

## Merger designs for ERLs<sup>☆</sup>

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### Abstract

Energy recovery linacs (ERLs) are potential candidates for the high power and high brightness electron beams sources. The main advantages of ERL are that electron beam is generated at relatively low energy, injected and accelerated to the operational energy in a linac, and after the use is decelerated in the same linac down to injection energy, and, finally, dumped. A merging system, i.e. a system merging together high energy and low energy beams, is an intrinsic part of any ERL loop. One of the challenges for generating high charge, high brightness electron beams in an ERL is development of a merging system. In this paper, we discuss merger system currently employed or planned to use for ERL as discuss their advantages and shortcomings. We also discuss analytical approach showing a way towards an optimal merger.

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### 1. Introduction

Energy recovery linacs (ERLs) is emerging accelerator technology, which promises to become a major driver for accelerator application requiring the high current and high brightness electron beams. The main advantages of ERL are that a fresh electron beam is generated at relatively low energy, injected and accelerated to the operational energy in a linac, *and after a single use* is decelerated in the same linac to injection energy, and is deposited of after taking its energy back.

This feature of ERL makes it especially attractive for the processes, which significantly affect quality of electron beam at a single pass, such as a significant energy spread growth in an FEL [1] or a significant increase of the transverse emittance in an interaction point (IP) of a collider [2]. Similarly, ERLs promise to maintain high

average and peak brightness of electron beams during acceleration and the use in future synchrotron radiation sources [3]. Higher brightness of these light sources will be achieved by limiting the affects of quantum fluctuations of spontaneous radiation on the energy spread and emittance growth to a few turns compared with continuous affect in storage rings. ERL also promise to bring to life X-ray sources with sub-picosecond durations.

A generic one-loop ERL is shown in Fig. 1. It has a gun system, a merger, a linac, a loop and, finally, a dump. In all cases, an ERL should preserve the high brightness electron beams generated at the gun through the entire process of acceleration, merging and transportation to the place of the use. Only after the use the preservation of beam quality becomes less important, unless it affects the energy recovery and lossless transportation to the dump.

ERL should operate with ampere-class beam currents to be competitive with storage rings. It translates into CW electron beams with average power from hundreds of megawatts to tens of gigawatts. Only *low injection energy* and very high efficiency energy recovery in superconducting RF (SRF) linacs makes the such ERL economically feasible. Furthermore, the use of *low injection and ejection*

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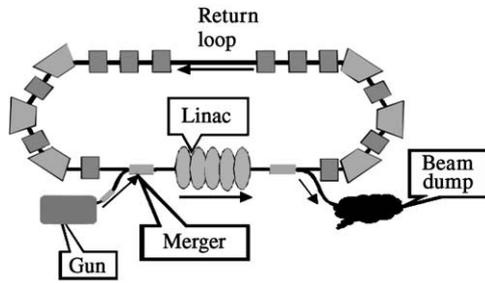


Fig. 1. Schematics view of ERL: electron beam is generated in the gun passes through a merger section, is accelerated to high energy, used and then is decelerated and dumped.

energies mitigates the radiation and environmental issues related to a dumping of megawatt class electron beam. Using electron energy well below 10 MeV dramatically reduces nuclei activation of the beam dump material which otherwise may become a major environmental and cost problem.

A merging system, i.e. a system merging together high energy and low energy beams, is an intrinsic part of any ERL loop located between the gun (which is desirably generates low emittance high quality electron bunches) and the main linac. It means that low energy electron, affected by space charge fields, will propagate through the merger before being accelerated to high energy where space charge effects are suppressed.

Ways of combating the emittance growth in high brightness electron accelerators is well understood both theoretically [4] and experimentally [5]. This method, called emittance compensation,<sup>1</sup> was developed for systems with axial symmetry. Any merger is using at least one dipole magnet, which both breaks the axial symmetry and strongly couples longitudinal and transverse (that in the plane of the bending) degrees of freedom. Hence, the traditional method of emittance compensation is no longer directly applicable to the merger system.

One of the challenges for generating high charge, high brightness electron beams in an ERL is development of a merging system, which provides achromatic condition for space charge dominated beam and which is compatible with the emittance compensation scheme.

In the absence of space charge forces, the coupling between longitudinal and transverse degrees of freedom is canceled by the use of an achromatic lattice for a merger—i.e. a magnetic system where transverse position of the particles at its exist does not depend on particles energy. The design of such achromatic system assumes that energy of the particles remains constant while they propagate through the system. There are varieties of such achromatic systems, some of which are currently used for ERLs [6–8].

