\alpha\mu\nu\lambda} \ell_\mu w_\nu S_{\perp\lambda} \quad (\ell^2 = w^2 = 0,\ \ell \cdot w = 1)$$

$$p_\alpha \epsilon^{\alpha \ell w S_\perp} = -p'_\alpha \epsilon^{\alpha \ell w S_\perp} = \sqrt{S} \frac{\sqrt{UT}}{|U + T|} \sin\varphi$$

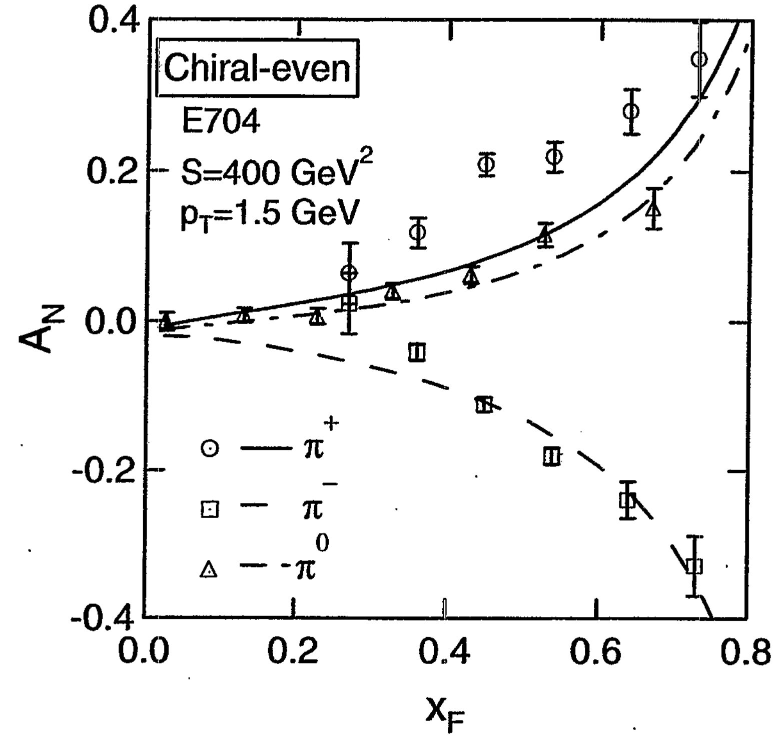
Put : $\hat{E}_F(z,z) \equiv \hat{K} \hat{f}_1(z) \underset{z\to 1}{\sim} (1-z)^\beta$

Then

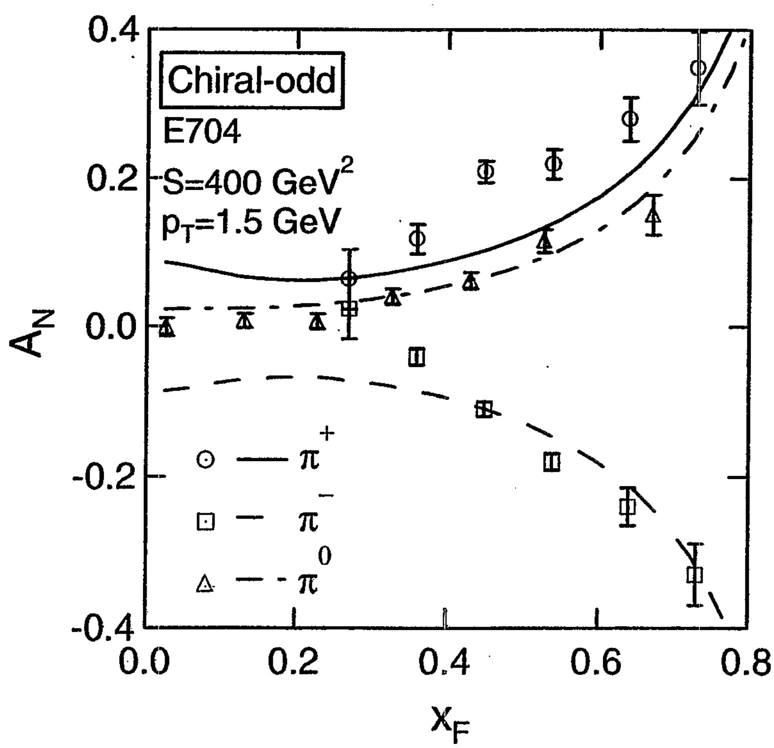
$$A_N = \frac{\Delta\sigma}{\sigma} \overset{x_F:\,\text{large}}{\sim} \frac{\hat{K}\pi M \ell_T}{(-U)} \left[1 + O\left(\frac{U}{T}\right)\right] \frac{1}{1 - x_F} \sin\varphi$$

