Stability with the 56 MHz system

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Nathan Towne did the transition work, Ilan Ben-Zvi supervised

Outline:

- Cavity parameters and comparison with existing impedance
- Transition
- Storage

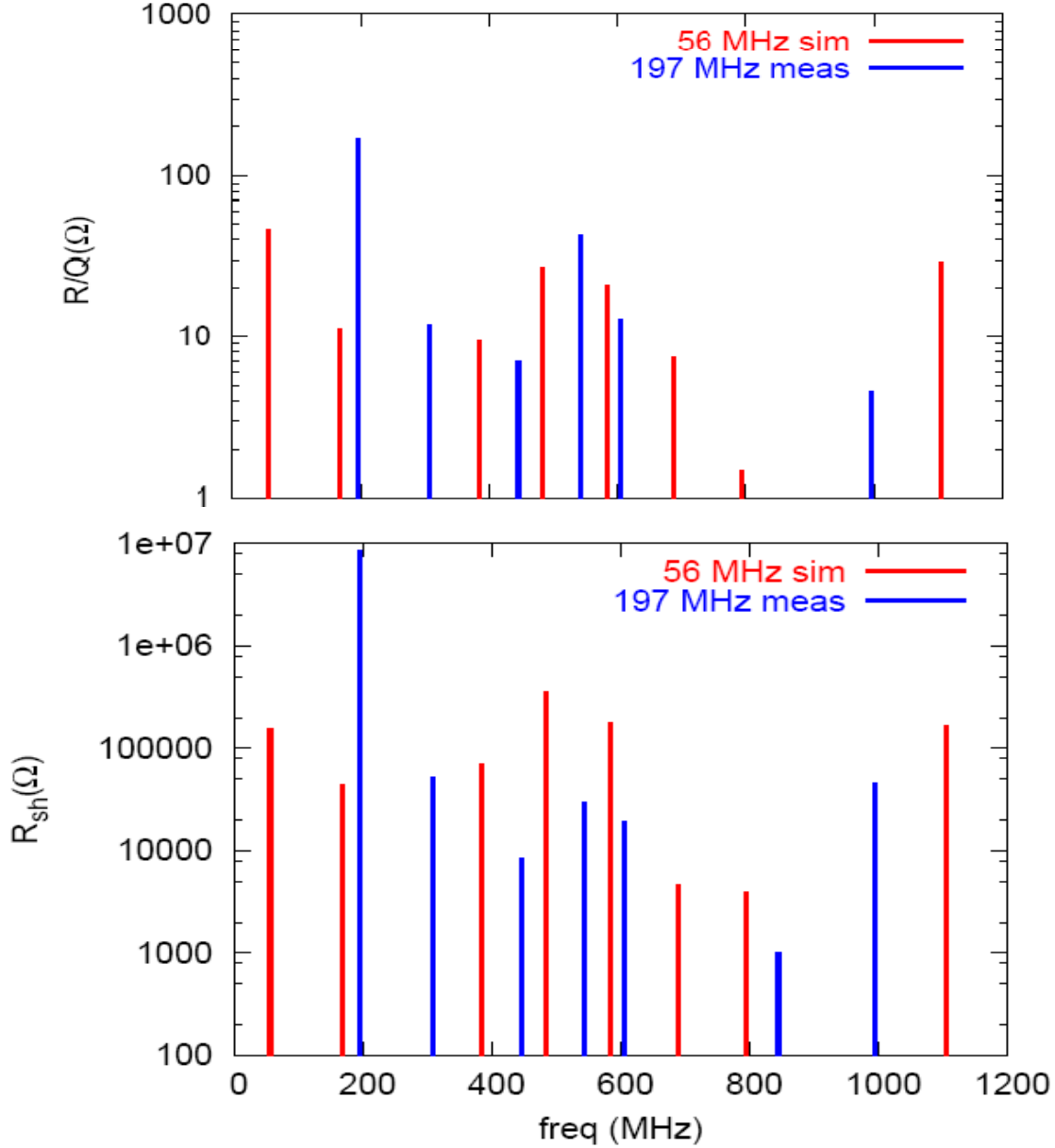
Impedance

A single 56 MHz cavity is used by both beams.

There are three 197 MHz cavities in each ring and four shared in the common section.

197 MHz impedance is for one cavity.

Measurements show that the HOM frequencies of the individual 197 MHz cavities drive different Coupled Bunch Modes (CBMs).



