The Relativistic Stern-Gerlach Interaction as a Tool for Attaining the Spin Separation

P. Cameron[1], M. Conte[2], A. Luccio[1],
W.W. MacKay[1], M. Palazzi[2], and M. Pusterla[3].

(1) RHIC Project, Brookhaven National Laboratory, Upton, NY 11973, USA.
(2) Dipartimento di Fisica dell'Università di Genova, INFN Sezione di Genova, Via Dodecaneso 33, 16146 Genova, Italy.
(3) Dipartimento di Fisica dell'Università di Padova, INFN Sezione di Padova, Via Marzolo 8, 35131 Padova, Italy.

15th International Spin Physics Symposium

2002 Spin

September 9-14, 2002
Brookhaven National Laboratory
Upton NY on Long Island

THE STERN-GERLACH FORCE
ACTING ON A CHARGED PARTICLE
ENDOWED WITH A MAGNETIC
MOMENT
WHICH TRAVELS INSIDE
A TIME VARYING E.M. FIELD
MUST BE FIRST EVALUATED
IN THE PARTICLE REST FRAME
OBTAINING

$$\vec{f}'_{SG} = \nabla'(\vec{\mu}^* \cdot \vec{B}')$$

$$\vec{f}'_{SG} = \frac{\partial}{\partial x'}(\vec{\mu}^* \cdot \vec{B}')\hat{x} + \frac{\partial}{\partial y'}(\vec{\mu}^* \cdot \vec{B}')\hat{y} + \frac{\partial}{\partial z'}(\vec{\mu}^* \cdot \vec{B}')\hat{z}$$

WHERE

$$\vec{\mu}^* = g\frac{e}{2m}\vec{S}.$$

IS THE MAGNETIC MOMENT

THEN THIS FORCE
MUST BE LORENTZ BOOSTED
FROM **PRF** FRAME (x', y', z', t')
TO **LAB** FRAME (x, y, z, t)

$$\vec{f}_{SG} = \frac{1}{\gamma}\frac{\partial}{\partial x}(\vec{\mu}^* \cdot \vec{B}')\hat{x} + \frac{1}{\gamma}\frac{\partial}{\partial y}(\vec{\mu}^* \cdot \vec{B}')\hat{y} + \frac{\partial}{\partial z'}(\vec{\mu}^* \cdot \vec{B}')\hat{z}$$

LORENTZ TRANSFORMATION OF
$\vec{E}, \vec{B}$ and $\vec{E}', \vec{B}'$

$$\vec{E}' = \gamma(\vec{E} + c\vec{\beta} \times \vec{B}) - \frac{\gamma^2}{\gamma + 1}\vec{\beta}(\vec{\beta} \cdot \vec{E})$$

$$\vec{B}' = \gamma\left(\vec{B} - \frac{\vec{\beta}}{c} \times \vec{E}\right) - \frac{\gamma^2}{\gamma + 1}\vec{\beta}(\vec{\beta} \cdot \vec{B})$$

$$\vec{f}_{\mathrm{SG}} = f_x\hat{x} + f_y\hat{y} + f_z\hat{z}$$

$$\begin{aligned}
f_x &= \mu_x^*\left(\frac{\partial B_x}{\partial x} + \frac{\beta}{c}\frac{\partial E_y}{\partial x}\right) + \mu_y^*\left(\frac{\partial B_y}{\partial x} - \frac{\beta}{c}\frac{\partial E_x}{\partial x}\right) + \frac{1}{\gamma}\mu_z^*\frac{\partial B_z}{\partial x}\\
f_y &= \mu_x^*\left(\frac{\partial B_x}{\partial y} + \frac{\beta}{c}\frac{\partial E_y}{\partial y}\right) + \mu_y^*\left(\frac{\partial B_y}{\partial y} - \frac{\beta}{c}\frac{\partial E_x}{\partial y}\right) + \frac{1}{\gamma}\mu_z^*\frac{\partial B_z}{\partial y}\\
f_z &= \mu_x^* C_{zx} + \mu_y^* C_{zy} + \mu_z^* C_{zz}
\end{aligned}$$

$$\begin{aligned}
C_{zx} &= \gamma^2\left[\left(\frac{\partial B_x}{\partial z} + \frac{\beta}{c}\frac{\partial B_x}{\partial t}\right) + \frac{\beta}{c}\left(\frac{\partial E_y}{\partial z} + \frac{\beta}{c}\frac{\partial E_y}{\partial t}\right)\right]\\
C_{zy} &= \gamma^2\left[\left(\frac{\partial B_y}{\partial z} + \frac{\beta}{c}\frac{\partial B_y}{\partial t}\right) - \frac{\beta}{c}\left(\frac{\partial E_x}{\partial z} + \frac{\beta}{c}\frac{\partial E_x}{\partial t}\right)\right]\\
C_{zz} &= \gamma\left(\frac{\partial B_z}{\partial z} + \frac{\beta}{c}\frac{\partial B_z}{\partial t}\right)
\end{aligned}$$

