The Basics of QCD Spin Physics

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Goal: To understand the structure of hadrons ($p, n, \pi, \ldots$) in terms of quarks and gluons (the QCD degrees of freedom)

In particular: the spin structure of hadrons

$$\left.\begin{array}{l}\text{quarks (spin-1/2)}\\ \text{gluons (spin-1)}\end{array}\right\} \Longrightarrow \text{proton (spin-1/2)}$$

QCD - the microscopic theory of the strong interactions - exhibits the following properties:

confinement	at low energies	:-(
asymptotic freedom	at high energies	:-)

$\Longrightarrow$ perform high energy scattering

For example: high energy ($\sqrt{s}$) $p\,p$ scattering at RHIC

Outline

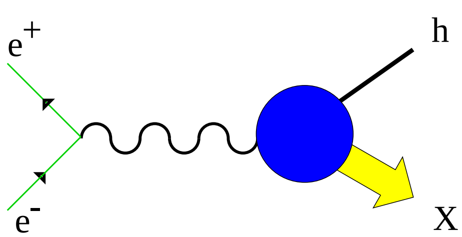
- Hard scattering processes relevant for spin physics
- Description of such processes:
  - Structure functions
  - Parton densities (distribution functions)
  - Relation between structure functions and distribution functions
  - Fragmentation functions
  - Importance of factorization: universality
- Description of polarized spin-1/2 and spin-1 particles:
  - Spin-1/2
  - Helicity and transverse spin
  - Massive versus massless particles
  - Polarized quark densities
  - Spin-1, polarized gluon densities
- From quarks and gluons to the proton: spin sum rule
- Very short overview of RHIC spin physics

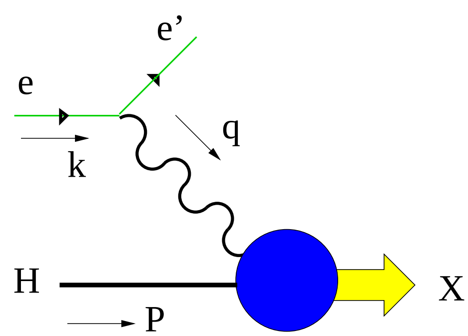
Hard Scattering Processes

Things that make life easy:

- Leptons — QED: clean & calculable
- Minimize # hadrons — e.g. inclusive processes

$e^+ + e^- \to X$
$\mu^+ + \mu^- \to X$
$e^+ + e^- \to$ hadron $+ X$
$e^+ + e^- \to$ jet $+ X$



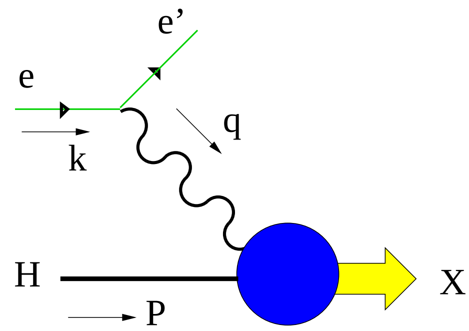


$\ell + H \to \ell' + X$ (DIS)
$\ell + H \to \ell' + h + X$ (Semi-Inclusive DIS)

Etc

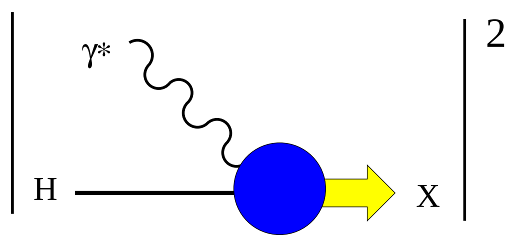
Structure Functions

Consider Deep Inelastic Scattering:



The cross section is proportional to the contraction of the lepton tensor with the hadron tensor: $\sigma \propto L^{\mu\nu} W_{\mu\nu}$

Hadron tensor: $W^{\mu\nu} = \Sigma_X$



$$W_{\mu\nu}(P,q) = \frac{1}{4\pi} \int d^4x \, e^{iq\cdot x} \, \langle P | [J_\mu(x), J_\nu(0)] | P \rangle$$

Parameterize $W^{\mu\nu}$ in terms of the available vectors: hadron momentum (P), photon momentum (q), hadron spin vector (S)

Structure Functions

Unpolarized hadron:

$$W_S^{\mu\nu}(P,q) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x_B, Q^2) + \frac{\tilde{P}^\mu \tilde{P}^\nu}{P \cdot q} F_2(x_B, Q^2)$$

$\tilde{P}^\mu = P^\mu - (P \cdot q) q^\mu / q^2$

$x_B = Q^2/2P \cdot q$ and $Q^2 = -q^2$

Polarized hadron:

$$W_A^{\mu\nu} = \frac{1}{P \cdot q} i\epsilon^{\mu\nu\rho\sigma} q_\rho \left[S_\sigma g_1(x_B, Q^2) + \left(S_\sigma - \frac{S \cdot q}{P \cdot q} P_\sigma \right) g_2(x_B, Q^2) \right]$$