Transition

$\gamma_{tr} = 23.2$ for nominal lattice, $\gamma_{tr} = 26.2$ for IBS suppression lattice

protons injected with γ=25.4, above transition. ions with γ=10

RHIC uses a first order transition jump with a full jump size of $\Delta\gamma$=1.

The jump duration is 30 milliseconds which is short compared to the 150 millisecond synchrotron period.

A multi-bunch simulation code which includes acceleration and an instantaneous jump was written and run by Nathan Towne.

Nine, symmetrically placed bunches were tracked. The narrow band impedance was multiplied by 120/9= 13.33 and the broad band impedance was unmodified.

A single ring was tracked (no cross-talk) and beam parameters were monitored.

The intensity threshold for instability was too small to be interesting.

Net growth during the process was calculable.

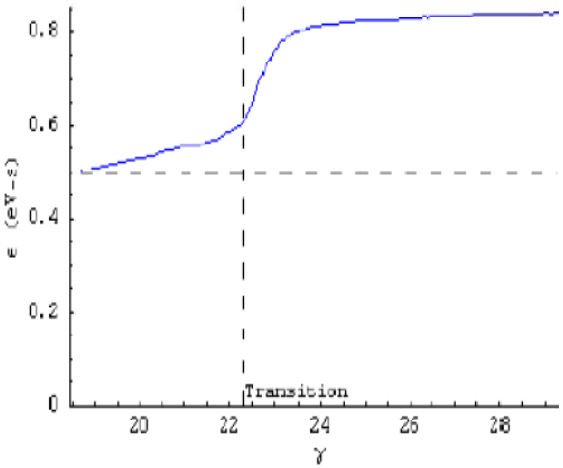
Simulations with 7 identical storage cavities, 197 MHz

We see emittance growth across transition.

This could be part of the problem.

Emittance growth across transition due to the NC HHC

- Simulation across transition shows substantial emittance growth.
- Are the NC HHCs responsible for some of the emittance growth before rebucketing? With SC HHCs and no rebucketing, would removal of some cavities reduce emittance at the top of the ramp?

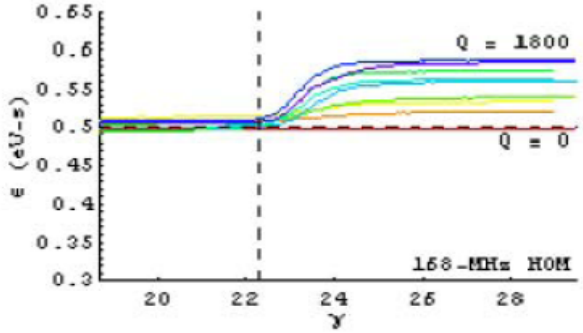


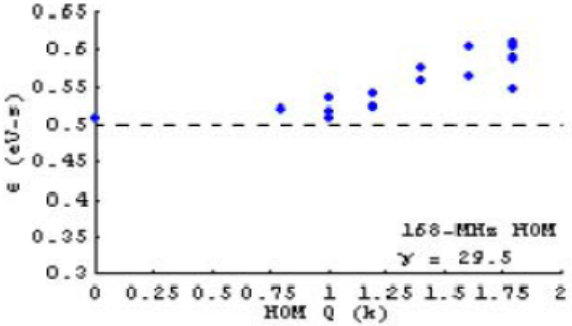
Nathan scanned the most worrisome mode in detail for the 56 MHz cavity

Several Simulations done

168-MHz HOM

- Longitudinal simulations of the ring with the 168-MHz HOM at different Qs show the continuum of variation of emittance growth. The growth of emittance starts at transition and persists for a time after.
- Emittance growth due to the 168-MHz HOM is modest at Q = 1000 and two cavities, even though well above threshold.
- With one cavity, Q between 2000 and 2500 has modest emittance growth.