SINCE WE ARE INTERESTED IN
THE $\vec{\mu}^* \iff \vec{B}$ INTERCATION
A **RF** CAVITY HAS TO BE
IMPLEMENTED
WITH IST AXIS
PARALLEL TO
THE MOTION DIRECTION $\hat{z}$
THEREFORE
IN ORDER TO EXPLOIT THE γ^2
FACTOR
PARTICLES MUST ENTER INTO THE
CAVITY
WITH THEIR MAGNETIC MOMENTS
PERPENDICULAR TO $\hat{z}$
AND PARALLEL TO $\hat{y}$
SINCE THIS IS THE “NATURAL”
ORIENTATION
OF BOTH SPIN-UP AND SPIN-DOWN
PARTICLES

The polarization P of a beam rotating in a ring with a field $\vec{B}_{\text{ring}}$ can be dsefined as

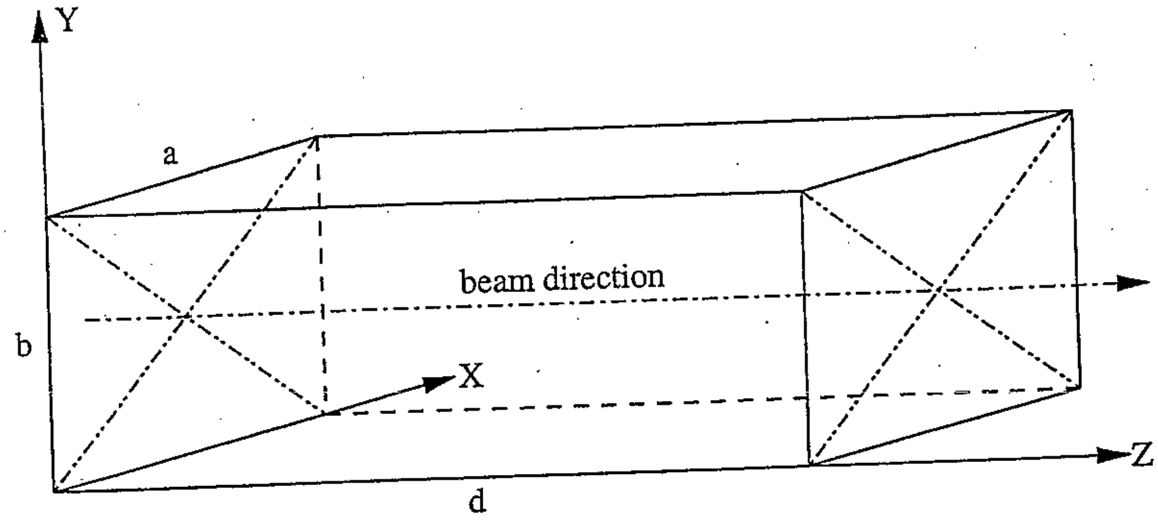
$$P = \frac{\tilde{N}_{\uparrow} - \tilde{N}_{\downarrow}}{\tilde{N}_{\uparrow} + \tilde{N}_{\downarrow}}$$

where

$$P,\ \tilde{N}_{\uparrow}, \tilde{N}_{\downarrow}$$

are the macroscopic averages
over the particle distribution in the beam
after computing
the quantum expectation values
of the observables

$N_{\uparrow}$ = No. particles - spin parallel to $\vec{B}_{\text{ring}}$
$N_{\downarrow}$ = No. part. - spin antiparallel to $\vec{B}_{\text{ring}}$