$S^2 = -1$ and $P \cdot S = 0$

F_1, F_2, g_1, g_2 are called structure functions

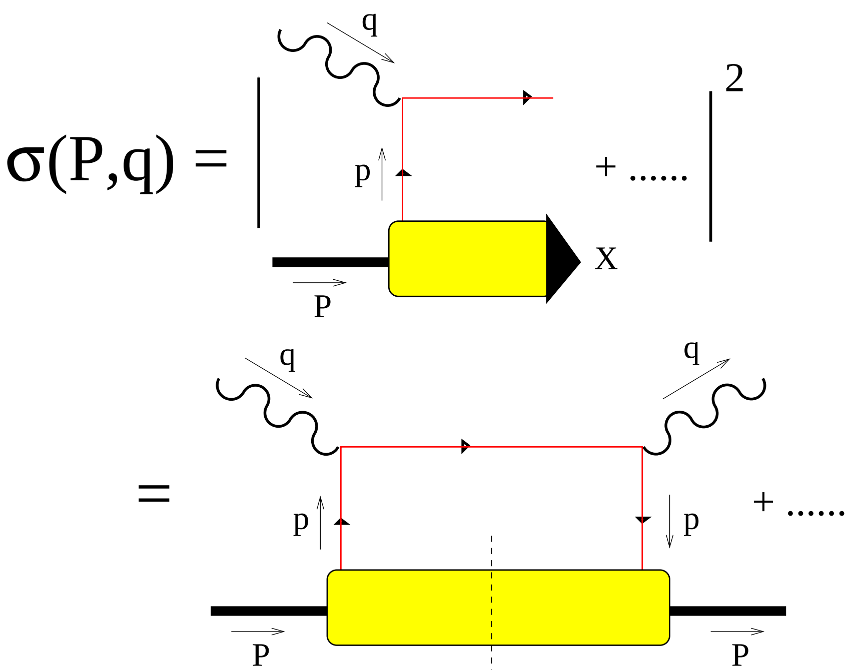
In the Bjorken limit ($Q^2 \to \infty, x_B$ fixed) they are independent of Q^2: scaling

Definition of structure functions is independent of the constituents of the hadron

Parton model idea:

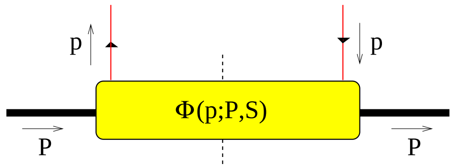
Deep inelastic scattering of a lepton (and hence virtual photon) off a hadron is actually scattering off a 'parton' inside that hadron

In first approximation that parton is free



Scattering off a parton $\otimes$ distribution of partons inside hadron

Distribution Functions

Parameterize 

in terms of its vectors: hadron momentum (P), quark momentum (p), hadron spin vector (S)

Most general parameterization in accordance with the symmetries:

$$\Phi(x) = \frac{1}{2}\left[f_1(x)\,\not{P} + g_1(x)\,\lambda\gamma_5\,\not{P} + h_1(x)\,\gamma_5\,\not{S}_T\,\not{P}\,\right]$$

The functions f_1, g_1, h_1 are called distribution functions

Other commonly used notation:

$$\begin{array}{lcl} f_1 & \to & q \\ g_1 & \to & \Delta q \\ h_1 & \to & \delta q \quad \text{or} \quad \Delta_T q \end{array}$$

Formal definition:

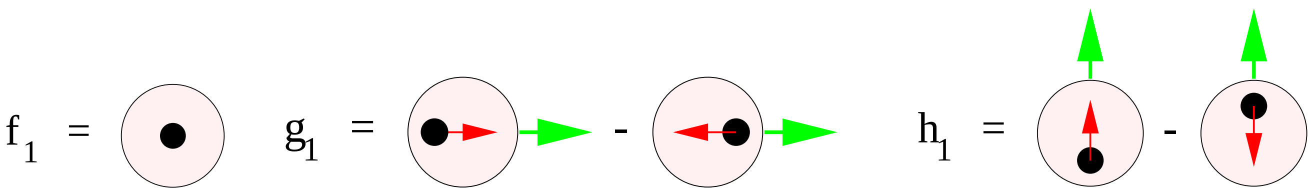
$$\Phi(x) = \int \frac{d\lambda}{2\pi}\, e^{i\lambda x}\, \langle P, S|\overline{\psi}(0)\,\mathcal{L}[0,\lambda]\,\psi(\lambda)|P, S\rangle$$

$$\mathcal{L}[0,\lambda] = \mathcal{P}\exp\left(-ig\int_0^{\lambda} d\lambda\, n_-^{\mu} A_{\mu}(\lambda n_-)\right) \qquad \text{(with } n_-^2 = 0\text{)}$$

Interpretation

Distribution functions can be interpreted as momentum densities:

Pictorially:



Observation: $g_1 \neq h_1$

To appreciate the difference between g_1 and h_1 we need to investigate further what a distribution function really is and how spin is defined in case of relativistic particles

In hard scattering processes one or more particles are highly relativistic so the natural coordinate system in which to describe the process is in lightcone coordinates:

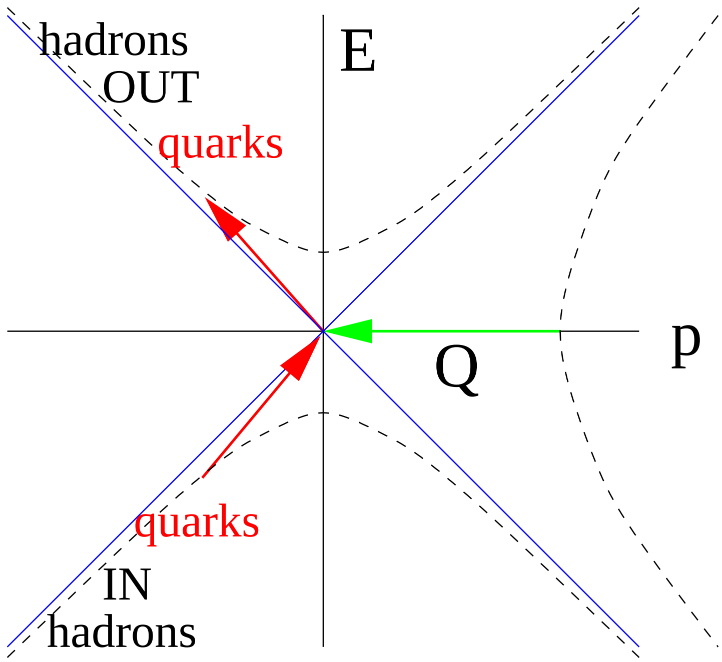
$$x^{\pm} = \frac{x^0 \pm x^3}{\sqrt{2}}, \quad \text{s.t.} \quad \begin{aligned} x \cdot y &= x^+ y^- + x^- y^+ - \boldsymbol{x}_T \cdot \boldsymbol{y}_T \\ x^2 &= 2x^+ x^- - \boldsymbol{x}_T^2 \end{aligned}$$

Light-Cone Coordinates

Boost in the z-direction: $x'^3 = \gamma(x^3 - \beta x^0), \quad x'^0 = \gamma(x^0 - \beta x^3)$

Therefore: $x'^{\pm} = x^{\pm}\,(\frac{1-\beta}{1+\beta})^{\pm\frac{1}{2}}$

In other words: ratios $x^{\pm}/y^{\pm}$ are invariant under these boosts

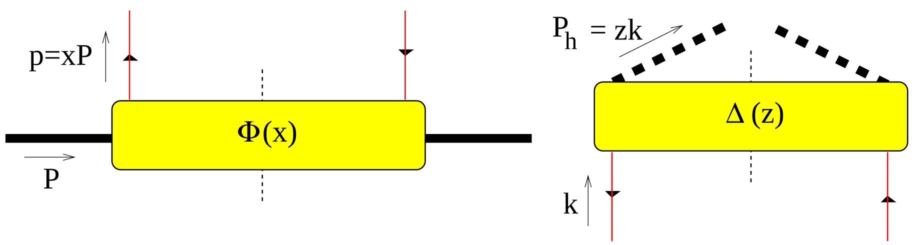


In the Bjorken limit $x_B = Q^2/(2P\cdot q) = x = p^+/P^+$, the light-cone momentum fraction of a quark (p) inside a hadron (P)

Fragmentation Functions

Apart from distribution functions there are fragmentation functions

Instead of describing the distribution of partons inside a hadron, they describe the fragmentation of a parton into hadrons



$$\Delta(z) = \frac{1}{2}\left[D_1(z)\,\not{P} + G_1(z)\,\lambda\gamma_5\,\not{P} + H_1(z)\,\gamma_5\,\not{S}_T\,\not{P}\right]$$

Here z is the light-cone momentum fraction of a produced hadron w.r.t. the fragmenting quark

Polarized fragmentation functions are essentially unknown
Relevant for polarized hyperon $(\Lambda, \Sigma, \Xi, \ldots)$ production

Note: magnitude of a distribution function (e.g. g_1) unrelated to that of the analogous fragmentation function (G_1)

Structure Functions in the PM

In first approximation the structure functions can be expressed in terms of the distribution functions, like:

$$F_2(x_B) = 2x_B\, F_1(x_B) = \sum_a e_a^2\, x_B\, [q_a(x_B) + \bar{q}_a(x_B)]$$

$q_a = f_1^a$

$q_a(x_B)$ = probability of finding a parton (quark) of flavor a in the proton with a (light-cone) momentum fraction x_B

$\bar{q}_a(x_B)$ = antiquark distribution function

e_a = quark charge

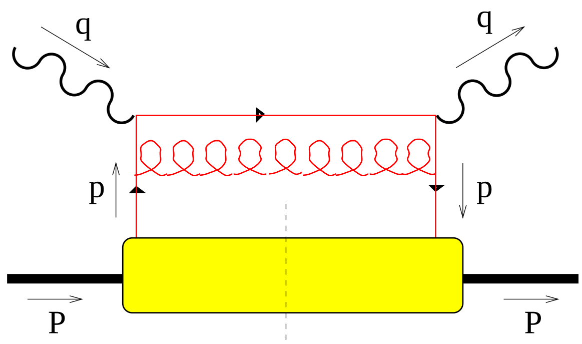
Polarized structure function g_1 in terms of distribution functions:

$$g_1(x_B) = \sum_a \frac{e_a^2}{2}\, [g_1^a(x_B) + \bar{g}_1^a(x_B)]$$

g_2 is not a simple function (so-called higher twist)
It cannot be expressed in terms of a probability distribution/density