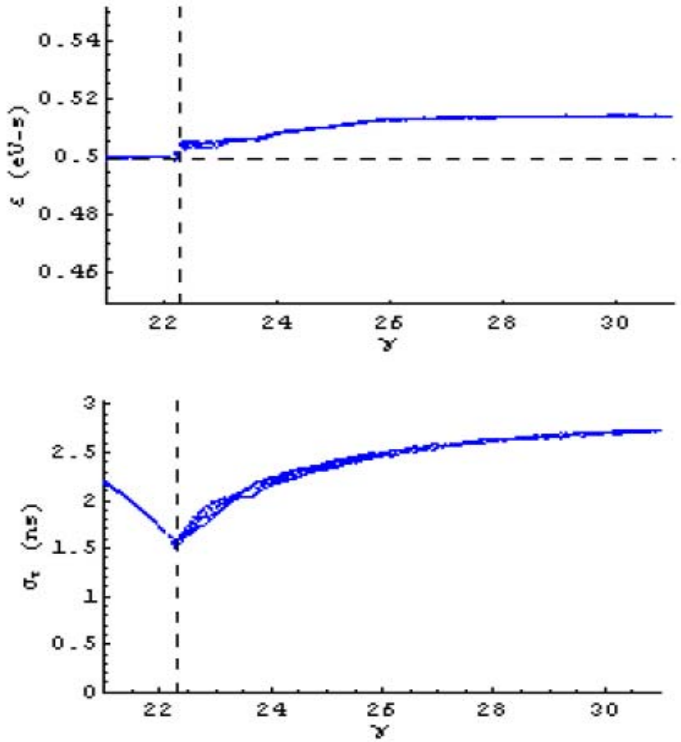


Two SC HHCs

All 56 MHz modes together but no 197s.
The 56 MHz cavity is no worse than the 197 MHz cavities.
Conclude that transition should not be an issue.

Acceleration across transition

- Longitudinal emittance (top) and bunch length (bottom) during acceleration with composite impedance model (seven HOMs) and BBI.
- Two runs are spliced together in the plots.
- Less than 5% emittance growth for 1.0e9 ions/bunch



Storage

During storage we have the typical situation.

The wide band impedance $Z/n \approx 3j\Omega$, causes a coherent frequency shift that negates Landau damping and a small resistive piece causes growth.

The growth times are quite long, of order a second.

Effects from the small number of macroparticles yield growth in the simulation code even though it is absent in the continuum (or large N) limit.

A frequency domain/Vlasov approach is required.

Basic idea of the new technique

Write down the Vlasov equation for a symmetric fill.

Include an RF kicker driving term

$$V_{kick} = \hat{V}\delta_p(\theta)\exp([in\omega_0 + i\Omega + \varepsilon]t)$$

The small positive part insures causal response and no funny business with Van-Kampen like modes.

Assume $|\Omega| << \omega_0$ so that only a single coupled bunch mode is driven. One gets