Y
a
b
beam direction
X
Z
d

TE_{011} Cavity

$$\vec{B} = \begin{cases} 0 \\ -B_o\left(\frac{b}{d}\right)\sin\left(\frac{\pi y}{b}\right)\cos\left(\frac{\pi z}{d}\right)\cos(\omega t) \\ B_o\cos\left(\frac{\pi y}{b}\right)\sin\left(\frac{\pi z}{d}\right)\cos(\omega t) \end{cases}$$

$$\vec{E} = \begin{cases} -\omega B_o\left(\frac{b}{\pi}\right)\sin\left(\frac{\pi y}{b}\right)\sin\left(\frac{\pi z}{d}\right)\sin(\omega t) \\ 0 \\ 0 \end{cases}$$

Spin-splitter

After a single cavity crossing
of two particles with opposite spin states
the following energy exchanges takes place

$$dU_{\uparrow} \simeq +2B_0\mu^*\gamma^2 \quad (\text{Spin} - \text{up particle})$$

$$dU_{\downarrow} \simeq -2B_0\mu^*\gamma^2 \quad (\text{Spin} - \text{down particle})$$

giving thus rise to an energy separation

$$dU = dU_{\uparrow} - dU_{\downarrow} \simeq 4B_0\mu^*\gamma^2$$

where

$$\mu^* = |\vec{\mu}^*| = \begin{cases} 1.41 \times 10^{-26}\ JT^{-1} & p(\bar{p}) \\ 9.28 \times 10^{-24}\ JT^{-1} & e^{\mp} \end{cases}$$

B_0 = Peak Magnetic Field in the TE_{011} Cavity

After having crossed $N_{\rm cav}$ cavities
and
completed $N_{\rm turns}$ revolutions,
particles with opposite spin states
should be gathered in couples of bunches
with a total energy separation

$$\Delta U \simeq 4 N_{\rm turns} N_{\rm cav} B_0 \mu^* \gamma^2$$

and a momentum spread

$$\frac{\Delta p}{p} = \frac{1}{\beta^2}\frac{\Delta U}{U} \simeq 4 N_{\rm turns} N_{\rm cav} \frac{B_0}{B_\infty}\gamma$$

$$N_{\rm SS} = \frac{B_\infty}{4 N_{\rm cav} B_0 \gamma}\left(\frac{\Delta p}{p}\right)_{\rm ring} \quad \text{and} \quad \Delta t = N_{\rm SS}\tau_{\rm rev}$$

$$B_\infty = \frac{mc^2}{\mu^*} = \begin{cases} 1.07 \times 10^{16}\ T & p(\bar{p}) \\ 8.82 \times 10^{9}\ T & e^{\mp} \end{cases}$$

Assessing $B_0 = 0.1\ T$ and $N_{\rm cav} = 200$,
we obtain for RHIC, HERA and LHC

	RHIC	HERA	LHC
E (GeV)	250	820	7000
γ	266.5	874.2	7462.7
$\tau_{\rm rev}(\mu s)$	12.8	21.1	88.9
Δp/p	4.1×10^{-3}	5×10^{-5}	1.05×10^{-4}
$N_{\rm SS}$	6.67×10^{9}	2.48×10^{7}	1.76×10^{6}
Δt	23.7 hr	523 s	156 s

Take note that, if we set

$B_0 = 1\ T$ {*} and $(\Delta\ p/p)_{\rm LHC} = 10^{-5}$ {**}

a factor of 100 could be gained thus reducing the LHC spin-splitting time to 1.56 seconds!

{*} Quite reasonable, since the associated $E_0 = 300\ MV/m$ is a sustainable value.
{**} Possible, perhaps, at the beam intensity expenses.

LHC Parameters at Collision

Picked up from

http://lhc-new-homepage.web.cern.ch/lhc-new-homepage/

	Values	Unit
Revolution frequency	11.2455	kHz
RF frequency	400.7	MHz
Harmonic number h	35640	
RF voltage V_{RF}	16	MV
Synchrotron period τ_s	0.042	s
Transition parameter η_{tr}	3.47×10^{-4}	
Bunch duration	0.28	ns
Bunch length	8.39	cm

Take note that
the LHC bunch length of 8.39 cm
fits very well the TE_{011} wavelength
$\lambda \simeq 10$ cm

SPURIOUS EFFECTS DUE TO THE TRANSVERSE ELECTRIC FIELD

$$\Delta U_E = -\frac{x' e\omega B_0\, bd}{\pi^2(\beta_{\rm ph}^2 - 1)} \sin\left(\beta_{\rm ph}\pi + \frac{\beta_{\rm ph}\pi}{2\gamma^2}\right) \simeq$$