<sup>1</sup>Meaning of this name is somehow confusing because the 6-D emittance is naturally preserved in the process according to Lowville's theorem. But areas projected onto  $x-p_x$  and  $y-p_y$  phase space planes are not invariants and, therefore, can both increase and decrease.

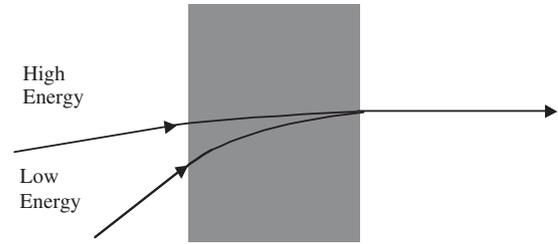


Fig. 2. Main function of a merger—combining two (or more) beams with different energies.

We will discuss these mergers and discuss the limits of their applicability in the following sections.

In the presence of strong space charge forces energy of particles changing while they propagate through a merger [9]. This changes, which can be rather significant, violate the achromatic conditions and blow out transverse emittance. In Section 4 we discuss a novel approach [10,11] providing for decoupling of the transverse and longitudinal motion and for attainment of low emittances typical for best emittance compensation schemes. We also present and discuss one practical design of such system.

## 2. Issues related to the merger design

Prior to discussing specific designs let us consider the basic requirements for a merger in high brightness ERL. First, the main goal of the merger is to merge two-electron beams with different trajectories and different energies onto a common orbit in the ERL's linac. This is possible with a use of a dipole magnet, which bend trajectories with the radii proportional of the beams relativistic moments  $p$ :

$$\rho = \frac{pc}{eB}, \quad E \equiv \gamma mc^2 = \sqrt{p^2 c^2 + m^2 c^4} \quad (1)$$

where  $E$  is the energy of electron,  $e$  is it electric charge,  $\gamma$  is relativistic factor,  $c$  is the speed of the light and  $B$  is the dipole's magnetic field. As the result, trajectories of beams are bent on different angles, hence providing for beam separations or conversion (see Fig. 2).

For generation of high brightness electron beams it is strongly desirable to decouple transverse and longitudinal motions. Decoupling provides the possibility of utilizing emittance compensation schemes and, most importantly, removes (or significantly reduce) undesirable nonlinear coupling terms. Let us find these conditions for a generic merger as follows [10,11].

Let us assume that the merger has magnets where bending occurs in horizontal plane.<sup>2</sup> Dipole field couples longitudinal and transverse motions. Electron can be described by a point in the 4-D phase space, a phase space

<sup>2</sup>Generalization for 3-D trajectory in the merger is straightforward [11], but goes beyond the subject of this paper.

vector  $Z$

$$Z = \begin{pmatrix} x \\ x' \\ \xi \\ \delta \end{pmatrix} \quad (2)$$

comprised of two Canonical pairs representing the transverse  $\{x, x' \equiv dx/ds = p_x/p_o\}$  and the longitudinal  $\{\xi = v_o(t_o(s) - t), \delta = E - E_o/p_o c\}$  degrees of freedom, where subscript “o” indicates parameter ( $v$  is velocity) of an ideal electron in the bunch and  $s$  is the length of the trajectory along the pass of ideal particle (i.e. standard independent variable in accelerator physics).

The motion of particles from azimuth  $s_1$  to  $s_2$  can be represented by a symplectic map  $M$  [12]

$$Z(s_2) = \mathbf{M}(s_1|s_2) : Z(s_1). \quad (3)$$

The symplectic condition can written as condition on the  $4 \times 4$  transport matrix  $\mathbf{T}$  using the map linearization in a vicinity if any arbitrary trajectory  $Z_o(s)$ :

$$Z = Z_o + \delta Z, \quad \delta Z(s_1) = \mathbf{T}(s_0|s_1, Z_o) \cdot \delta Z(s_0).$$