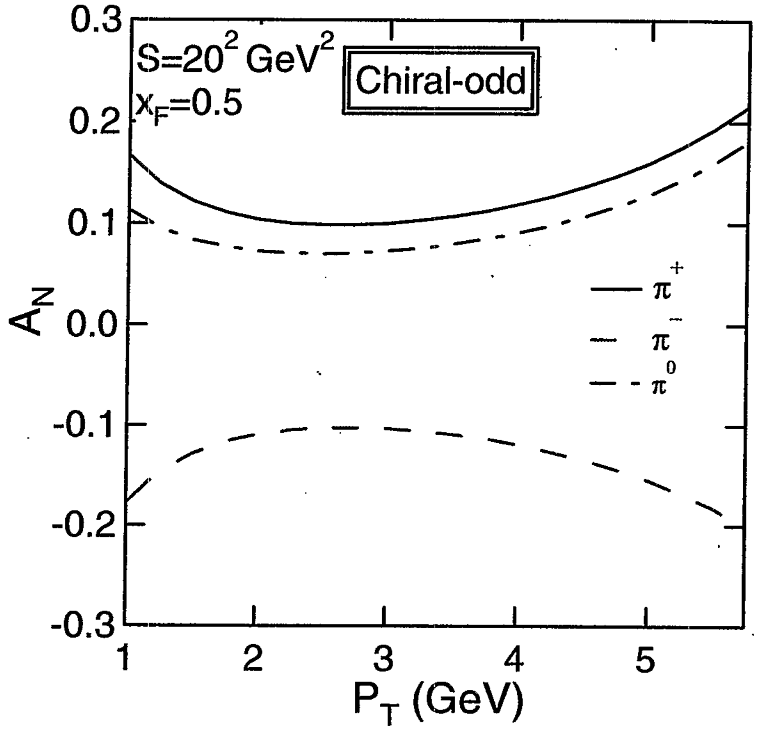


+ (mirror

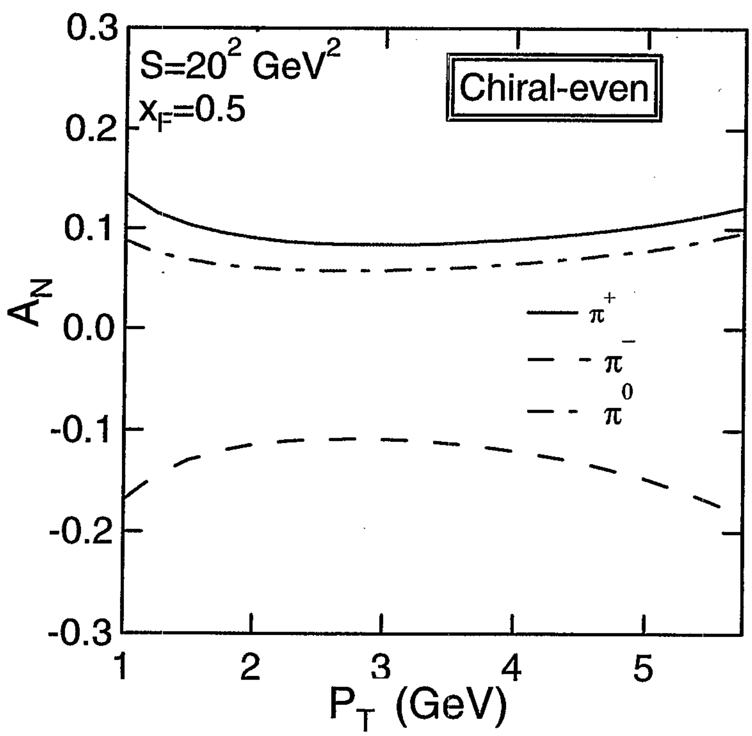


same as
Qin & Sterman



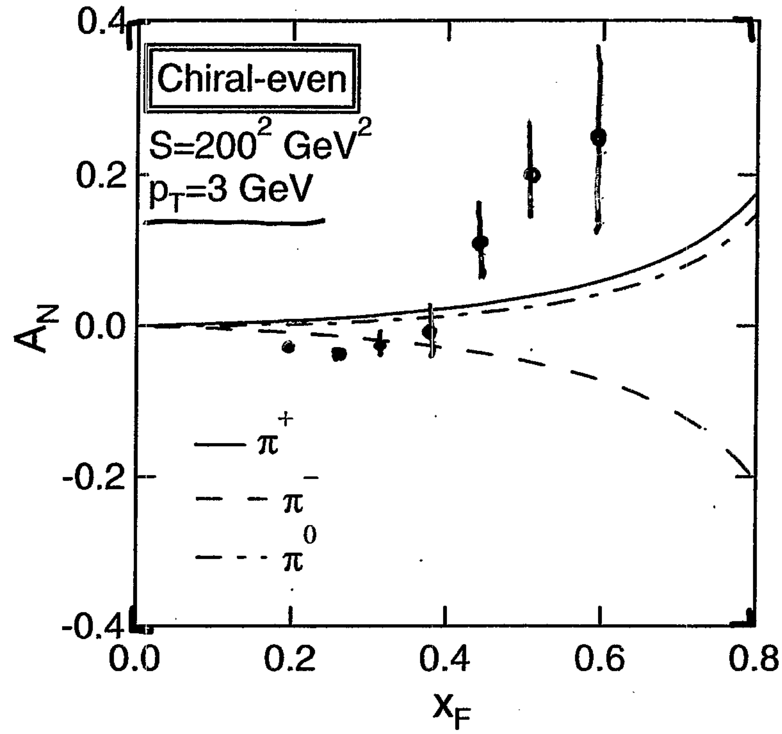


S=20^2 GeV^2
