

Determination of the Polarization
of the Decay Positrons
in Polarized Muon Decay

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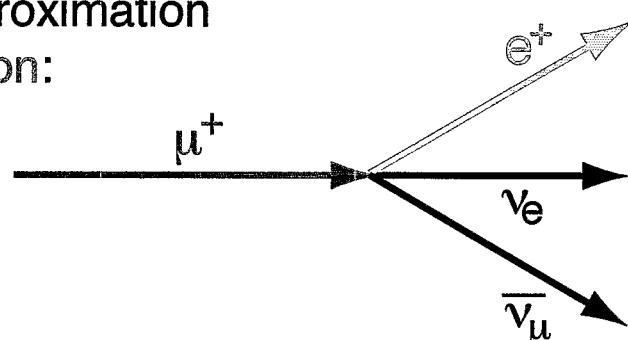
Contents

- Motivation and Theoretical Background
- Experimental Setup and Principles of Measurements
- Preliminary Results
- Methods for further Data-Analysis (Monte-Carlo based)
- Conclusion and Outlook

Theory of Muon-Decay

In the limit of small momentum transfer $m_W \gg q^2$:

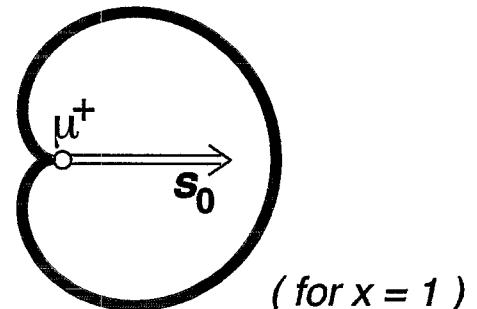
μ -decay described using approximation
of four fermion point interaction:



Decay Asymmetry:

polarized positive muons emit the
decay positron preferably in the
direction of the muon spin

$$\frac{d\Gamma}{d \cos\theta} \sim 1 + 1/3 P_\mu \cos\theta$$



$x = \frac{E_{e^+}}{E_{\max}}$ is the reduced positron energy

$$E_{\max} = \frac{m_e^2 + m_\mu^2}{2 m_\mu}$$

$$x_0 \leq x \leq 1$$

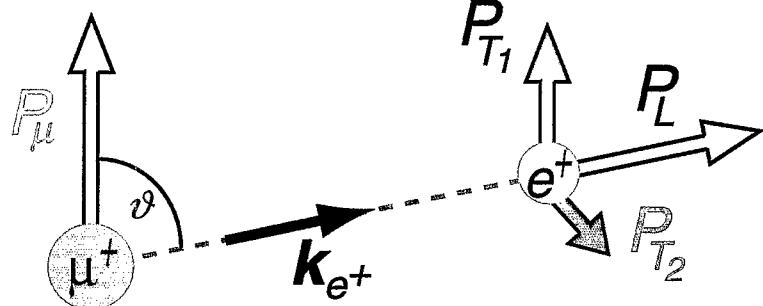
$$x_0 = \frac{m_e}{E_{\max}}$$

Observables in the Decay of Polarized Muons: Polarization Components of the Positrons

$$P_{T_1} = f_1(E, \vartheta, \eta, \eta''),$$

$$P_{T_2} = f_2(E, \vartheta, \frac{\alpha'}{A}, \frac{\beta'}{A}),$$

where $\eta, \eta'', \frac{\alpha'}{A}$ and $\frac{\beta'}{A}$
are Michel Parameters.



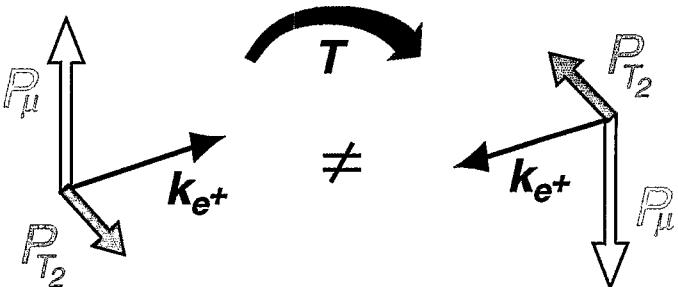
For $\vartheta = \pi/2$:

$$P_{T_1}(x) = \frac{x_0(1-x)}{2x^2 - 3x + x_0^2} - \frac{1}{2} \cdot \frac{\eta \cdot 3x + \eta''(3x-4)}{3-2x}$$

$$P_{T_2}(x) = 0 + 2 \frac{3\frac{\alpha'}{A}(1-x) + 2\frac{\beta'}{A}}{3-2x}$$

SM predictions: $\eta = \eta'' = \frac{\alpha'}{A} = \frac{\beta'}{A} = 0$
 $\Rightarrow \langle P_{T_1} \rangle_E = 0.003, P_{T_2} \equiv 0$

A non-zero P_{T_2} violates
time reversal invariance:



Matrix element for Muon Decay

$$\mathcal{M} = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T \\ \varepsilon,\mu=R,L}} g_{\varepsilon\mu}^{\gamma} \langle \bar{e}_{\varepsilon} | \Gamma^{\gamma} | (\nu_e)_n \rangle \langle \bar{\nu}_m | \Gamma_{\gamma} | (\mu)_{\mu} \rangle$$

γ labels the type of interaction:

Γ^S	= 4-scalar
Γ^V	= 4-vector
Γ^T	= 4-tensor

ε, μ indicate the chiralities of the spinors of the observed (charged) leptons.

n, m indicate the chiralities of the neutrinos, which are uniquely determined for given γ, ε and μ .

Measuring P_{T_1} yields the low energy parameter η *without* the suppression factor m_e/m_{μ} :

$$\eta = \frac{1}{2} \text{Re} \left\{ g_{LL}^V g_{RR}^{S*} + g_{RR}^V g_{LL}^{S*} + g_{LR}^V (g_{RL}^{S*} + g_{RL}^{T*}) + g_{RL}^V (g_{LR}^{S*} + g_{LR}^{T*}) \right\}$$