The condition has well-known asymmetric matrix form [13]:

$$\mathbf{T}^T \cdot S \cdot \mathbf{T} = S, \quad S = \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix}, \quad \sigma = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad (4)$$

which corresponds to six Poincare’ invariants of motion. Using  $2 \times 2$  block presentation of the transport matrix

$$\mathbf{T} = \begin{bmatrix} M & P \\ Q & N \end{bmatrix}, \quad M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \quad (5)$$

one can rewrite six symplectic condition as follows:

$$\begin{aligned} \det M + \det Q &= 1, \\ \det M &= \det N, \quad \det Q = \det P, \\ M^T \sigma P + Q^T \sigma N &= 0. \end{aligned} \quad (6)$$

Decoupling of longitudinal and horizontal motions is equivalent to requirement of matrix  $\mathbf{R}$  to be a block-diagonal matrix with  $P = 0$  and  $Q = 0$ . In this case  $\det M = 1$  ( $\det N = 1$  as well) and last of Eqs. (6) can be solved explicitly by multiplying last equation by  $M\sigma$  and taking into account the identity  $M\sigma M^T = \sigma \cdot \det M$ :

$$P = -M\sigma Q^T \sigma N. \quad (7)$$

It means that satisfying four explicit conditions  $Q = 0$  is both necessary and sufficient for full decoupling, i.e. it makes  $P = 0$  as well.

Let us derive these conditions using 1-D equation of motion for the horizontal direction. The betatron part of linearized Hamiltonian can be written as a symmetric bilinear form

$$H = \frac{1}{2} X^T H X, \quad X^T = (x, x'), \quad H^T = H \quad (8)$$

with typical form of the Hamiltonian matrix of

$$H(s) = \begin{bmatrix} 1 & 0 \\ 0 & K_1(s) \end{bmatrix} \quad (9)$$

where  $K_1(s)$  is the focusing (de-focusing) strength which must include both the magnets and the space-charge forces [4,11,13].

Inclusion of space-charge forces makes the system significantly more complicated even for elliptical beams with homogeneous density because the focusing depends on the beam charge and the beam sizes, which in return depend on the focusing. Therefore, these equations should be solved self-consistently. In many cases it is possible to solve the system iteratively. Our considerations here do not depend on the method of solution but focused on the approach. Note also, that the space-charge forces can depend on the longitudinal position of the particle. This dependence is one of the major reasons why emittance compensation [4] is very important for maintaining low emittance of electron beam. Detailed discussions of compatibility of emittance compensation with decoupling requirements in a merger can be found elsewhere [11].

The horizontal homogeneous differential equation of motion in vector form are also well known

$$X' = D(s) \cdot X, \quad D = \sigma \cdot H \quad (10)$$

which are solvable in the form of transport matrix:

$$X(s) = M(s_0|s) \cdot X_o$$

which also satisfy the equations of motion (10):

$$M' = D(s) \cdot M, \quad M \equiv M(s_0|s). \quad (11)$$

Let us consider now a particle whose energy is deviated from that of ideal particles as [11]

$$E(s) = E_o \cdot (1 + \delta(s)) \quad (12)$$

and write a modification of well known inhomogeneous (i.e. transverse dispersion) differential equation:

$$X' = D(s) \cdot X + \delta(s) \cdot \begin{bmatrix} 0 \\ K_o(s) \end{bmatrix} \quad (13)$$

where  $K_o(s) = 1/\rho(s)$  is the curvature of trajectory defined by the dipole field (1). Eq. (13) differs from traditional equations [13] for transverse dispersion only by a fact that energy of the particle is no longer a constant and is changing along the pass of the electron. General solution of Eq. (13) can be found using traditional variation method for ordinary linear differential equations [14]  $X = M(s) \cdot A(s)$ :

$$\begin{aligned} X(s) &= M(s_0|s) \cdot \\ &\left\{ X_o + \int_{s_0}^s \delta(s_1) \cdot M^{-1}(s_0|s_1) \cdot \begin{bmatrix} 0 \\ K_o(s_1) \end{bmatrix} ds_1 \right\} \end{aligned} \quad (14)$$

with obvious specific solution with zero initial conditions  $R(s_0) = 0$ , which we will call generalized dispersion [11]:

$$R(s) = \begin{bmatrix} \int_{s_0}^s K_o(s_1) \cdot \delta(s_1) \cdot m_{12}(s_1|s) ds_1 \\ \int_{s_0}^s K_o(s_1) \cdot \delta(s_1) \cdot m_{22}(s_1|s) ds_1 \end{bmatrix}. \quad (15)$$