x_F=0.5
Chiral-odd
A_N
P_T (GeV)
π^+
π^-
π^0

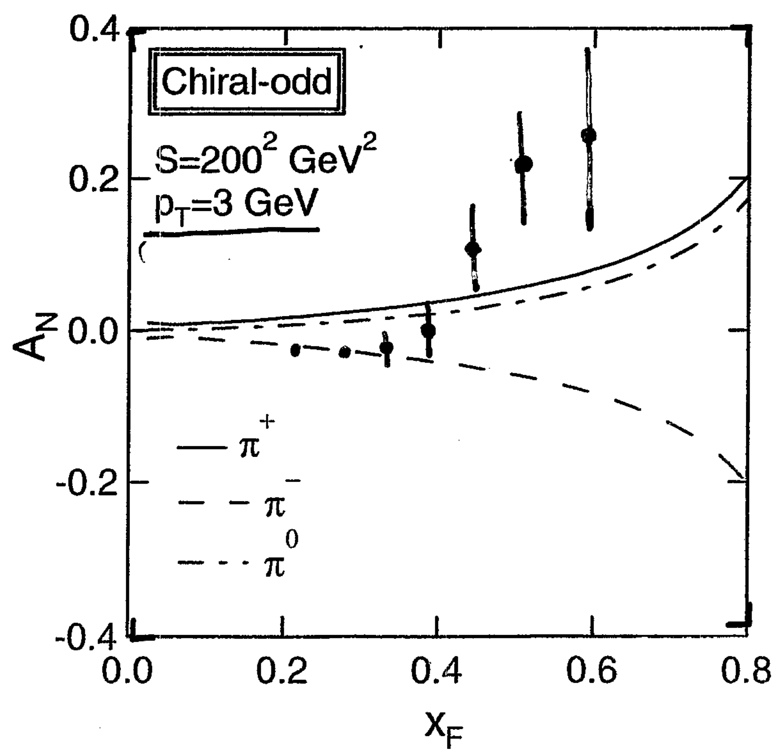


S=20^2 GeV^2
x_F=0.5
Chiral-even
A_N
P_T (GeV)
π^+
π^-
π^0