In the Standard Model:

$$g_{LL}^V = 1$$

$$g_{\varepsilon\mu}^{\gamma} = 0 \quad (\text{all other interactions})$$

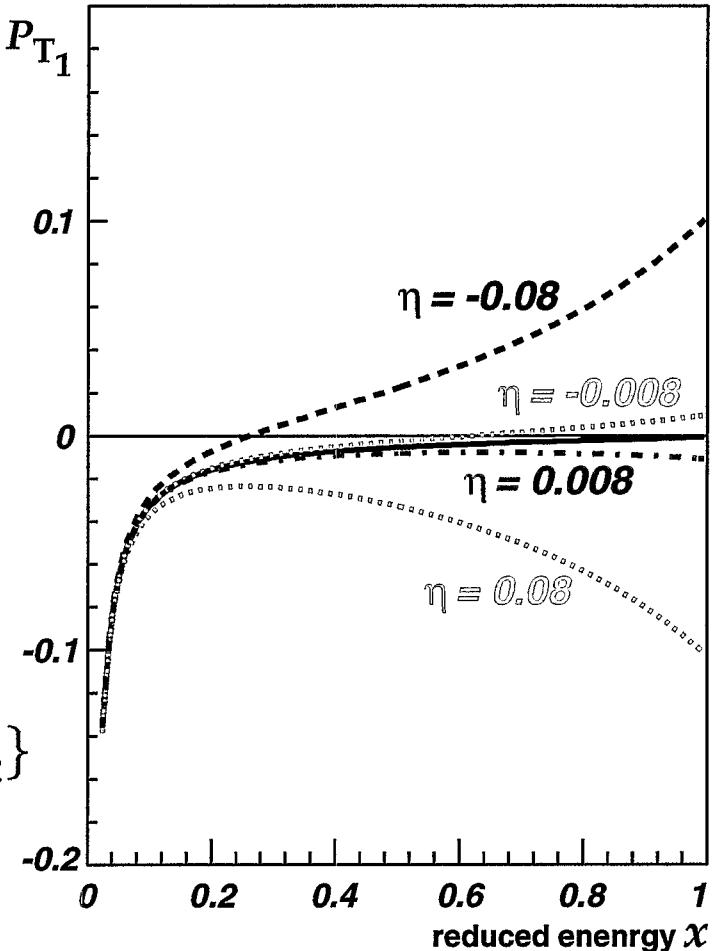
Beyond the Standard Model

If one assumes that there is one additional coupling, a non-standard result for P_T would correspond to a non-zero g_{RR}^S .

Assuming $g_{RR}^S \neq 0$ and knowing that $g_{LL}^V \approx 1$, one deduces:

$$P_{T_1}(E_e) \rightarrow \eta \approx \frac{1}{2} \operatorname{Re}\{g_{RR}^S\}$$

$$P_{T_2}(E_e) \rightarrow \frac{\beta'}{A} \approx \frac{1}{4} \operatorname{Im}\{g_{RR}^S\}$$



current limits

$$\langle P_{T_1} \rangle = 0.016 \pm 0.021^1)$$

experimental aim

$$\Delta \langle P_{T_1} \rangle = 0.003$$

$$\langle P_{T_2} \rangle = 0.007 \pm 0.022^1)$$

$$\Delta \langle P_{T_2} \rangle = 0.003$$

$$\langle P_L \rangle = 1.00 \pm 0.04^2)$$

1): H. Burkhard et al., 1985, 2): average for several experiments

Main Scientific Interests, Motivation for the Experiment at PSI

1. search for new couplings beyond $V - A$:

assuming the presence of one additional coupling, the coupling constant g_{RR}^S can be determined from the Michel parameters η and $\frac{\beta}{A}$

2. violation of time reversal invariance :

a non-zero P_{T_2} violates time reversal invariance !

3. model independent determination of the Fermi coupling constant G_F :

$$G_F = \sqrt{192 \pi^3 \frac{1}{\tau_\mu m_\mu^5} \left(1 - 4\eta \frac{m_e}{m_\mu}\right)}$$

contributions to $\left(\frac{\Delta G_F}{G_F}\right)^2$: $\Delta\tau_\mu : 8 \cdot 10^{-11}$
 $\Delta m_\mu : 6 \cdot 10^{-13}$
 $\Delta\eta : 1,6 \cdot 10^{-8}$

η can be determined via measurement of P_{T_1}

→ measure P_{T_1} and P_{T_2}

+ measure P_L as a check of consistency
and to complete the polarization vector

Experimental Methods for measuring all 3 Components of the Positrons' Polarization Vector

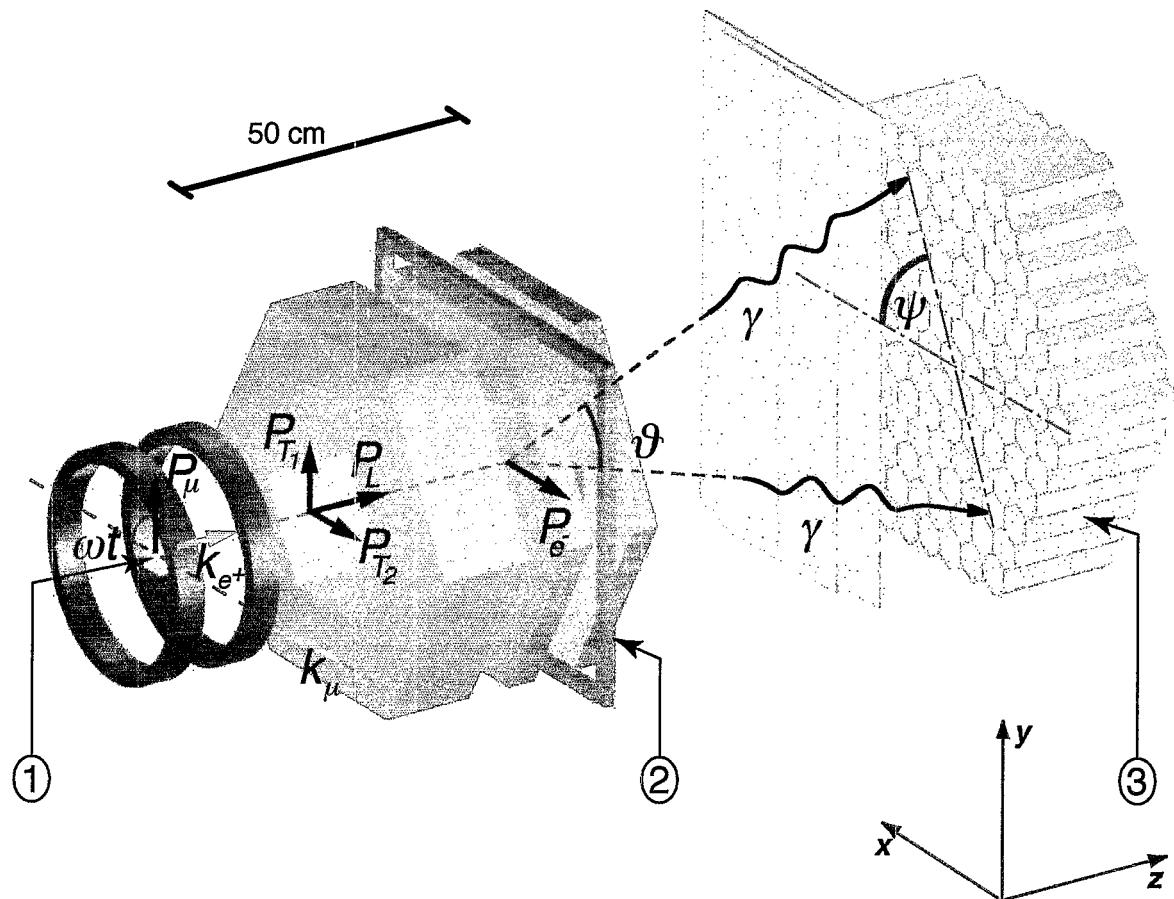
$$\mathbf{P}_{e^+} = \begin{pmatrix} P_{T_1} \\ P_{T_2} \\ P_L \end{pmatrix} \equiv \begin{pmatrix} P_T \cdot \cos \varphi \\ P_T \cdot \sin \varphi \\ P_L \end{pmatrix}$$

with 3 simultaneous and independent measurements:

Observable	Method
P_T	Time dependence of annihilation
φ	Remnant μ SR ¹⁾ effect
P_L	Spatial dependence of annihilation

1) : μ SR = “Muon Spin Rotation”

Setup of the Experiment and Principle of Measurement

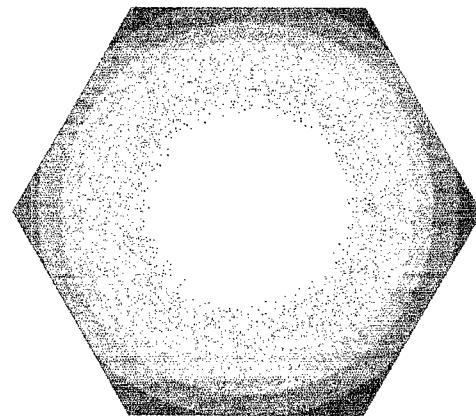


- ① : Beryllium stop target within spin precession magnet
- ② : magnetized Vacoflux foil within iron return yoke
- ③ : calorimeter consisting of 127 BGO crystals

Measurement of $|P_T|$

using the Dependence of the Annihilation Cross-Section
on the Relative Orientation of Spins

simulations of photon intensity
distributions on BGO - wall :

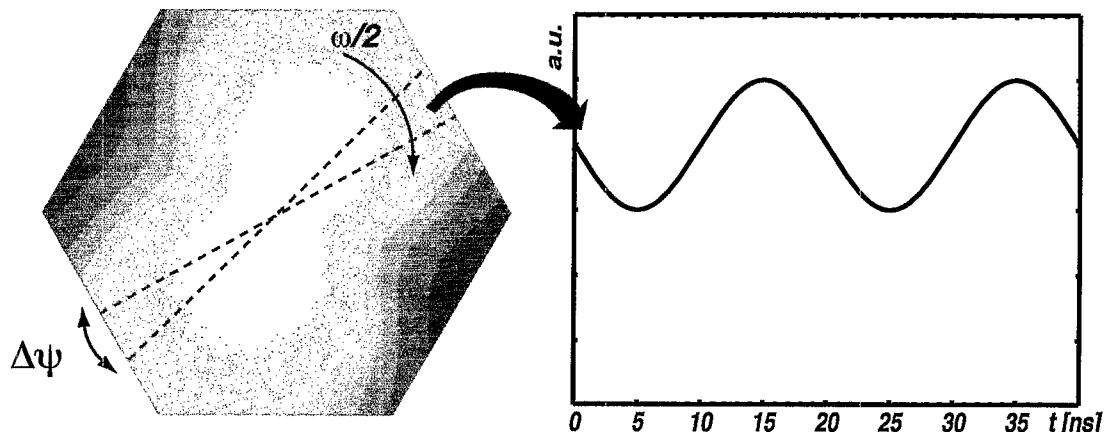


Assumptions:

$P_T @ 45^\circ$, $E_{\gamma_1} = E_{\gamma_2} = 10 \text{ MeV}$,

e^- -polarisation in magn. foil = 100%

$$|P_T| = 0$$



$$|P_T| = 1$$

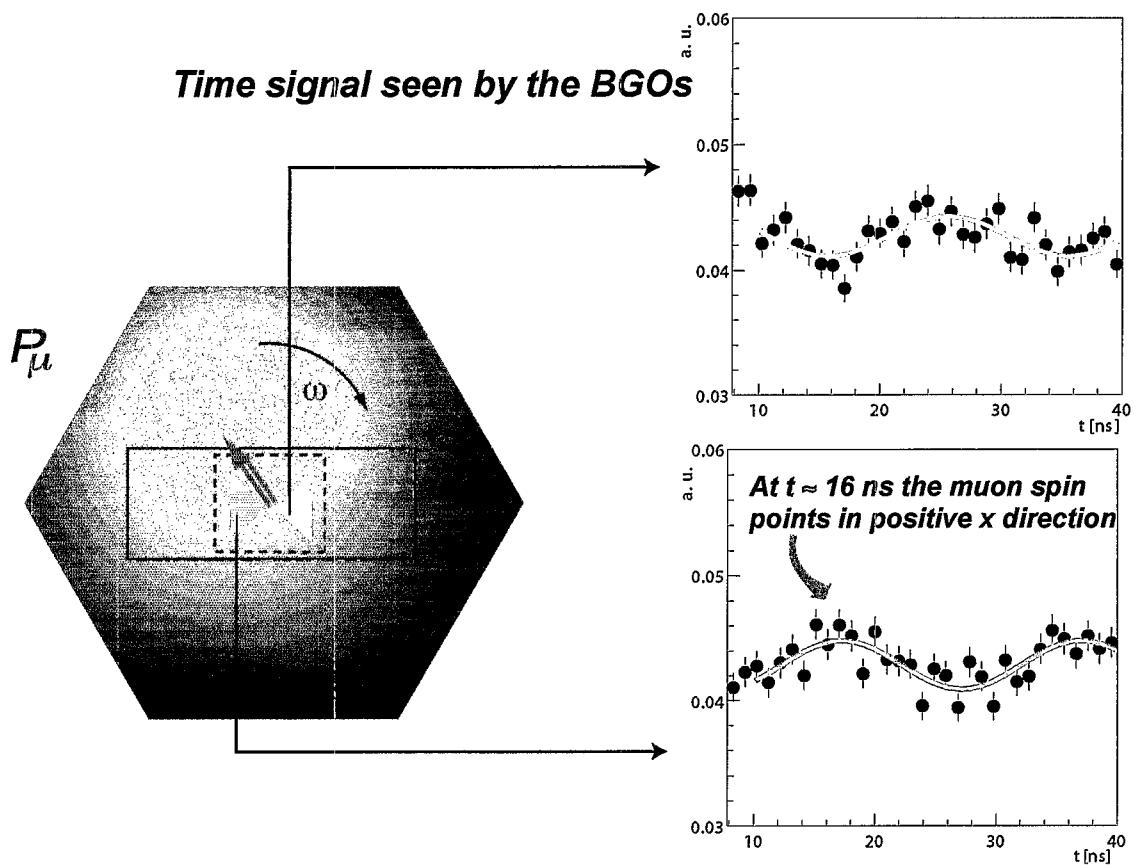
time spectrum for a given $\Delta\Psi$
 \mathcal{A} and α are functions of P_{T_1} and P_{T_2}