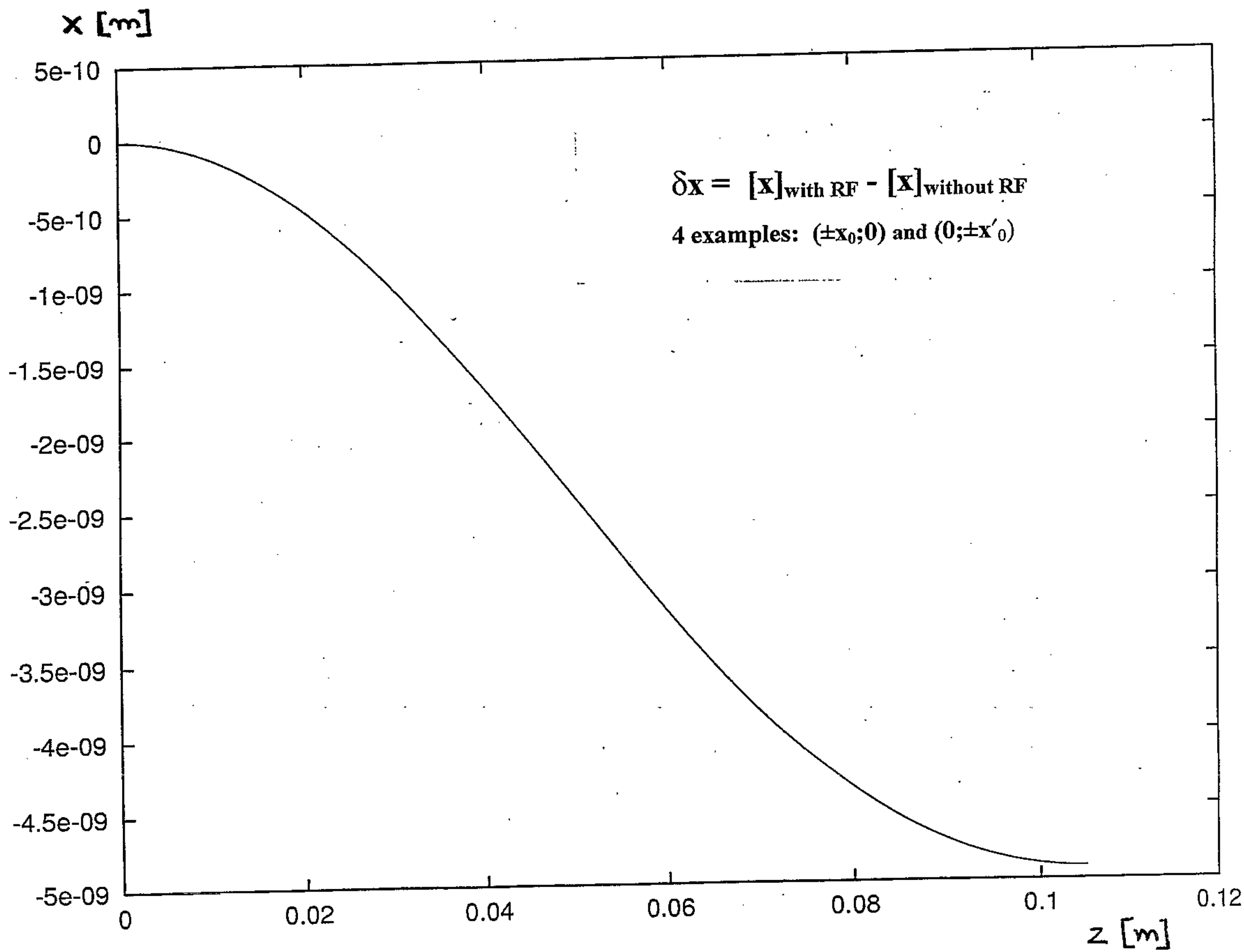
$$\pm\left[\frac{bd}{2\pi}\,\frac{\beta_{\rm ph}}{\beta_{\rm ph}^2 - 1}\,\frac{e\omega B_0}{\gamma^2}\right] x'$$