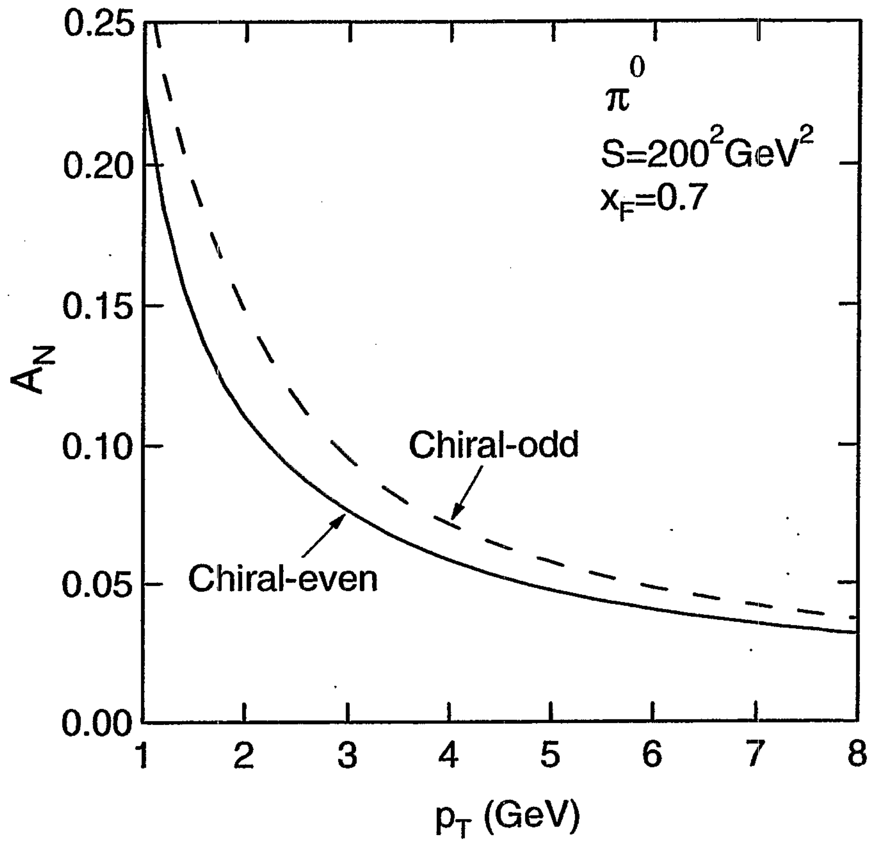
STAR π^0



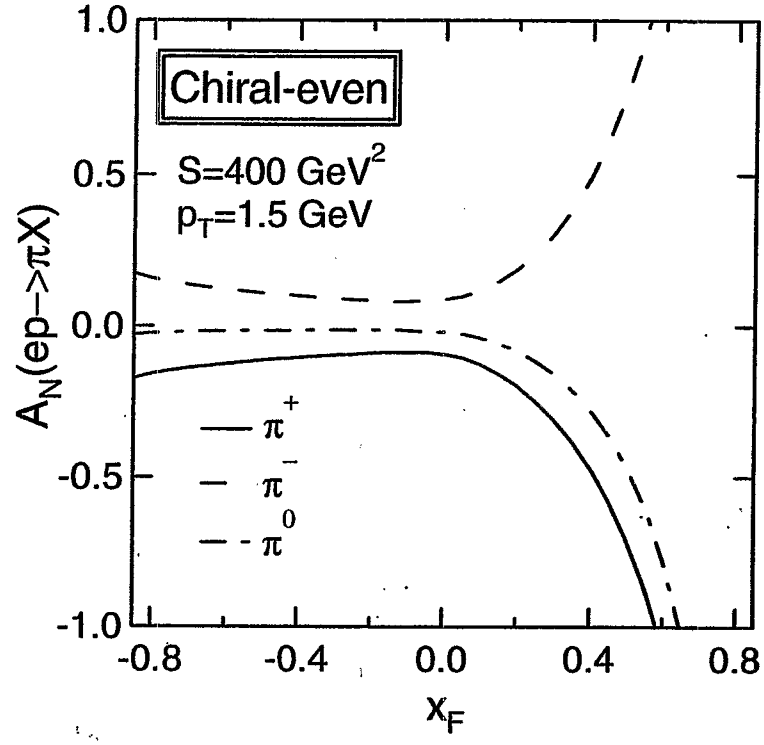
Chiral-even
S=200² GeV²
p_T=3 GeV
A_N
x_F
π⁺
π⁻
π⁰
0.4
0.2
0.0
-0.2
-0.4
0.0
0.2
0.4
0.6
0.8