Perturbative QCD Corrections

Parton model is a first approximation (no Q^2 dependence)
It receives corrections:



$\alpha_s \log(Q^2/\mu^2)$
pQCD corrections
[DGLAP]

These corrections yield the energy scale dependence of distribution functions, i.e. the dependence on the scale at which they are probed

Leading order (LO) Q^2 dependence of f_1:

$$\frac{\partial\ f_1(x;Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} C_F \int_x^1 \frac{dy}{y} \left[\frac{3}{2}\,\delta(1-x/y) + \frac{1+(x/y)^2}{(1-x/y)_+} \right] f_1(y;Q^2)$$

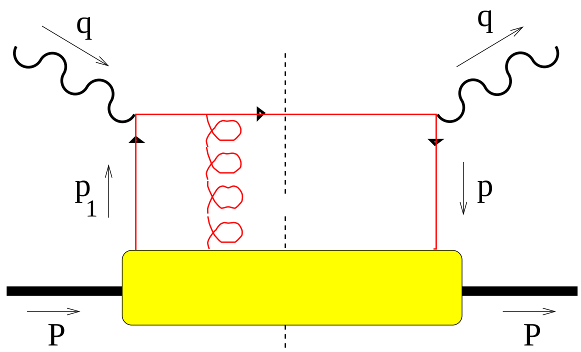
This so-called evolution allows one to make predictions for different energies if known at one energy scale

Rigorously tested, one of the great successes of QCD

Spin Asymmetries in Inclusive DIS

At subleading order in $1/Q$, $\Phi(x)$ must be expanded further

$$\Phi(x) = \frac{1}{2}\left[f_1(x)\,\not{P} + g_1(x)\,\lambda\gamma_5\,\not{P} + h_1(x)\,\gamma_5\,\not{S}_T\,\not{P}\right] + \frac{M}{2}g_T(x)\gamma_5\,\not{S}_T + \text{other higher twist}$$



Dynamical power corrections $1/Q^n$

The function $g_T = g_1 + g_2$ is obtained from DIS (SLAC, SMC)

$$\frac{d\sigma(\vec{\ell}\vec{H} \to \ell' X)}{dx_B\,dy} = \frac{4\pi\alpha^2 s}{Q^4}\sum_{a,\bar{a}} e_a^2 \left\{ \left(\tfrac{y^2}{2}+1-y\right) x_B\, f_1^a(x_B) + y(1-\tfrac{y}{2})\,\lambda_e\,\lambda\, x_B g_1^a(x_B) - 2y\sqrt{1-y}\,\lambda_e\,|\boldsymbol{S}_T|\cos(\phi_s)\,\frac{M}{Q}\,x_B^2\, g_T^a(x_B) \right\}$$

Higher twist generates an azimuthal spin asymmetry

Factorization

Parton model is a factorized description of the hadron tensor

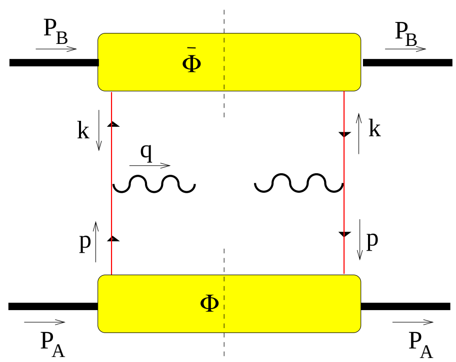
The hard scale Q serves to separate hard from soft parts of the process

Distribution functions describe the soft parts, which are not calculable from first principles at present

Distribution functions are universal (factorization theorems)

The Drell-Yan Process

$H_1 + H_2 \to \ell + \bar{\ell} + X$

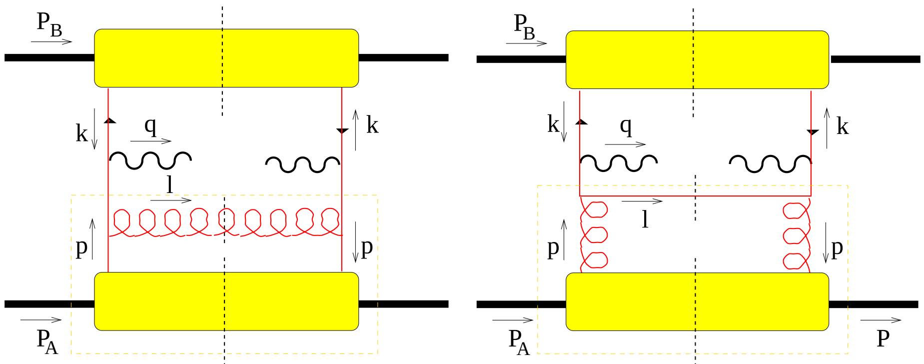


This process factorizes into a hard part and two soft parts
Soft parts the same as the one appearing in DIS: $\Phi_{DY} = \Phi_{DIS}$