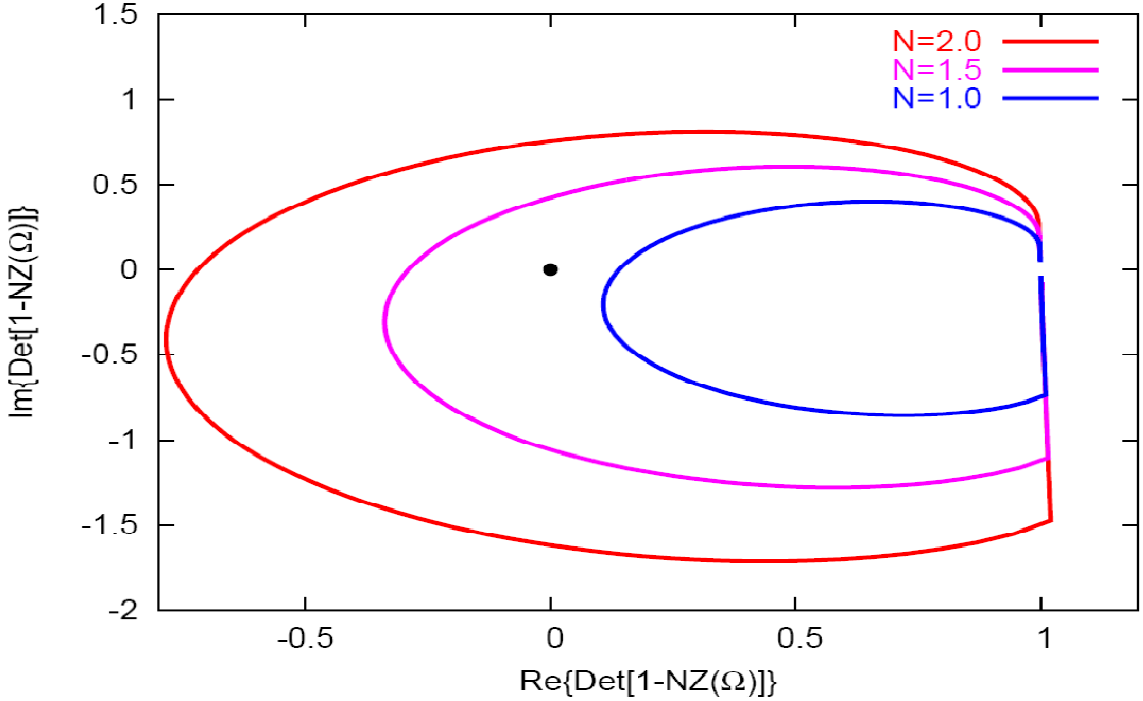
$$[-i\Omega + im\omega_s(r)]R_m(r) =$$

$$F_0' \frac{\bar{\eta} q m \omega_0}{2\pi r \omega_{s0}} \left\{ \frac{\hat{V} J_m[(k_0 M + s)r]}{k_0 M + s} - \sum_{k \neq 0} \frac{\rho_k Z_k}{kM + s} J_m[(kM + s)r] \right\}$$

Theory, continued

$$\rho_p - \frac{\bar{\eta} q^2 \omega_0^2 M N}{2\pi\omega_{s0}} \sum_k \frac{Z_k}{kM+s} \rho_k \sum_m m \int_0^{\hat{\phi}} \frac{dr J_m[(pM+s)r] J_m[(kM+s)r]}{-i\Omega + im\omega_s(r)} F_0'(r) = drive. \quad (7)$$

$$[\mathbf{1} - N\mathbf{Z}(\Omega)]\mathbf{P} = \mathbf{D}$$



m=1 used almost exclusively

Tracking versus NYQUIST

- Comparison with direct particle tracking and low frequency impedance yields good agreement, as do formulas.
- When the broad-band Z/n becomes important the agreement is OK, shown on the right.
- The factor of 2 discrepancy might be due to different truncation of the broad band impedance.
- More likely is that simulation behavior near the threshold is funny, linear RF growth rate is 4.8/second.