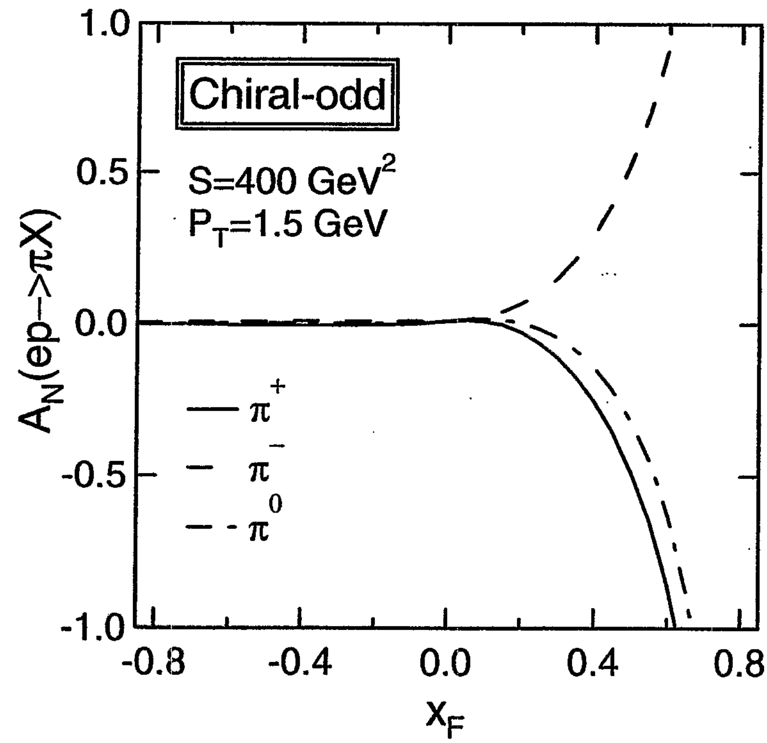


Chiral-odd
S=200² GeV²
p_T=3 GeV
A_N
x_F
π⁺
π⁻
π⁰
0.4
0.2
0.0
-0.2
-0.4
0.0
0.2
0.4
0.6
0.8



p_T-dependence large in $p_T \lesssim 4$ GeV.





· Week S dependence.

Comparison between PP and eP collisions

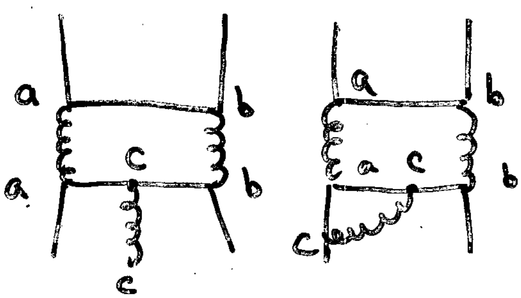
★ Signs of A_N^{ep} are opposite to A_N^{pp}.

At large x_F ; $|A_N^{ep}| \gg |A_N^{pp}|$.

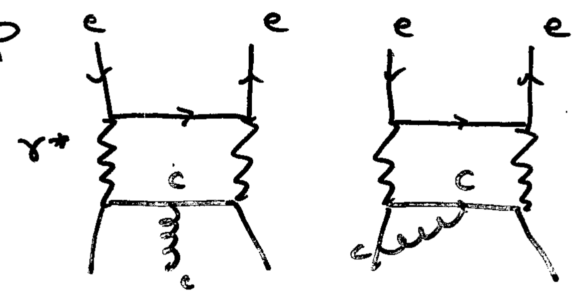
This is due to the color factor in polarized cross sectio

Main diagrams at large x_F :

PP



ep



$Tr[t^b t^c t^a t^c] = \underbrace{\frac{-1}{2N_c}}\, Tr[t^b t^a]$

Suppression & Sign change!

