Microphonics, LLRF

- Microphonic requirements:
  - The requirement for the amplitude stabilization is $<10^{-4}$ at double the synchrotron frequency of RHIC (about 240 Hz with the 56 MHz cavity at 2.5 MV).
  - At this frequency the noise content is expected to be much less than 0.5 Hz.
  - The cavity is beam driven, off resonance, thus frequency errors get translated into amplitude errors.

The current in the circuit is given by

$$\vec{I} = I_b e^{j\omega t} = \frac{d}{dt}(\vec{V}C) + \frac{\vec{V}}{R} + \frac{1}{L}\int \vec{V}dt$$

where the capacitance is time dependant

$$C = C_0\left(1 + \mu\left(e^{j\omega_m t} + e^{-j\omega_m t}\right)\right),$$

where μ is half the amplitude of the sinusoidal modulation of the capacitor, which has a frequency ω_m.

$$\frac{d}{dt}C = jC_0\mu\omega_m\left(e^{j\omega_m t} - e^{-j\omega_m t}\right)$$

We assume that the voltage is composed of the carrier and two sidebands, at angular frequencies

$$\omega_\pm = \omega \pm \omega_m$$

$$\vec{V} = V_0 e^{j\omega t} + V_- e^{j\omega_- t} + V_+ e^{j\omega_+ t}$$

Therefore the time derivative and integral of the voltage are given by

$$\frac{d}{dt}\vec{V} = j\omega V_0 e^{j\omega t} + j\omega_- V_- e^{j\omega_- t} + j\omega_+ V_+ e^{j\omega_+ t}$$

$$\int \vec{V}dt = \frac{V_0}{j\omega} e^{j\omega t} + \frac{V_-}{j\omega_-} e^{j\omega_- t} + \frac{V_+}{j\omega_+} e^{j\omega_+ t}$$

We can substitute the values given above for the voltage, capacitance and their derivatives or integrals. Then we collect the terms for a particular sideband, say the upper sideband, and this results in the following connection between the amplitude of the carrier and that of the sidebands:

$$\frac{V_\pm}{V_0} = -\frac{jRC_0\mu\omega}{1 + 2jQ\dfrac{\Delta\omega \pm \omega_m}{\omega}}$$

where $\Delta\omega=\omega-\omega_r$ is the detuning of the resonator from the beam frequency, and this is a positive quantity since the resonator is below the beam frequency. With a bit more manipulation we get

$$\left|\frac{V_\pm}{V_0}\right| = \frac{\mu Q}{\sqrt{1 + 4Q^2\left(\dfrac{\Delta\omega \pm \omega_m}{\omega}\right)^2}}$$

Thus, to avoid a large disturbance of the voltage by the sidebands, μ should be small compared to 1/Q, particularly when a sideband nulls the detuning.

Assuming a frequency deviation of $\pm$0.56 Hz, then $\mu \sim 10^{-8}$. For a loaded Q of 4×10^{7}, the worst value of the sideband amplitude ratio is 0.4, and this is only for a small band of mechanical frequencies for which the mechanical frequency ω_m cancels $\Delta\omega$ within about one Hertz. The requirement for the amplitude stabilization is that it be better than 10^{-4} at frequencies of about double the synchrotron frequency of RHIC (about 240 Hz with the 56 MHz cavity at 2.5 MV). At this frequency the noise content is expected to be much less than 0.5 Hz.

C-AD will use digital LLRF in all its systems

System capable of $2*10^{-7}$ with 4 kHz sampling or 10^{-4} direct (with averaging)