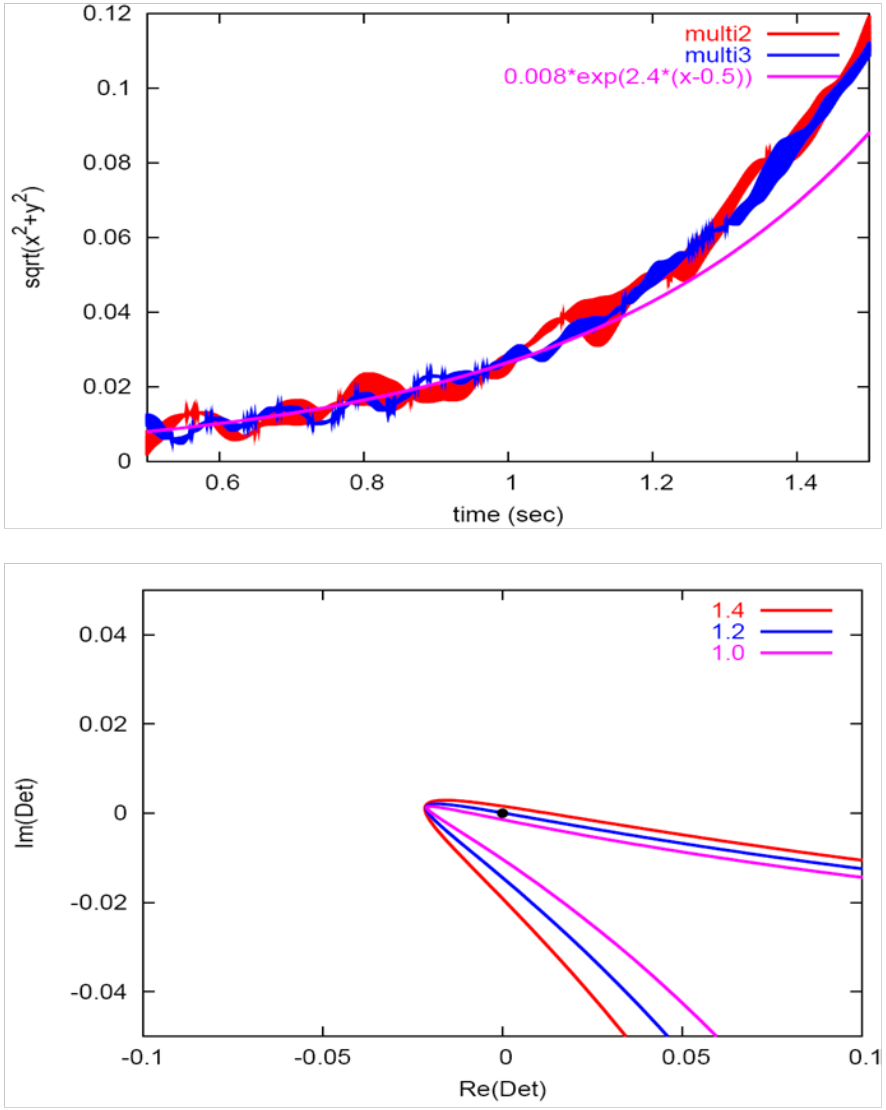


FIG. 4: Curves for $f_{max} = 10^8 s^{-1}$ and different growth rates.

seven 197s instead of one

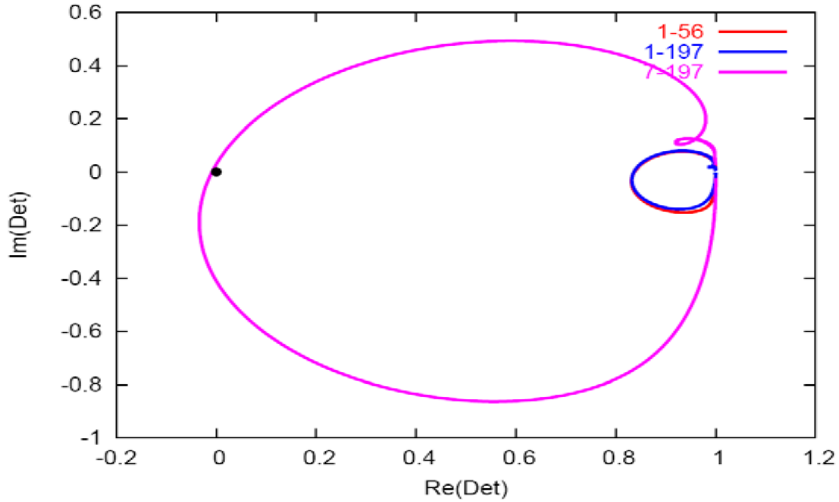


FIG. 5: Plot of the determinant for $Im(\Omega) = 1\ s^{-1}$ and 2×10^{11} protons with 2.4 MV on 56 MHz cavity. The $m = 1$ dipole mode is plotted. There were 720 bunches in the calculation but the narrow band Z was reduced by a factor of 6. Broad band Z unchanged.

The above is without the broad band Z/n.