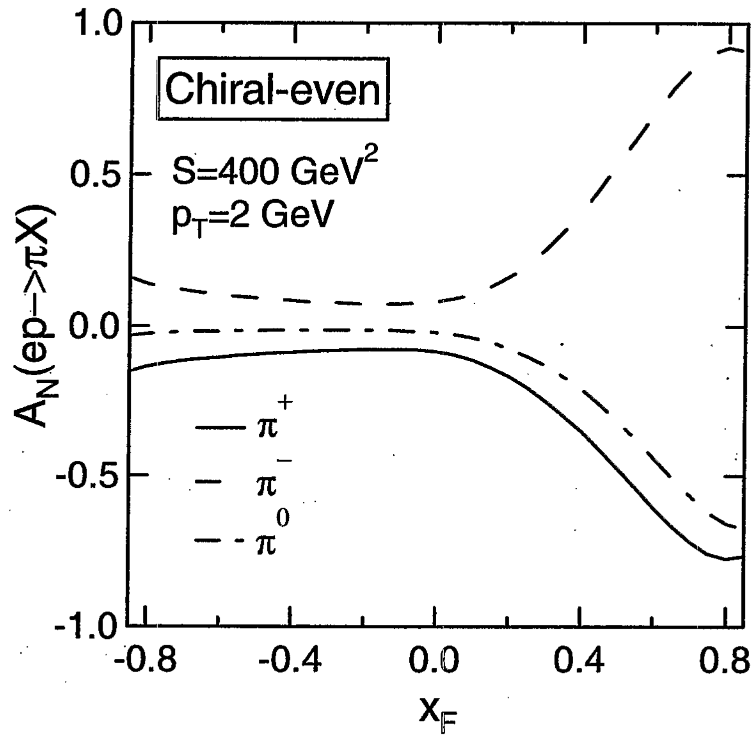
NO difference from unpolarized case!

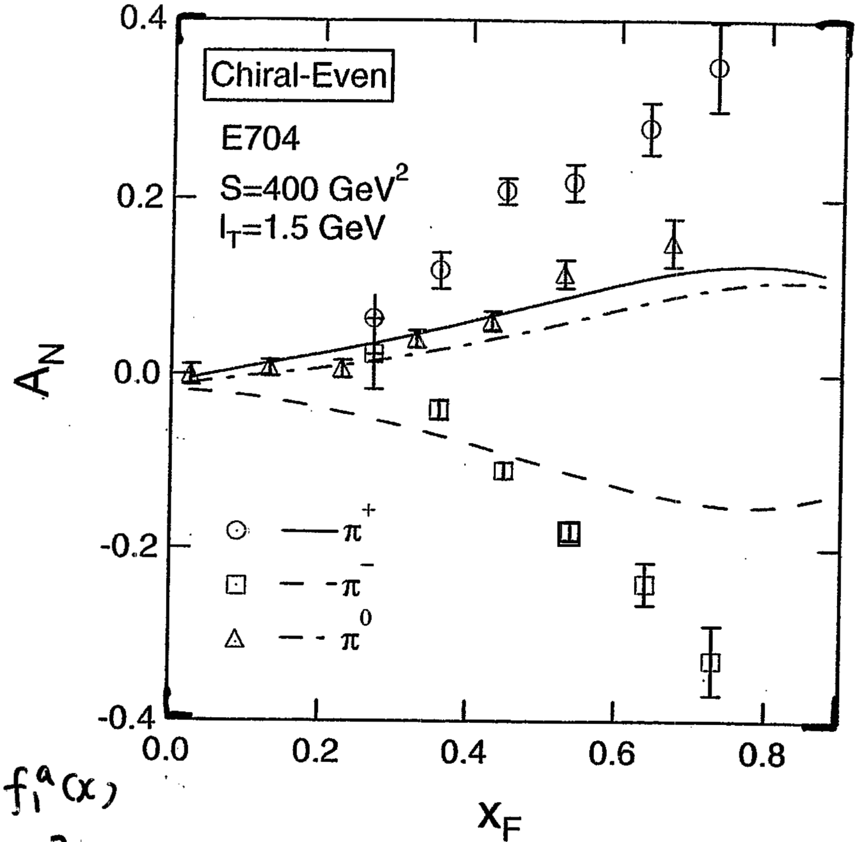
★ If we determine $G_F(xx)$ and $\hat{E}_F(zz)$ to reproduce E704 it gives $|A_N^{ep}| > 1$!

$\Rightarrow$

① Applied kinematic region is not good.

② Our model for the twist-3 functions not good. $\frac{1}{1-x_F}$ behavior has to be fixed.