Note that we used the identity relation for transport matrices as:  $M(s_0|s) \cdot M^{-1}(s_0|s_1) = M(s_1|s)$ . Specific solution for  $\delta = 1$  is well known in accelerator physics as (transverse) dispersion function with two typical notations:  $\eta(s)$  and  $D(s)$  [13,15]. In this paper, we will use  $\eta(s)$ . We assume  $\eta, \eta'$  to be zero at initial azimuth  $s_0$ :

$$\begin{aligned} \eta(s) &= \int_{s_0}^s K_o(s_1) \cdot m_{12}(s_1|s) ds_1, \\ \eta'(s) &= \int_{s_0}^s K_o(s_1) \cdot m_{22}(s_1|s) ds_1. \end{aligned} \quad (16)$$

Naturally, generalized dispersion (15) can be expressed using  $\eta(s)$  by a simple integration in Eq. (14) by parts:

$$R(s) = \delta(s) \cdot \begin{bmatrix} \eta(s) \\ \eta'(s) \end{bmatrix} - \int_{s_0}^s \delta'(s_1) \cdot M(s_1|s) \cdot \begin{bmatrix} \eta(s_1) \\ \eta'(s_1) \end{bmatrix} ds_1. \quad (15')$$

Conventional achromats have  $\eta(s) = 0$  and  $\eta'(s) = 0$  at the exit of the system, which provide for required decoupling in the absence of the longitudinal space charge forces (LSCFs) when  $\delta' = 0$ .

In the presence of the space charge forces  $\delta' \neq 0$  and conventional achromats are no longer sufficient for the required decoupling. In a general case energy variation along the pass depends on the parameters of the merger and is a function of initial phase space coordinates at its entrance:

$$\delta(s) = f(s, \delta_o, \xi_o, X_o, Y_o), \quad Y^T = [y, y'] \quad (17)$$

and it is hard to expect that for an arbitrary function  $f$ ,  $R(s_f) = 0$  at the merger exit,  $s_f$ .

It seems natural that in a decoupled case (see Eq. (5) with  $P = Q = 0$ ) there should be only two parameters, which define the energy evolution in the system. Detailed studies of a specific schemes [10,11] found that there are number of correlations, which allow the use of some simpler functional dependencies in Eq. (17), at least for the longitudinal space charge effects in the mergers. The accuracy of these assumptions should be judged by the success or the failures of direct simulation using 3-D codes, which include space charge effects (see following sections).

As suggested in Refs. [10,11], in a number of cases one can use parametric dependence of the energy variation on initial phase space coordinate as follows:

$$\delta(s) = f_1(\delta_o, \xi_o, X_o, Y_o) \cdot g_1(s) + f_2(\delta_o, \xi_o, X_o, Y_o) \cdot g_2(s) \quad (17')$$

where  $f_{1,2}(\delta_o, \xi_o, X_o, Y_o)$  are arbitrary functions of the initial coordinates and  $g_{1,2}(s)$  are some known function of the azimuth. In this case the generalized dispersion at the

exit of the merger can be written using (15) as

$$\begin{aligned} R(s) &= f_1(\delta_o, \xi_o, X_o, Y_o) \cdot R_1(s) \\ &\quad + f_2(\delta_o, \xi_o, X_o, Y_o) \cdot R_2(s), \\ R_{1,2}(s) &= \begin{bmatrix} \int_{s_0}^s K_o(s_1) \cdot g_{1,2}(s_1) \cdot m_{12}(s_1|s) ds_1 \\ \int_{s_0}^s K_o(s_1) \cdot g_{1,2}(s_1) \cdot m_{22}(s_1|s) ds_1 \end{bmatrix} \end{aligned} \quad (18)$$

and to define requirement for the decoupling in the merger in the form of four integral to be zeroed:

$$R_1(s_f) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad R_2(s_f) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (19)$$

It is worth noting that conditions (19) satisfy the decoupling (i.e. emittance minimization) conditions independently of specific functional dependence  $f_{1,2}(\delta_o, \xi_o, X_o, Y_o)$ , which may strongly depend on the initial beam distribution, its charge, etc. The method we propose is solely based on the knowledge of two functions  $g_1(s)$  and  $g_2(s)$ .