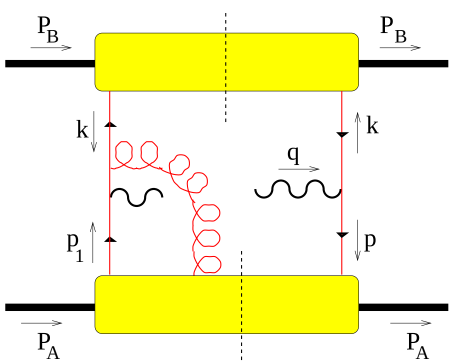
This universality can be used to make predictions

Factorization

Examples of corrections to the parton model description of the Drell-Yan process:



$\alpha_s \log(Q^2/\mu^2)$
pQCD corrections
[DGLAP]



$1/Q^n$ power corrections
"Higher twist"

Proofs of factorization theorems are complicated, but essential

Spin-1/2

Spin-1/2 particle: two pure states $|i\rangle$ (spin 'up' and spin 'down')

Impure states are described by a density operator: $\rho = \sum_i p_i \, |i\rangle\langle i|$
where p_i are the probabilities

For a spin-1/2 particle one can use the Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\rho = \frac{1}{2}\left(\mathbf{1} + \boldsymbol{S}\cdot\boldsymbol{\sigma}\right) = \frac{1}{2}\begin{pmatrix} 1+S^3 & S^1 - iS^2 \\ S^1 + iS^2 & 1 - S^3 \end{pmatrix}$$

$\boldsymbol{S}$ is the polarization (or spin) vector of a state (three-vector)

For a pure state $|\boldsymbol{S}| = 1$

The relativistic generalization of the spin three-vector $\vec{S}$:
The four-vector S^μ, satisfying $S^2 = -1, \; P \cdot S = 0$

Pauli-Lubanski Vector

Rotations of three-vectors are generated by the angular momentum operators $\vec{J}$, satisfying: $[J_i, J_j] = i\epsilon_{ijk} J_k$

$$\vec{J}^2 |j,m\rangle = j(j+1)|j,m\rangle, \quad J^3|j,m\rangle = m|j,m\rangle$$

The relativistic generalization of intrinsic angular momentum is the Pauli-Lubanski operator:

$$W_\mu = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma} M^{\nu\rho} P^\sigma,$$

where $M^{\nu\rho}$ is the generator of Lorentz transformations

$[W_\mu, W_\nu] = i\epsilon_{\mu\nu\rho\sigma} W^\rho P^\sigma$

$W^2 = -M^2 S(S+1)$, where S is integer or half integer

The Lorentz invariant measure of intrinsic angular momentum

W_μ generates transformations that leave four-vectors p^μ invariant

Rest frame: $\quad W^\mu = (0, M\epsilon_{ijk} M^{jk}/2) = (0, M\vec{J})$

Helicity

Helicity $=$ projection of spin on the 3-momentum: $\vec{J} \cdot \vec{p}/|\vec{p}|$

But helicity is a Lorentz invariant quantity:

$$\frac{W_\mu S^\mu}{M} |PS\rangle = \lambda |PS\rangle$$

with $\lambda = -S, \ldots, S-1, S$

S^μ is the four-vector generalization of the spin quantization axis

$S^\mu = (|\vec{p}|/M, p^0 \hat{p}/M) \quad \Longrightarrow \quad \lambda = \vec{J} \cdot \vec{p}/|\vec{p}|$ is the helicity

This is the only choice of S^μ, s.t. $W_\mu S^\mu$ contains only J

In the rest frame: $\vec{J} \cdot \vec{S}$ denotes the helicity

Transverse Spin

Transverse spin state = off-diagonal state in the helicity basis

Transverse spin states in the $\hat{y}$ and $-\hat{y}$ directions are defined as

$$|\uparrow\rangle = [\,|+\rangle + i\,|-\rangle\,]\,/\sqrt{2}, \quad |\downarrow\rangle = -\,[\,|+\rangle - i\,|-\rangle\,]\,/\sqrt{2}$$

$$\Longrightarrow \begin{cases} |\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| & = \quad |+\rangle\langle+| \,+\, |-\rangle\langle-| \\ |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow| & = \quad -i\,|+\rangle\langle-| \,+\, i\,|-\rangle\langle+| \quad \longleftarrow \sigma_y \end{cases}$$

Transverse spin states of spin-1/2 particles are helicity flip states

Transverse spin states in the $\hat{x}$ and $-\hat{x}$ directions:

$$|\uparrow\rangle = [\,|+\rangle + |-\rangle\,]\,/\sqrt{2}, \quad |\downarrow\rangle = [\,|+\rangle - |-\rangle\,]\,/\sqrt{2}$$

Transverse Spin

In the rest frame transverse polarization is the same as longitudinal polarization:

take $S^\mu = (0,0,0,1) \Longrightarrow W_\mu S^\mu/M = J_x p_t/M - K_y p_z/M$
($M^{k0} = K^k$ boosts)

In the rest frame: $W_\mu S^\mu/M|PS\rangle = J_\perp|PS\rangle = \lambda|PS\rangle$

rotational invariance!