ω : frequency of muon spin precession

t : time between stop of muon and its decay

Determination of the Phase φ of P_T

μ SR Effect is used to find the direction of the muon spin



results from 1999 data : $t_0 = 15.717 \pm 0.517$ ns

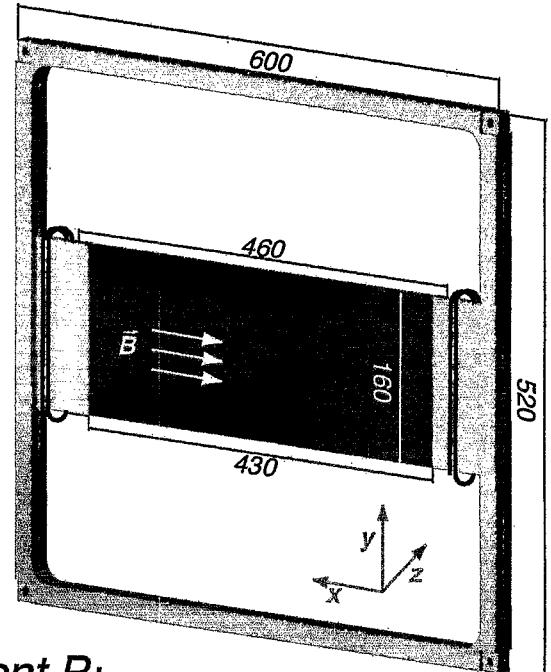
→ determination of "time zero", resp. the Phase φ of the transverse polarization

Measurement of the Longitudinal Polarization

using information about position
on magnetized Vacoflux foil
(determined by tracks reconstructed
from drift-chamber data)
where annihilations take place

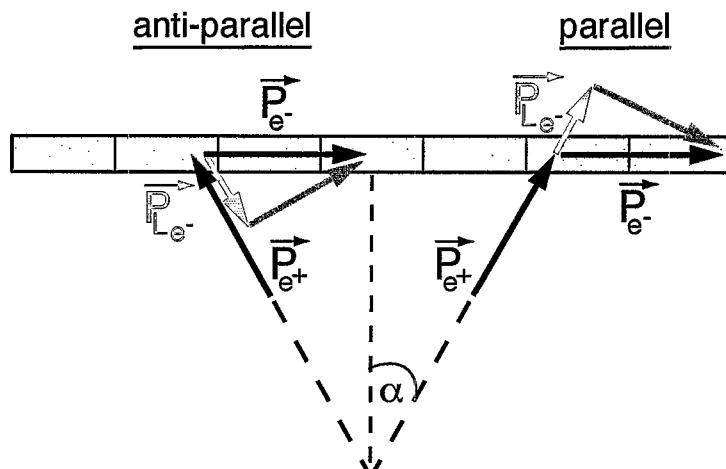
area on foil taken into account: 140^2 mm^2

area divided into rectangular bins (ij),
17 bins in x- and y-direction, respectively



*Tracks that do not hit the center
of the foil 'see' a longitudinal component $P_{L_e^-}$
of the polarization of the electrons in the foil.*

*This $P_{L_e^-}$ can either be parallel or anti-parallel
to the positron polarization :*



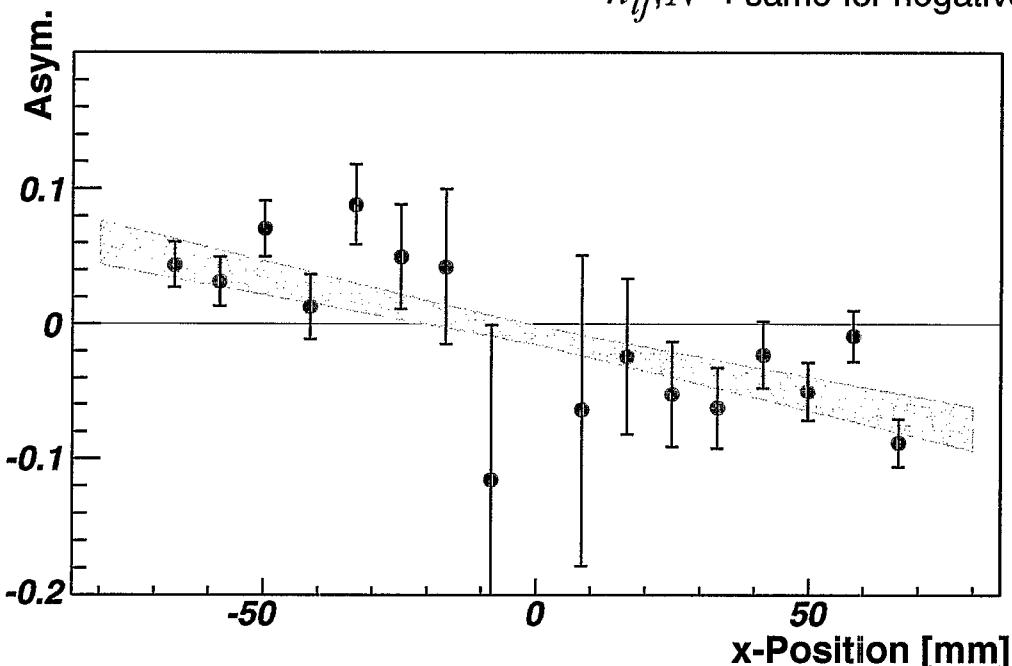
Measurement of the Longitudinal Polarization

The annihilation cross section depends on the relative orientation of the spins of positrons and electrons in the foil. It is larger if both spins are anti-parallel.



$$\text{Asymmetry: } A_{ij} = \frac{\frac{n_{ij}^-}{N^-} - \frac{n_{ij}^+}{N^+}}{\frac{n_{ij}^-}{N^-} + \frac{n_{ij}^+}{N^+}}$$

where n_{ij}^+ : number of annihilations in bin ij
for positive foil polarization
 N^+ : total number of annihilations
for positive polarization
 n_{ij}^-, N^- : same for negative polarization



- angle α
- elektron polarization in foil ($P_{e^-} = 7.2\%$)
- analysing power of 0.79
- background factor of 0.75
(backgr. ratio 25 %, mainly due to bremsstrahlung)

$$\langle P_L \rangle_E$$

The following Results are based on the Data from the Datataking-Run in Fall 1999

Rates [s⁻¹] :

μ^+ stopped in target :	20×10^6
e^+ on magnetized foil : (startcounters)	175×10^3
startcounters but not vetocounters :	37×10^3
events triggered as annihilations :	260
(T \wedge A \wedge Σ BGO \wedge CRU)	

about 18 days of datataking ...

