The Relativistic Stern-Gerlach Interaction and Quantum Mechanics Implications

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The impulse of a transverse magnetic gradient generated by the quadrupolar field

$$\vec{B} = \begin{cases} Gx \\ Gy \\ 0 \end{cases}$$

acting on a particle with its spin parallel to $\hat{y}$ and moving along the z-axis, is

$$\delta p_x = \mu^* G \tau = \mu^* G \frac{\ell_Q}{\beta c}$$

(where ℓ_Q is the quadrupole length) which makes the trajectory-slopes, of particles with opposite spin states, vary by the amounts

$$\eta_\pm = \frac{\delta p_x}{p} = \pm \frac{\mu^* G \ell_Q}{\beta^2 \gamma m c^2} \quad (p = \text{particle momentum})$$

The classical transverse motion of a particle, which crosses a focusing quadrupole while circulating in a ring, coincides with the motion of the centre of its quantum-mechanical wave packet represented by the wave-function of the "corresponding state".

$$\Delta x_0 = \sqrt{\frac{\hbar}{\sqrt{mveG}}} \quad \text{and} \quad \Delta p_0 = \sqrt{\hbar\sqrt{mveG}}$$

Outside the quad region, no discrete energy levels exist. This implies a dilation of the wave-packet size:

$$\Delta x_{wp} = \Delta x_0 \sqrt{1 + \frac{\lambda^2}{(\Delta x_0)^4} \frac{z^2}{4\beta^2}} \simeq \frac{\lambda}{2\beta \Delta x_0} z$$

where z is the space covered by the particle outside the focusing quadrupole and $\lambda = h/mc$ is the Compton wave length of the particle.

The momentum uncertainty generates an angular deflection

$$x'_q = \frac{\Delta p_0}{mv}$$

which, over a length z, gives rise to a spatial increment

$$\Delta x_q = \frac{\Delta p_0}{mv} z$$

Comparing this growth to the size of the swollen wave-packet, we obtain:

$$\frac{\Delta x_q}{\Delta x_{wp}} = \frac{\Delta p_0}{mv} z \frac{2mv \Delta x_0}{\hbar z} = \frac{2\Delta p_0 \Delta x_0}{\hbar} = 2$$

Choosing 5 MeV protons ($v = 3.095 \times 10^7\ ms^{-1}$ or $\beta = 0.103$) and setting $G = 10\ Tm^{-1}$, we obtain

$$\Delta x_0 = 1.93 \times 10^{-8}\ m$$
$$\Delta x_{wp} = 5.37 \times 10^{-7}\ m \quad (z = 10\ m)$$
$$\Delta x_q = 1.07 \times 10^{-6}\ m$$
$$\delta p_x = 4.56 \times 10^{-33}\ kgms^{-1} \quad (\ell_Q = 1\ m)$$

or $\delta p_x \Delta x_q \simeq 5 \times 10^{-39}\ Js \ll \hbar \simeq 10^{-34}\ Js$, which agrees with the Bohr/Pauli arguments about the impossibility to observe the spin of a free fermion. This statement holds for a single quadrupole crossing, i.e. for a single kick.

In the case of repetitive impulses of the Stern-Gerlach force, we may find a coherent adding up, so far the wave-packet growth Δx_{wp} has the same order of magnitude of the displacement Δx_q. Therefore the small spatial increments

$$\delta x = \frac{\delta p_x}{p} z = 8.80 \times 10^{-13}\ m$$

will sum up till reaching the value of Δx_q after about 10^6 revolutions. Take note that

$$p(W = 5\ MeV) = 96.99\ MeV/c = 5.18 \times 10^{-20}\ kgms^{-1}$$

In the case of the energy gain/loss, the situation is simpler and straightforward. In fact it suffices to recall the

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$$\Delta E\, \Delta t \geq h = 6.625 \times 10^{-34}\ Js$$

In our case we have $\quad \Delta E = dU \simeq \gamma^2 2\mu^* B_0, \quad$ with

$$\mu^* = \begin{cases} 9.27 \times 10^{-24}\ JT^{-1} & (e^\pm) \\ 1.41 \times 10^{-26}\ JT^{-1} & (p, \bar{p}) \end{cases}$$

Combining these equations and assessing $B_0 = 0.1\,T$, we obtain:

$$\frac{\Delta E}{\gamma^2} = \begin{cases} 2 \times 9.27 \times 10^{-24} \times 10^{-1} = 1.85 \times 10^{-24}\ J & (e^\pm) \\ 2 \times 1.41 \times 10^{-26} \times 10^{-1} = 2.82 \times 10^{-27}\ J & (p, \bar{p}) \end{cases}$$

which, for

$$\Delta t = \frac{1}{2}\tau_{RF} \simeq \frac{1}{2}\frac{1}{(3\ GHz)} \simeq 10^{-10}\ s$$

gives rise to

$$\Delta E\, \Delta t \simeq \begin{cases} \gamma^2 1.85 \times 10^{-34}\ Js \\ \gamma^2 2.82 \times 10^{-37}\ Js \end{cases}$$

i.e. the uncertainty principle requirements are always fulfilled, provided that

$$\gamma \geq \sqrt{\frac{6.625}{1.85}} = 1.84 \quad (e^\pm)$$

and

$$\gamma \geq \sqrt{\frac{66.25}{2.82} \times 100} = 48.47 \quad (p, \bar{p})$$

i.e. for absolutely sensible values!