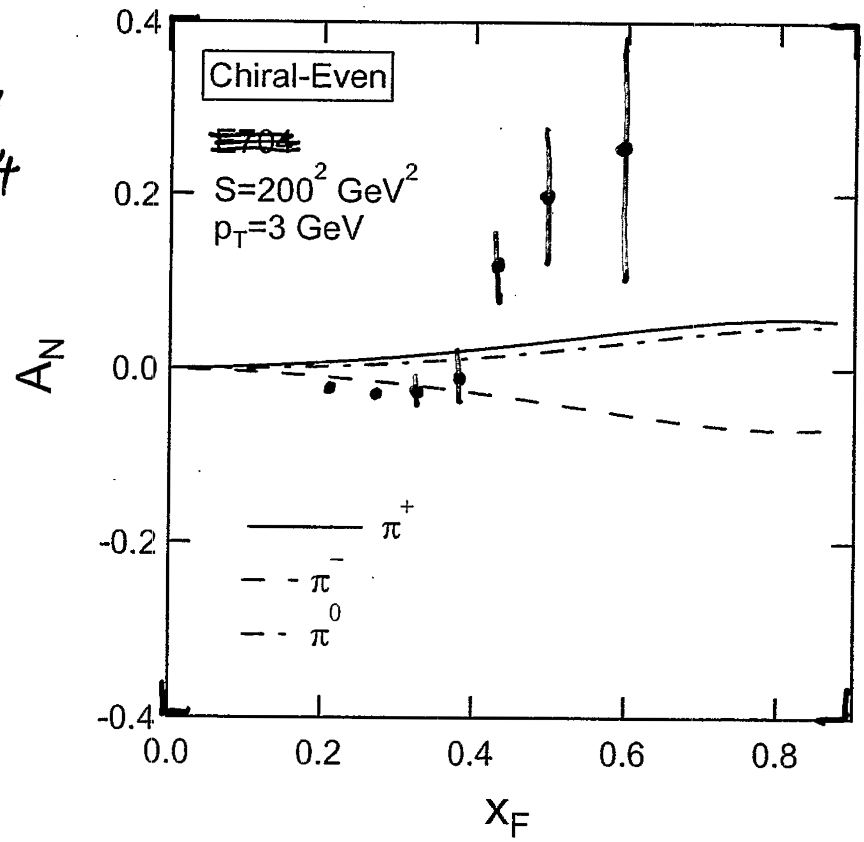

Chiral-even
S=400 GeV^2
p_T=2 GeV
$A_N(ep\to\pi X)$
x_F
π^+
π^-
π^0
1.0
0.5
0.0
-0.5
-1.0
-0.8
-0.4
0.0
0.4
0.8



$$G_F^a(x,x) = K^a f_1^a(x) \overset{x\to 1}{\sim} (1-x)^{\beta_a}$$

$$\beta_a \to \beta_a + x^3$$

$$\begin{cases} \beta_u = 3.027 \\ \beta_d = 3.774 \end{cases}$$



Summary

(1) A_N for $p^\uparrow p \to \pi X$ and $e p^\uparrow \to \pi X$ is studied within QCD factorization theorem. All twist-3 contributions are identified and the formulas are given.

(2) $p^\uparrow p \to \pi X$: Valence-quark soft-gluon approximation keeping only derivatives of the twist-3 functions is adopted. A model :

$$G_F(x\,x) \sim f_1(x), \quad \hat{E}_F(z\,z) \sim \hat{f}_1(z)$$ is used to see qualitative behavior of A_N.

→ $h_1(x) \otimes \frac{d}{dz}\hat{E}_F(z\,z)$ can be an equally good source of A_N as $\frac{d}{dx} G_F(x\,x)$

(3) $e p^\uparrow \to \pi X$: The formula include all the soft-gluon poles. A model determined by $p^\uparrow p \to \pi X$ is tested.

→ Signs of A_N^{ep} are opposite to A_N^{pp}, and $|A_N^{ep}| \gg |A_N^{pp}|$ at large x_F due to color factor.

$|A_N^{ep}| > 1$ at large x_F

→ Applied kinematic region of the formula and the model for the twist-3 functions should be reconsidered.

★ Origin of A_N in $p^\uparrow p \to \pi(l) X$

	l_T : large ($\sim \sqrt{s}$) Twist-3 "T-even"	l_T : small ($\ll \sqrt{s}$) Intrinsic $\vec{k}_\perp$ "T-odd"
Distribution chiral-even	$\frac{d}{dx} G_F(x, x)$ (Qiu-Sterman)	$f_{1T}^\perp(x, \vec{k}_T)$ (Sivers)
fragmentation chiral-odd	$\frac{d}{dz} \hat{E}_F(z, z)$	$H_1^\perp(z, \vec{k}_\perp)$ (Collins)