Number of Events :

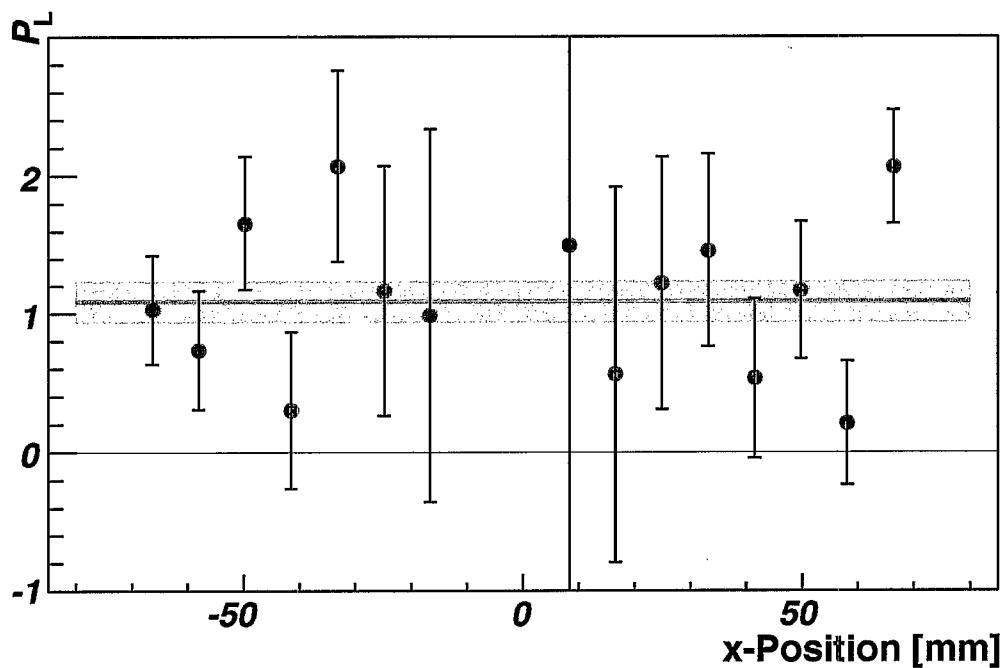
total no. of raw events recorded as annihilations :	240×10^6
after all cuts :	27×10^6
(reconstructed track from target to foil, hit-info. of driftchambers to locate annihilation, kinematic consistency for annihilation events to exclude background)	
used events :	11×10^6
(technical problem with a TDC, exclusion of run periods with changes in setup parameters)	

Results: Longitudinal Polarization P_L

Asymmetry A_{ij}



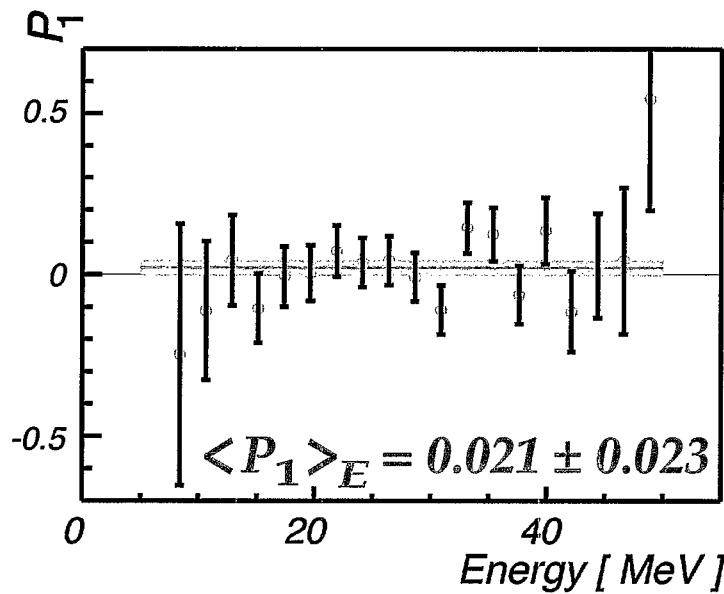
- angle α
- elektron polarization P_{e^-}
- analysing power
- background factor



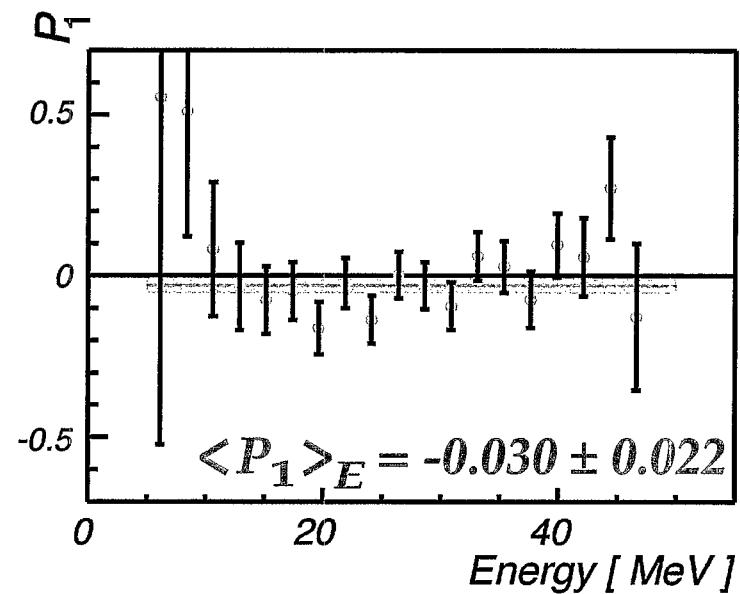
$$\langle P_L \rangle_E = 1.09 \pm 0.15$$

Results: P_T at the Time of Annihilation

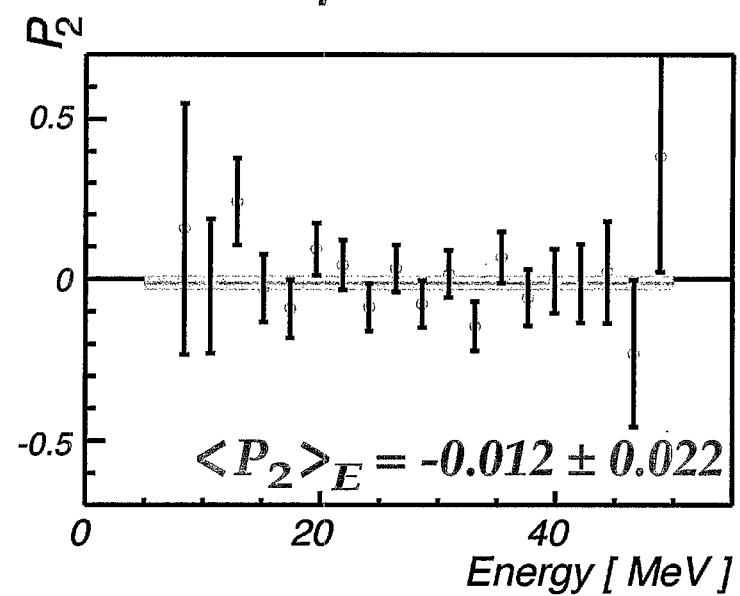
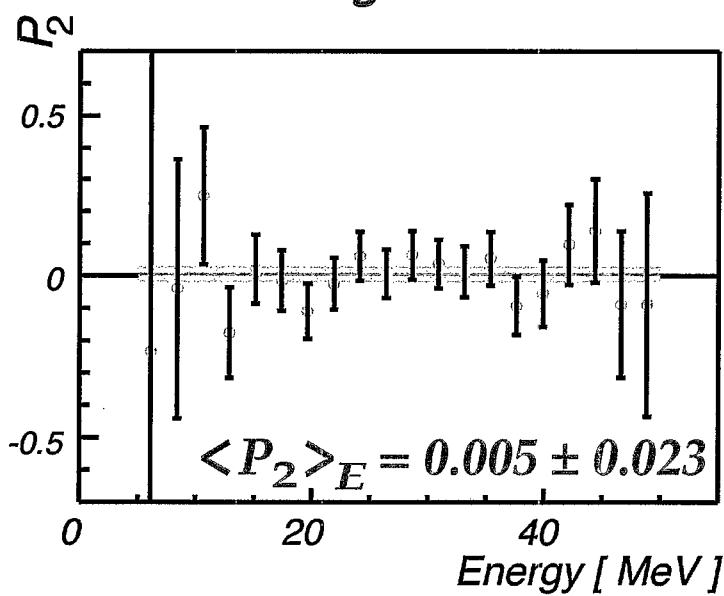
Log Likelihood parameter estimation leads to the following results for the two different orientations of electron polarization in the magnetized foil:



negative



positive



Next Steps in Data-Analysis:

How to get from P_1, P_2 and φ to P_{T_1}, P_{T_2} ?

P_1, P_2 : transverse polarization components
at the time of annihilation
 in the global coordinate system

P_{T_1}, P_{T_2} : transverse polarization components
at the time of muon decay
 relative to \mathbf{P}_μ and \mathbf{k}_{e^+}

Differential decay probability for an e^+
 emitted at an angle of $\vartheta = \pi/2$ with respect to \mathbf{P}_μ :

$$\frac{d^2\Gamma}{dx} \sim G_F \cdot \sqrt{x_2 - x_0^2} \cdot F_{IS}(x) \cdot (1 + \mathbf{P}_e \cdot \hat{\zeta}),$$

where $x = E_{e^+} / E_{\max}$, $\mathbf{P}_e = P_{T_1} \cdot \hat{x} + P_{T_2} \cdot \hat{y} + P_L \cdot \hat{z}$
 ($\hat{x}, \hat{y}, \hat{z}$: muon rest system; $\mathbf{P}_\mu \parallel \hat{x}$)

Neglecting x_0 (electron mass), the transverse polarization components become

$$P_{T_1}(x) \approx \frac{\eta \cdot 3x + \eta''(3x - 4)}{6 - 4x}$$

$$P_{T_2}(x) \approx \frac{\frac{3}{4} \frac{\alpha'}{A} (1 - x) + \frac{1}{2} \frac{\beta'}{A}}{6 - 4x}$$

These distributions are linear in η , η'' , $\frac{\alpha'}{A}$ and $\frac{\beta'}{A}$!

Next Steps in Data-Analysis

In Monte-Carlo simulations, we generate positrons having polarization distributions $Q_i(x)$, $i = 1..4$

For each $i \neq 0$ only one of the four Michel Parameters is $\neq 0$:

- Q_1 : $\eta_0 = 0.4$, all others are $= 0$
- Q_2 : $\eta_0'' = 0.4$, all others are $= 0$
- Q_3 : $\alpha'_0/A = 0.25$, all others are $= 0$
- Q_4 : $\beta'_0/A = 0.125$, all others are $= 0$

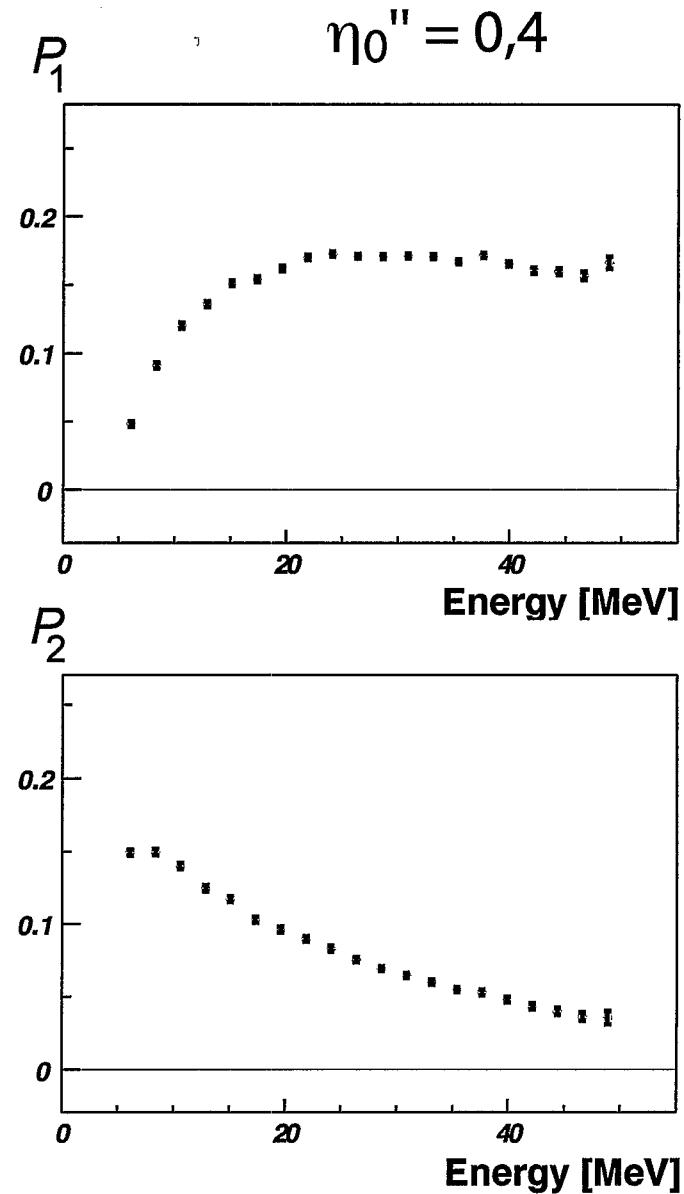
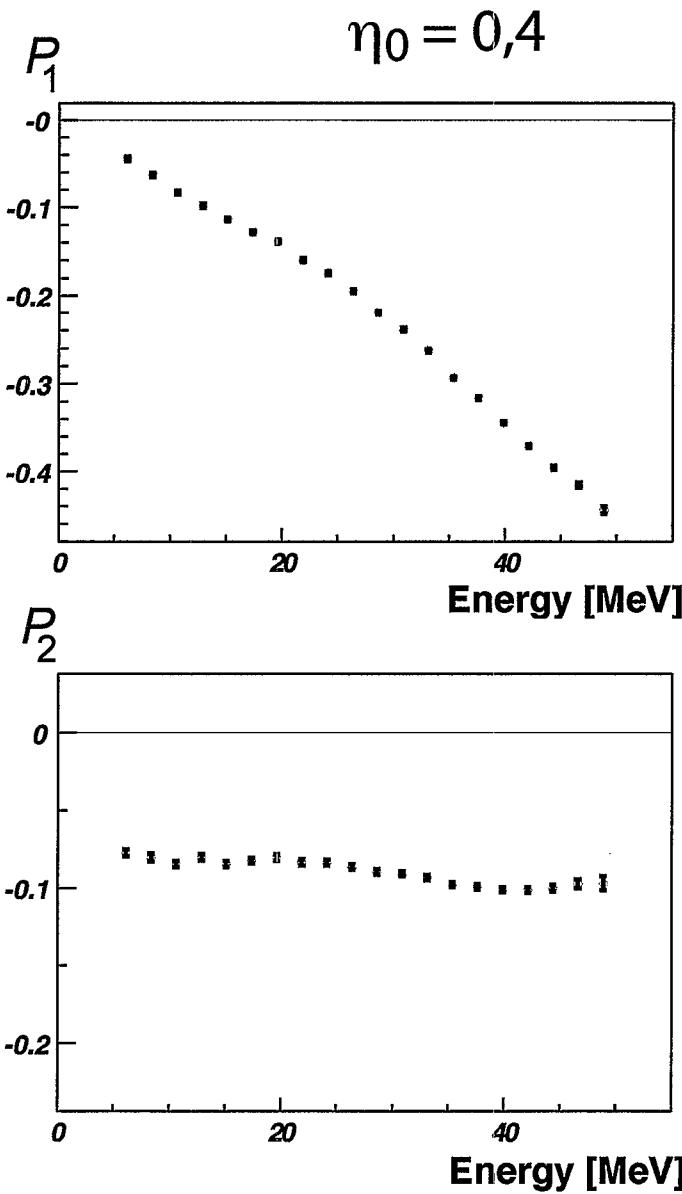
→ The tracking of the positrons and dealing with their polarization in the Monte-Carlo-simulations leads to distributions $V_i(x)$ for the polarization at the time of annihilation.