Rest frame polarization vector $S^\mu = (0,0,0,S_x)$
Boost in $\hat{z}$ direction: S^μ not an eigenvector of J_x anymore, since $[K_z, J_x] \neq 0$

$[K_z, W_x] = 0$ as opposed to $[K_z, J_x] \propto [M_{03}, M_{23}] \propto M_{02} = K_y$

Hence, $S^\mu = (0,0,\vec{1}_\perp)$ is an eigenstate of $W_\perp$ and not of $J_\perp$

That's why transverse spin state is also called transversity to distinguish it from eigenstates of $\boldsymbol{J}_T$

Example

Consider the spin state $|\vec{S}\,\rangle = 0.8|+\rangle + 0.6|-\rangle$

$$\Longrightarrow \langle \sigma_x \rangle = 0.48, \langle \sigma_z \rangle = 0.14$$

$$|\vec{S}\,\rangle\langle\vec{S}\,| = \frac{1}{2}\left(\mathbf{1} + 0.96\,\sigma_x + 0.28\,\sigma_z\right)$$

Polarization three-vector:

$$S_x = 0.96,\ S_z = 0.28 \Longrightarrow |\vec{S}\,| = 1 \qquad \text{(note: pure state)}$$

$$\lambda = \langle \vec{\sigma} \cdot \vec{S}\,\rangle = \frac{1}{2}$$

Boost in $\hat{z}$ direction: state stays $0.8|+\rangle + 0.6|-\rangle$, even though for components of the polarization four-vector S^μ:

$S_x/S_z \sim 1/\gamma \stackrel{\gamma\to\infty}{\longrightarrow} 0$ $\quad$ S_x seems to become less important

Behavior of polarization vector under boosts often leads to confusion

$M/E \to 0$ and $M = 0$

A boost of a polarized particle at rest to a high momentum:
S^μ acquires a longitudinal component proportional to its momentum while the transverse component is invariant

High energy particle: $S^\mu = \lambda P^\mu / m + S_T + \mathcal{O}(m/E)$

Divergent as $m \to 0$ or $\gamma \to \infty$, but helicity is Lorentz invariant

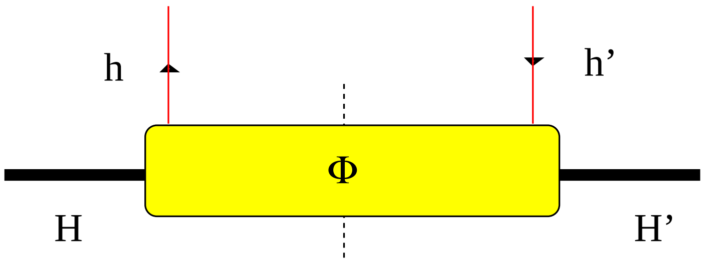
For massless spin-1/2 particles: $P^2 = 0$, $W^2 = 0$, $W^\mu = \lambda P^\mu$
Polarization vector $S^\mu = \lambda P^\mu$, but spin states stay the same

Transverse spin is defined as off-diagonal state in the helicity basis

One cannot go to the rest frame to connect it to S_T and it is not an eigenstate of $\boldsymbol{J}_T$

Helicity and transverse spin are the correct quantities to consider

Parton Densities as Helicity Amplitudes



$(H, h; H', h')$

Helicity conservation: $H + h = H' + h'$

Parity flips sign of helicities

Time reversal switches $(H, h) \leftrightarrow (H', h')$

Imposing these requirements leaves three possibilities:

$$
\begin{aligned}
f_1 &= (+,+;+,+) \quad + \quad (+,-;+,-) \\
g_1 &= (+,+;+,+) \quad - \quad (+,-;+,-) \\
h_1 &= (+,+;-,-)
\end{aligned}
$$

helicity flip

Positivity of the density matrix leads to bounds:

- $|g_1(x)| \leq f_1(x)$
- Soffer's inequality $|h_1(x)| \leq \frac{1}{2}\left[f_1(x) + g_1(x)\right]$

FAQ 1

Why is $h_1 \neq 0$? (Or at least why not $h_1 \approx 0$?)

Reasoning behind the question:

1) $S_T/S_{\|} \to 0$ for ultrarelativistic particles
   For massless spin-1/2 particles: $S^\mu = \lambda P^\mu$

2) Transverse spin effects are suppressed in DIS, since the photon cannot flip helicity except with a suppression factor m_q/E_γ

Answers:

1) Behavior of $S_{\|}$ is irrelevant for spin state; helicity and transverse spin are the concepts to consider also in the limit $M/E \to 0$

2) Another hadron can "see" the transverse spin unsuppressed: this is one reason to do $p\,p$ scattering

FAQ 2

Why is $h_1 \neq g_1$?

Reasoning behind the question:

In the rest frame transverse polarization is the same as longitudinal polarization: rotational invariance!