Lowest frequency HOMs, broad band Z/n, $\varepsilon = 0.1 s^{-1}$

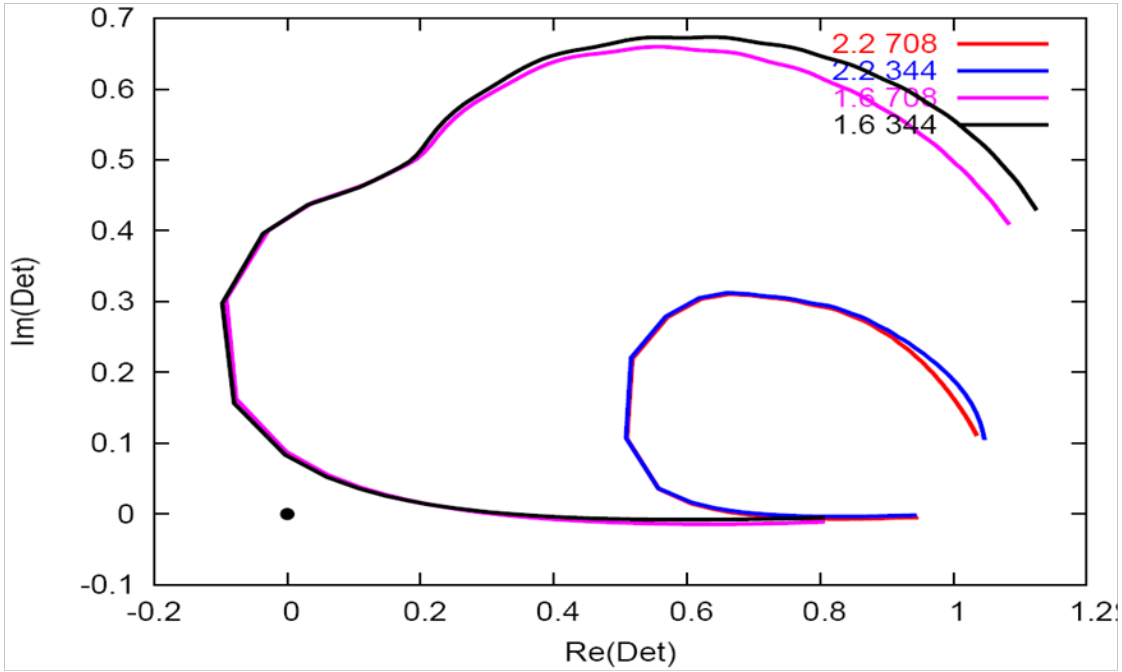


FIG. 6: Plot of the determinant for $Im(\Omega) = 0.1$ s^{-1} and 1×10^{11} protons with 2.4 MV on the 56 MHz cavity , mode 344 is for the 197 MHz HOM mode 708 is for the 56 MHz cavity. There were 720 bunches in the calculation so the narrow band Z was reduced by a factor of 6. The broad band Z was unchanged.

rms bunch length is important

Threshold with lowest frequency HOMs

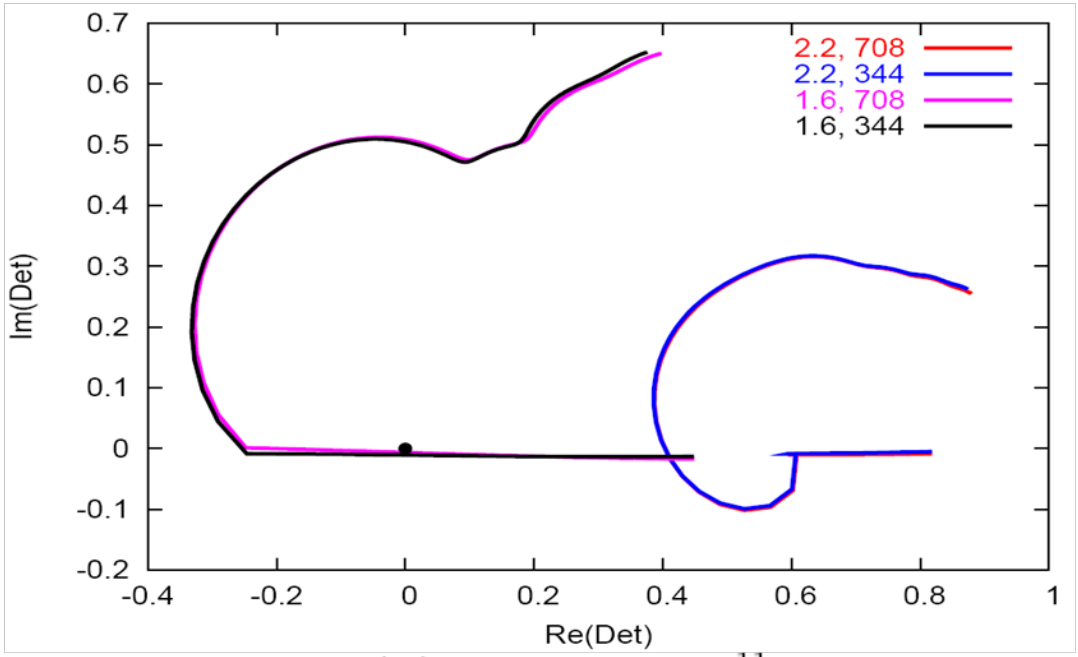


FIG. 7: Plot of the determinant for $Im(\Omega) = 0$ and 1×10^{11} protons with 2.4 MV on the 56 MHz cavity. Bunch lengths with $\sigma_t = 1.6$ ns and 2.2 ns are shown. Other details are like in Figs 5 and 6