⇒ The distribution $V(x)$ as measured is fitted using the $V_i(x)$:

$$V(x) = \sum_{i=0}^4 \alpha_i V_i$$

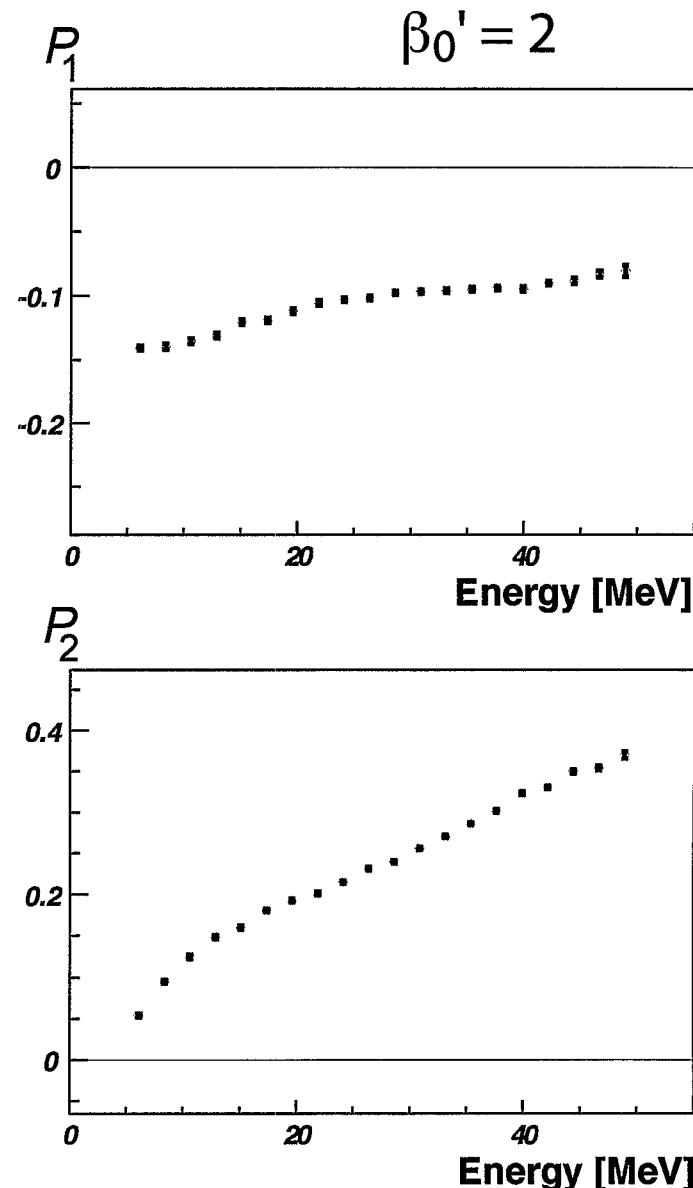
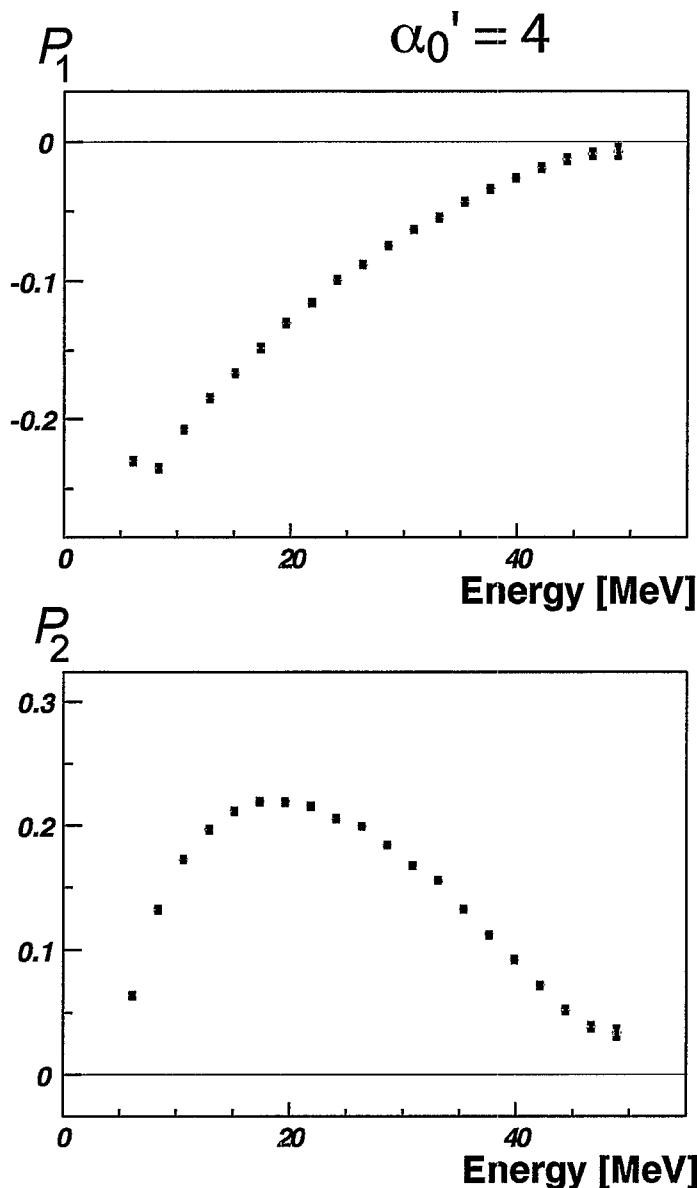
The resulting values for the α_i yield the values for the Michel Parameters as determined by means of our data-taking.