Answer:

From the perspective of a relativistic probe transverse and longitudinal polarization are not the same

The direction defined by the probe is in the definition of the distribution functions

Distribution functions are properties of a hadron being probed by a highly relativistic probe

These properties are not less fundamental than the hadron structure as seen by a low energetic probe

Massless spin-1 particles (on-shell)

Spin-1 state is described by a set of polarization four-vectors $\epsilon_\mu^{(\lambda)}$ satisfying $\epsilon^\mu q_\mu = 0$ (current conservation)

To satisfy $\epsilon^\mu q_\mu = 0$ one could take $\epsilon^\mu \sim q^\mu$, however this is an unphysical state, not affecting $\vec{E}$ or $\vec{B}$

Massless spin-1 particles have two spin states ($\lambda = \pm 1$)
This corresponds to transverse fields ($\epsilon_x^{(\lambda=\pm 1)} \neq 0, \epsilon_y^{(\lambda=\pm 1)} \neq 0$)
Not transversely spinning states; they are eigenstates of J_z
Correspond to right- and left-handed circular polarization vectors:

$$\begin{aligned} \epsilon_R^\mu &= (0, -1, -i, 0)/\sqrt{2} \\ \epsilon_L^\mu &= (0, 1, -i, 0)/\sqrt{2} \end{aligned}$$

Off-diagonal states in the massless photon case ($\lambda = \pm 1$), are associated with transverse directions, but these are the directions of the field polarization ϵ_μ,

$$|\hat{x}\rangle = [\,|+1\rangle + |-1\rangle\,]/\sqrt{2}, \quad |\hat{y}\rangle = [\,|+1\rangle - |-1\rangle\,]/\sqrt{2}$$

These are states of linear polarization in the $\hat{x}$ and $\hat{y}$ directions

Massive spin-1 particles

Massive spin-1 states, including off-shell (gluon) states, have three spin states ($\lambda = 0, \pm 1$)

Massive spin-1 states (e.g. deuteron, ρ) can be characterized by a polarization vector S^μ and tensor $T^{\mu\nu}$:

$$S^\mu = -\frac{i}{M}\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu \epsilon_\rho P_\sigma$$

This shows that polarization vector ϵ^μ is not the spin vector S^μ (transverse ϵ_T^μ gives $S^\mu = S^z$ in the rest frame)

For pure tensor polarization state:

$$T^{\mu\nu} = \frac{1}{2}\left(\epsilon^\mu \epsilon^{*\nu} + \epsilon^\nu \epsilon^{*\mu}\right) - \frac{1}{3}\left(g^{\mu\nu} - \frac{P^\mu P^\nu}{M^2}\right)\epsilon^\rho \epsilon^*_\rho$$

A general spin-1 polarization density is often written as

$$\rho = \frac{1}{3}\left(\mathbf{1} + \frac{3}{2} S_i \mathbf{\Sigma}_i + 3 T_{ij}\mathbf{\Sigma}_{ij}\right)$$

Contains generalizations of the Pauli matrices

Transverse spin and gluon distributions

Gluon distribution in the proton

At leading order there are two gluon distribution functions:
$g(x)$ and $\Delta g(x)$

Since the gluon helicity is ± 1, $\delta\lambda = \{0, \pm 2\}$, while for protons helicity flip means $\delta\lambda = \pm 1$, they are incompatible. So there is no gluon helicity flip function $\delta g(x)$ at leading order

In other words, off-shellness is higher twist
Leads to a twist-3 helicity flip gluon distribution function $(\Delta_T G, G_{3T}, \ldots)$

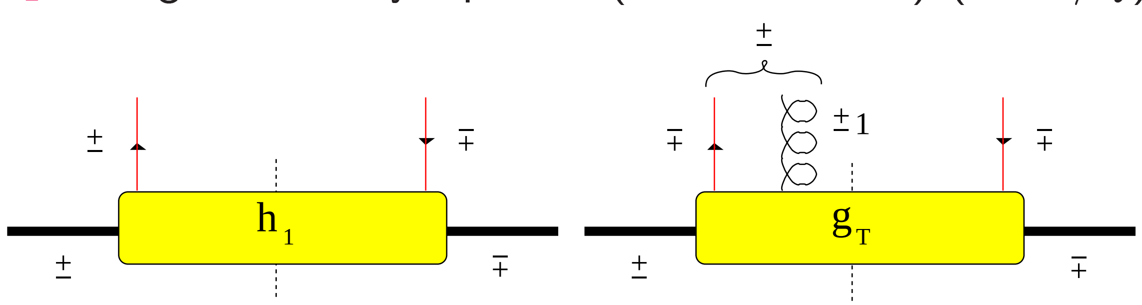
What is the difference between g_T, h_1 and $\Delta_T G$?