Worst possible world

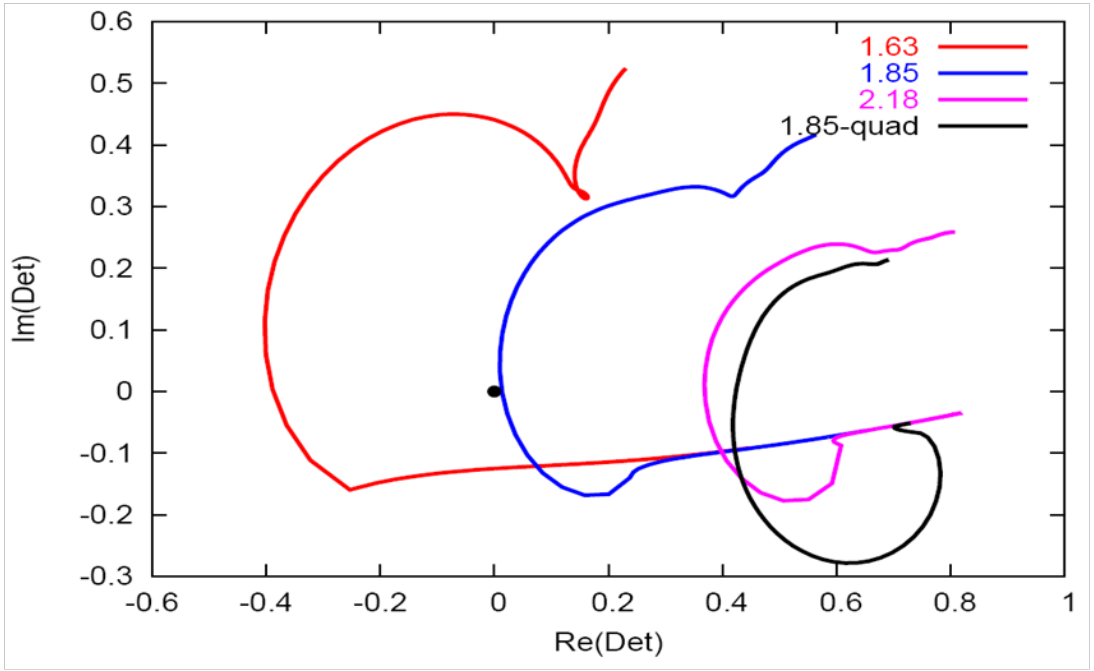


FIG. 8: Plot of the determinant for $Im(\Omega) = 0$ and 1×10^{11} protons with 2.4 MV on the 56 MHz cavity. Bunch lengths with $\sigma_t = 1.6$ ns, 1.8 ns and 2.2 ns are shown. Also, the quadrupole ($m = 2$) mode for the 1.8 ns mode is shown. All the 56 MHz modes below 600 MHz where adjusted slightly in frequency to drive the same, 708, CBM. There were 720 bunches used for the calculation and the narrow band impedance was reduced by a factor of 6 to simulate 120 bunches.

Conclusions

1) Behavior at transition and store were considered.
2) Near transition the 56 MHz cavity will cause no more emittance growth than the 197 MHz cavities already do.
3) During store the 56 MHz cavity has no significant impact on the stability threshold.

Backup: Granularity and simulation scaling with N

$$\ddot{x}_j + \Omega_j^2 x_j = -\frac{2g\Omega_0}{N}\sum_{k=1}^{N}\dot{x}_k$$

$$\Omega_j = \Omega_0 + \omega_j, \qquad |\omega_j| << \Omega_0$$

$$x_j = a_j \exp(-\lambda t - i\Omega_0 t)$$

$$(\lambda - i\omega_j)a_j = \frac{g\Omega_0}{N}\sum_{k=1}^{N} a_k$$

$$1 = \frac{g\Omega_0}{N}\sum_{k=1}^{N}\frac{1}{\lambda - i\omega_k}$$

$$\int_{-\infty}^{\omega_k} f(\omega)d\omega = \frac{k-1/2}{N}$$

$$\lambda \approx i\omega_K, \quad \Delta\omega = \frac{1}{Nf(\omega_K)}$$

$$\begin{aligned}
& \sum_{m=1}^{N}\frac{1}{\lambda - i\omega_m} \\
= & \sum_{|m-K|<M}\frac{1}{\lambda - i\omega_m} + \sum_{|m-K|\geq M}\frac{1}{\lambda - i\omega_m} \\
\approx & \sum_{|m|<M}\frac{1}{\lambda - i\omega_K - im\Delta\omega} + \sum_{|m-K|>M}\frac{i}{\omega_m - \omega_K} \\
\approx & \sum_{k=-\infty}^{\infty}\frac{1}{\lambda - i\omega_K - ik\Delta\omega} \\
+ & iN\int_{-\infty}^{\infty}\frac{\omega - \omega_K}{0^+ + (\omega - \omega_K)^2}f(\omega)d\omega. \qquad (13)
\end{aligned}$$