Simulated Polarization Distributions Q_i



For each Q_i : 200 times 600 000 generated events
 → about $2,4 \cdot 10^6$ events on FeCo foil

Simulated Polarization Distributions Q_i



Test of the Method

χ^2 method to fit the simulated distributions to the measured one

Test of the method and the fit:

- simulating polarization distributions (“pseudo data”) for which the four Michel parameters take general (small, “unknown”) values
- fitting the simulated distributions to the pseudo data and see if the Michel parameters can be correctly deduced

For example, the following distribution was simulated for this purpose:

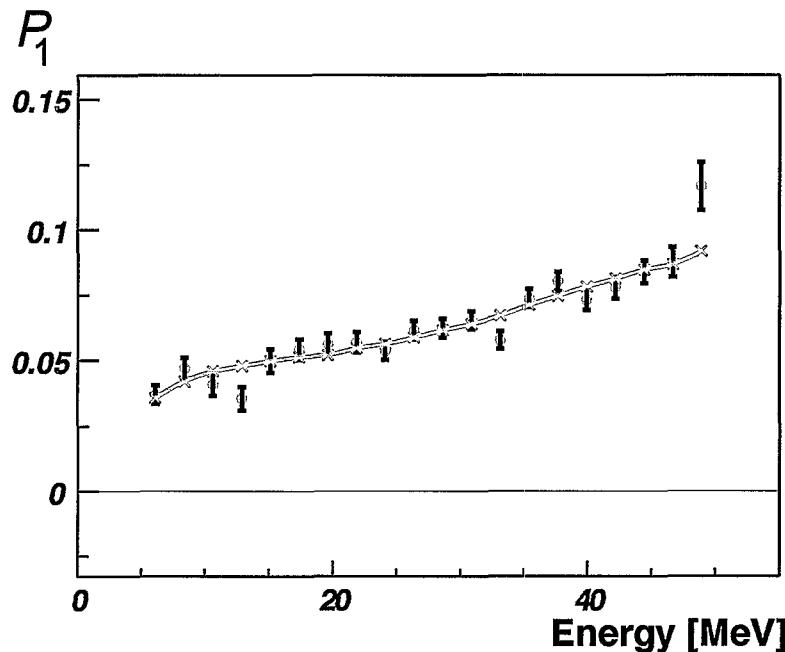
60 times $5 \cdot 10^5$ events generated ($\rightarrow \approx 6 \cdot 10^5 e^+$ on the foil)

$$\eta = -57,0 \cdot 10^{-3}, \quad \eta'' = 36,0 \cdot 10^{-3},$$

$$\alpha'/A = -13,1 \cdot 10^{-3}, \quad \beta'/A = -11,9 \cdot 10^{-3}$$

Test of the Method

given: $\eta = -57,0 \cdot 10^{-3}$, $\eta'' = 36,0 \cdot 10^{-3}$
 $\alpha'/A = -13,1 \cdot 10^{-3}$, $\beta'/A = -11,9 \cdot 10^{-3}$



Fit results:

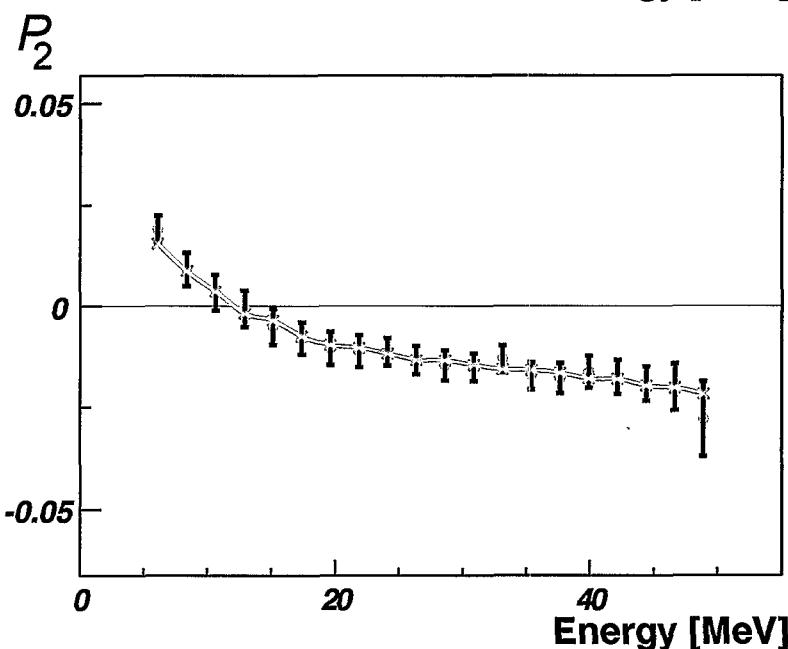
$$\eta = (-63 \pm 5) \cdot 10^{-3}$$

$$\eta'' = (32 \pm 7) \cdot 10^{-3}$$

$$\alpha'/A = (-11 \pm 3) \cdot 10^{-3}$$

$$\beta'/A = (-13 \pm 1) \cdot 10^{-3}$$

The χ^2 of this fit is 18,6



Conclusion and Outlook

- Successful measurement of P_L demonstrates that the apparatus is sensitive to polarization effects
- Determination of the Transverse Polarization P_T at the time of annihilation based on the data taken in 1999; statistical errors in the range of the previous exp. limits
- A method to deduce the actual components P_{T1} and P_{T2} is being developed and has been cross-checked
 - first measurement of all three positron polarization components
 - we will soon be able to set new limits on Michel parameters η , η'' , α'/A , β'/A , and on additional couplings beyond the Standard Model

up to now, the transverse polarization is within the errors compatible with zero → no hints for physics beyond the SM

- More Data was taken in Nov. 2000 :
 - 29 days of datataking,
 - ≈ 3 times higher event rate than in 1999
 - 1.37×10^9 raw annihilation events recorded,
 - ≈ 13 times more "good" annihilations than in 1999
- reduction of $\Delta \langle P_T \rangle$ to the order of 3×10^{-3}