with

$$\beta_{\rm ph} = \sqrt{1 + \left(\frac{d}{b}\right)^2}$$

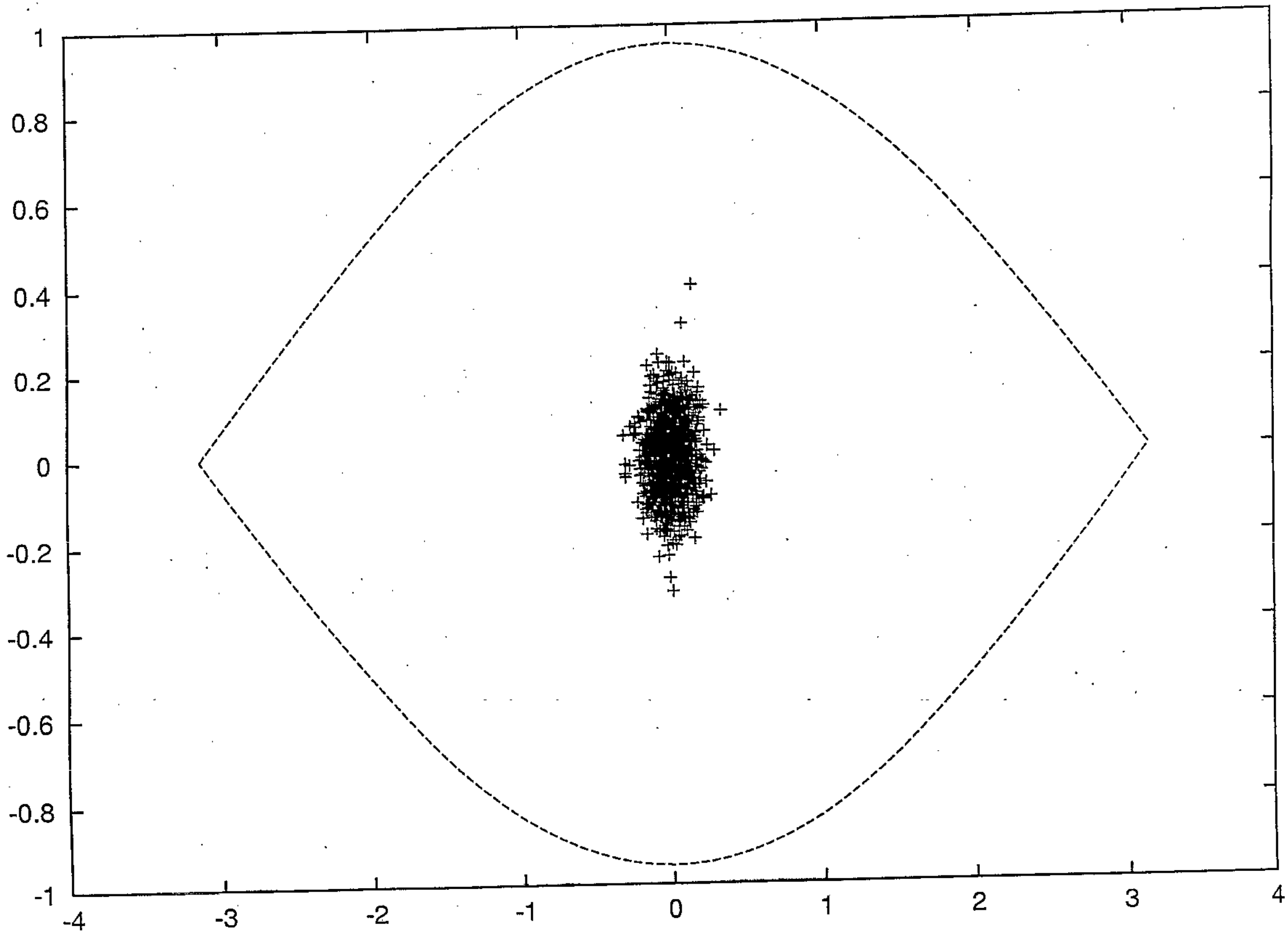
$$(\Delta U_E)_{\rm tot} = \Sigma\,\Delta U_E = 0$$