$$\lim_{M\to\infty}\sum_{k=-M}^{M}\frac{1}{z - ik} = \pi\frac{\exp(2\pi z) + 1}{\exp(2\pi z) - 1},$$

The infinite spike corresponds to the Vlasov mode.
Others are like stochastic cooling modes, but growing instead of damping.

$$R(\omega_K) = \pi\Omega_0 f(\omega_K) \qquad X(\omega_K) = \Omega_0 \int_{-\infty}^{\infty} \frac{\omega - \omega_K}{0^+ + (\omega - \omega_K)^2} f(\omega) d\omega$$

$$\exp[2\pi N f(\omega_K)(\lambda - i\omega_K)] = \frac{1 + gR - igX}{1 - gR - igX}$$

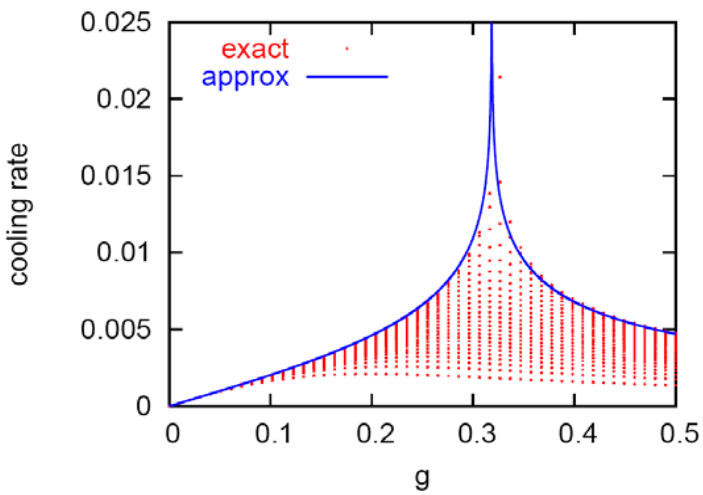


Figure 4: Comparison of actual values of $Re(\lambda)$ versus gain with those obtained from equation (14) with $X = 0$ for a rectangular frequency distribution with $N = 51$. The numerical solution had one eigenmode with a monotonically growing eigenvalue, which is not fully shown.

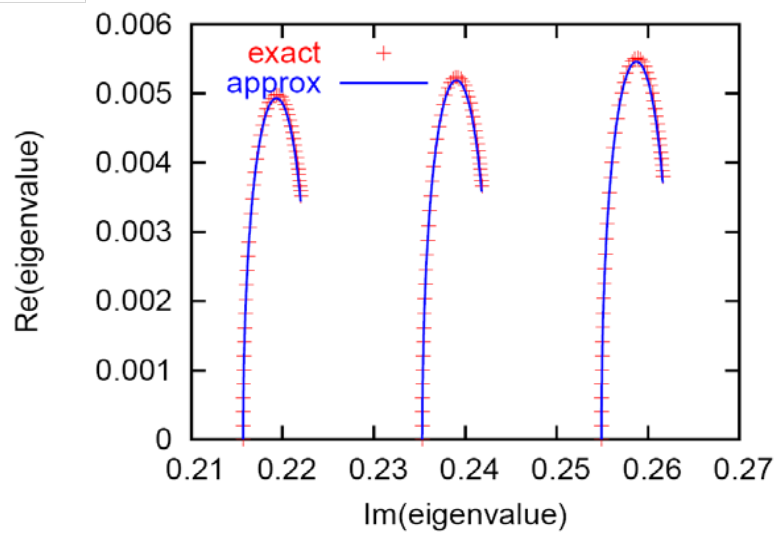


Figure 5: Evolution of λ as a function of gain for the exact, numerical solution and equation (14). The oscillator frequencies were uniformly spaced with $\omega_j = j/N$ and $N = 51$.