Different quark/gluon states inside the transversely polarized proton

g_T: a diagonal quark helicity state (chiral-even) $(\to M/Q)$

h_1: an off-diagonal quark helicity state (chiral-odd) (no mass)

$\Delta_T G$: a gluon helicity flip state (not chiral-odd!) $(\to M/Q)$



Spin Sum Rule

Sum of the contributions to the proton spin add up to 1/2:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_z + .\overset{?}{.}.$$

$\Delta\Sigma = \Delta u + \Delta d + \Delta s$

$\Delta q = \int_0^1 dx\, [(q_+ - q_-) + (\bar{q}_+ - \bar{q}_-)]$
$\Delta g = \int_0^1 dx\, [g_+ - g_-]$

$\Delta\Sigma$ not uniquely defined, often people take: $\Delta\Sigma' = \Delta\Sigma + N_f \frac{\alpha_s}{2\pi}\Delta g$
There are two operators with the same quantum numbers

Ellis-Jaffe sum rule: $\Gamma = \int_0^1 g_1^p(x) dx$

Naive parton model: $\Delta\Sigma = 1$

$$\begin{aligned} \Gamma &= \sum_a \frac{e_a^2}{2} \int_0^1 \Delta q^a(x) dx \\ &= \frac{1}{2}\left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right) \end{aligned}$$

Hyperon decay data & $\Delta s = 0 \longrightarrow \Delta\Sigma \sim 0.6$

Experiment (SMC): $\Delta\Sigma \sim 0.3, \quad \Delta s \sim -0.1$

Spin Sum Rule

$$\begin{aligned} \frac{1}{2} &= \frac{1}{2}\Delta\Sigma + \Delta g + L_z \\ &= \frac{1}{2}\Delta\Sigma + L_q + J_g \end{aligned}$$

Problem: Is $L_z = L_q + L_g$ or $J_g = \Delta g + L_g$?

Appears not to be the case, unless one chooses a specific frame, not Lorentz invariant, but hard processes have a preferred direction anyway: the direction of the probe momentum

Proposal to measure the quark orbital angular momentum contribution in Deeply Virtual Compton Scattering: $\gamma^* + p \to \gamma + p'$, where $(p - p')^2$ is small

There is no transverse spin sum rule; the tensor charge $\int_0^1 h_1^a(x)dx$ is not a conserved quantity (depends on the dynamics)

There exists no experimental information on h_1!

RHIC Spin Physics

Older (fixed target) $p\,p$ (and $p\,\bar{p}$) experiments: $\sqrt{s} \lesssim 30$ GeV

RHIC: $\sqrt{s} = 200 - 500$ GeV (50 GeV?)

Notation for asymmetries:

$$A_{LL} = \frac{\sigma_{++} - \sigma_{+-}}{\sigma_{++} + \sigma_{+-}} \quad A_L = \frac{\sigma_{-} - \sigma_{+}}{\sigma_{-} + \sigma_{+}} \quad A_{TT} = \frac{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}}{\sigma_{\uparrow\uparrow} + \sigma_{\uparrow\downarrow}}$$

$$A_{LL}^{PV} = \frac{\sigma_{--} - \sigma_{++}}{\sigma_{--} + \sigma_{++}} \quad \bar{A}_{LL}^{PV} = \frac{\sigma_{-+} - \sigma_{+-}}{\sigma_{-+} + \sigma_{+-}}$$

$\underline{\Delta g(x)}$:

- $\vec{p}\,\vec{p} \to \gamma\, X$
  A_{LL}^{γ} dominated by $q\,g \to q\,\gamma$, whereas $q\,\bar{q} \to g\,\gamma$ is negligible
  $A_{LL}^{\gamma}\, d\sigma \sim \Delta q\, \Delta g\, \hat{a}_{LL}\, d\hat{\sigma}$

- $\vec{p}\,\vec{p} \to$ jet X as function of p_T
  A_{LL}^{jet} at $p_T \sim 10$ GeV/c dominated by $g\,g$,
  at $p_T \sim 20 - 60$ GeV/c by $g\,q$ and at large p_T by $q\,q$

- $\vec{p}\,\vec{p} \to \gamma$ jet; jet jet, ...

- $\vec{p}\,\vec{p} \to \pi^0\, X;\ c\bar{c}\, X;\ b\bar{b}\, X;\ J/\psi\, X;\ \chi_2\, X;\ \ldots$

RHIC Spin Physics

$\Delta q(x), \Delta\bar{q}(x)$:

- $\vec{p}\,p \to W^{\pm}\,X \quad (A_L^{W^\pm})$
  By varying the x and y (rapidity) dependence one can isolate different contributions: $\Delta u/u, \Delta d/d, \Delta\bar{u}/\bar{u}, \Delta\bar{d}/\bar{d}$

δq:

- $p^{\uparrow}\,p^{\uparrow} \to \ell\,\bar{\ell}\,X, \quad A_{TT}$ in the Drell-Yan process
- $p\,p^{\uparrow} \to \pi\,\text{jet}\,X; \pi^+\pi^-\,X$ using nontrivial fragmentation functions

Parity violation:

- $\vec{p}\,p \to \text{jet}\,X$ (A_L^{jet}) and $\vec{p}\,\vec{p} \to \text{jet}\,X$ $(A_{LL}^{PV}, \bar{A}_{LL}^{PV})$

Other interesting processes/observables:

- Single transverse spin asymmetries A_N in inclusive $\pi, K, \Lambda, \ldots$ production, prompt photon production, Drell-Yan
- Polarized fragmentation functions via longitudinal and transverse Λ polarization