due to the betatron oscillations inchoerence

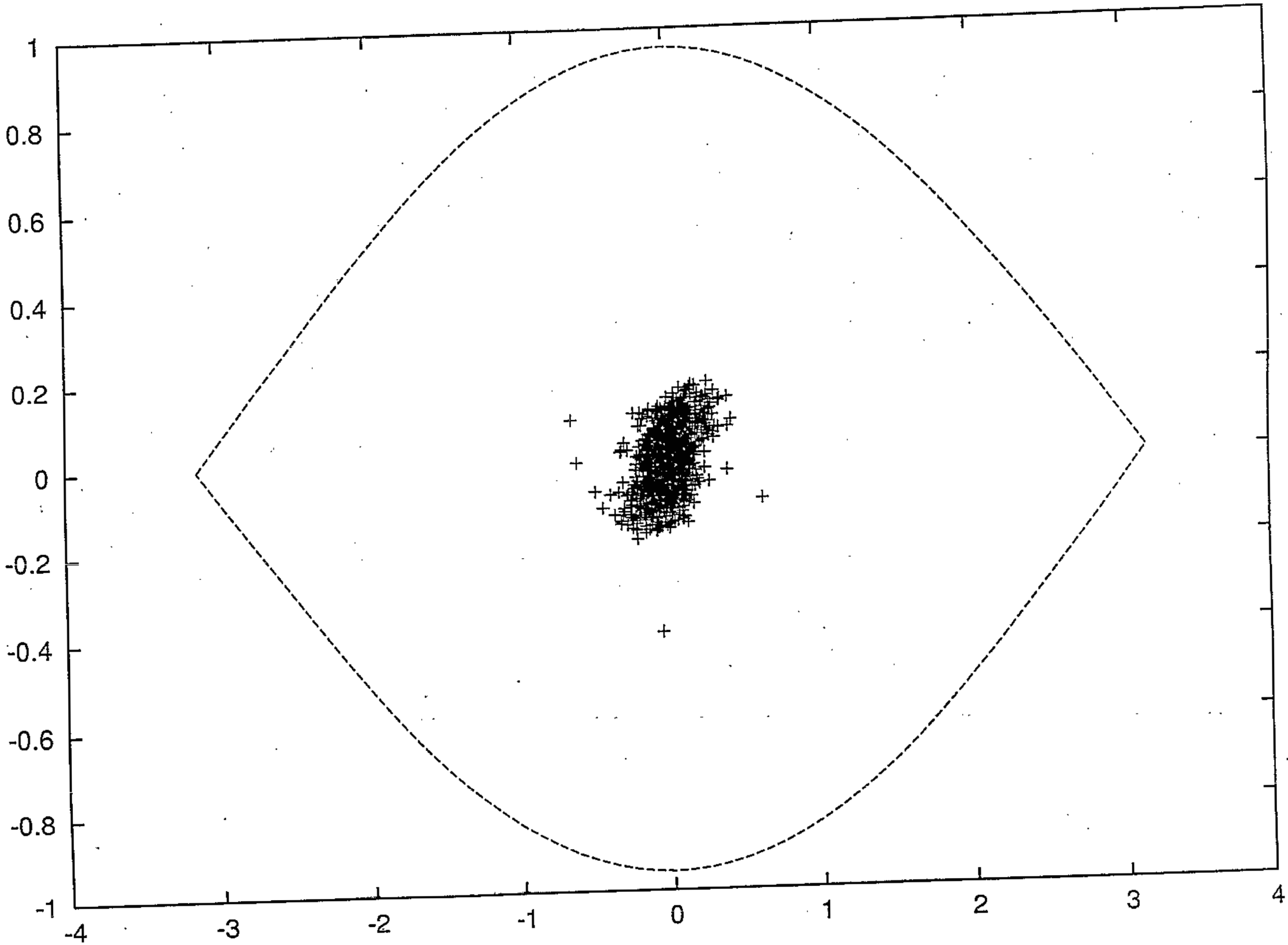


x [m]
5e-10
0
-5e-10
-1e-09
-1.5e-09
-2e-09
-2.5e-09
-3e-09
-3.5e-09
-4e-09
-4.5e-09
-5e-09
0
0.02
0.04
0.06
0.08
0.1
0.12
z [m]
δx = [x]with RF - [x]without RF
4 examples: (±x0;0) and (0;±x'0)

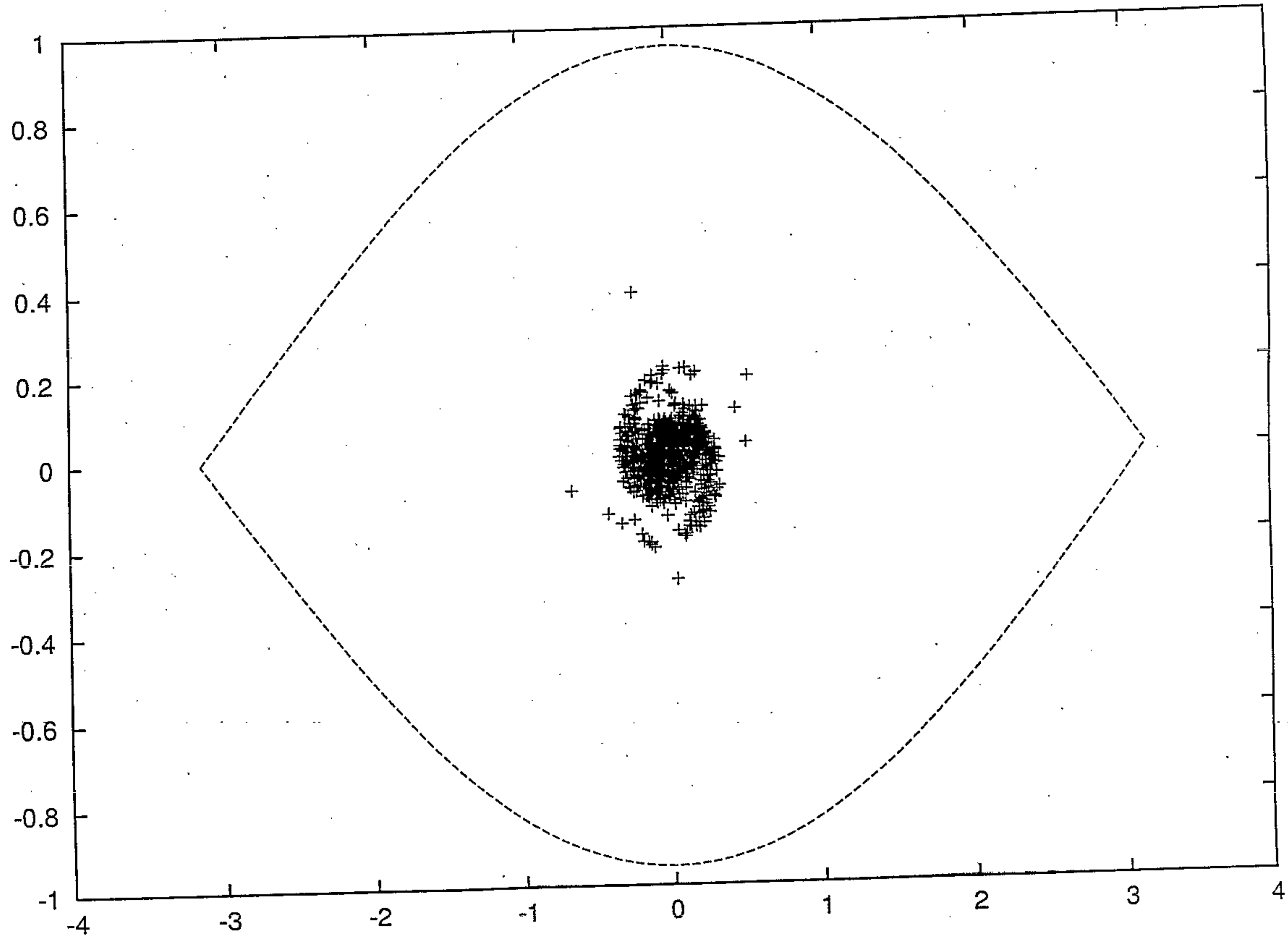
"Initial" bunch



After 10 τ_s



After 40 τ_s



WHAT PRESENTED SO FAR
SEEMS TO SOUND
“RATHER” PROMISING!

NEVERTHELESS

Further deeper and detailed studies are needed in the meaning that groups belonging to particle accelerator laboratories should strictly collaborate in this venture.

PROVIDED THAT

An actual scientific interest does exist, e.g. such as

Easing the production of polarized protons
Producing antiprotons

THEREFORE

An experimental verification of the γ^2-law is becoming more and more important independently of its application to a polarimeter

Last but not least some urgency has to be considered since

Three of us are BNL staff members and are heavily committed with their duties.

Two Italians are approaching their seventies and therefore will retire within 2003.

So that just one of us could work at full time: a PhD student who will discuss his thesis next 2003 spring.