One of the popular test choices for the parametrization (17) is

$$\begin{aligned} f_1 &= \delta_o, & g_1(s) &= 1, \\ f_2 &= f_2(\xi_o), & g_2 &= s \end{aligned} \quad (20)$$

which we will call further a “frozen case”, i.e. the case when the longitudinal force of each electron remains constant during propagation in the merger. In this case, four conditions are reduced to a conventional achromatic conditions  $\eta(s_f) = 0$  and  $\eta'(s_f) = 0$  at the exit of the system:

$$\begin{aligned} \int_{s_0}^{s_f} K_o(s) \cdot m_{12}(s|s_f) ds &= 0, \\ \int_{s_0}^{s_f} K_o(s) \cdot m_{22}(s|s_f) ds &= 0 \end{aligned} \quad (21a)$$

plus two non-trivial conditions of:

$$\begin{aligned} \int_{s_0}^{s_f} K_o(s) \cdot s \cdot m_{12}(s|s_f) ds &= 0, \\ \int_{s_0}^{s_f} K_o(s) \cdot s \cdot m_{22}(s|s_f) ds &= 0. \end{aligned} \quad (21b)$$

Their equivalent form for a conventional achromat is as follows:

$$\int_{s_0}^{s_f} M(s|s_f) \cdot \begin{bmatrix} \eta(s) \\ \eta'(s) \end{bmatrix} ds = 0. \quad (21'b)$$

Analysis of the emittance growth caused by a merger for the “frozen case” of the LSCF can be also derived using by first-order beam transport theory [16]. This method is summarized in Appendix A.

### 3. Mergers used in operational ERLs

At present time there are three operational ERLs at Tomas Jefferson National Accelerator Facility (TJNAF), Newport News, VA, USA, at Budker Institute of Nuclear

Physics (BINP), Novosibirsk, Russia and at Japan Atomic Energy Research Institute (JAERI), Tokai-mura, Ibaraki, Japan. Two ERLs at BINP and TJNAF use conventional achromatic lattices for their mergers. A conventional achromat is designed in assumption of single particle optics, i.e. in the absence of both transverse and longitudinal space charge effects.

Novosibirsk’s ERL [6] is using a thermionic gun with 1.5 nC bunches with normalized emittance of 30 mm mrad. The beam is accelerated to about 2 MeV before entering the merger chicane. Rather large normalized emittance of the beam allows the use of such system, which does not satisfy achromatic condition for space charge dominated beam (see next sections). In this case the growth of the normalized emittance for a few mm mrad was considered as acceptable. Fig. 3 shows one of a simplest achromatic system—a chicane. Advanced modification of chicane is used at BINP’s ERL [6].

ERL at TJNAF [7] is using a DC photo-injector gun with rather low, 135 pC, charge per 2 ps bunch and a rather large normalized emittance of 10 mm mrad. Electrons are accelerated to 9.1 MeV before entering a merger. The merger is a conventional three-dipole achromat (see Fig. 4), which uses focusing strength of the middle dipole to make have  $\eta(s) = 0$  and  $\eta'(s) = 0$  at the exit.

As shown in Fig. 4, the TJNAF’s merger has very large non-compensated dispersion related to the space charge (and as explained in Appendix A, also to coherent synchrotron radiation). As shown in Appendix A, such

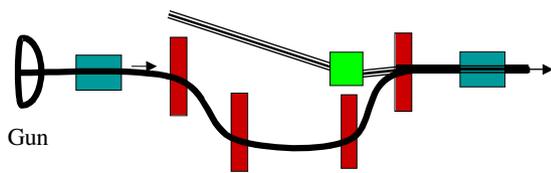


Fig. 3. An achromatic chicane comprises of four dipole magnets with parallel edges.

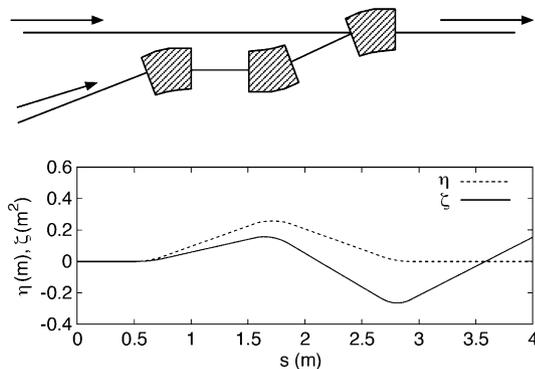


Fig. 4. Lattice of three dipole merger similar to that employed at Jlab’s ERL [7]. On the graph one can see the conventional momentum dispersion function  $\eta$  and the space charge dispersion function  $\zeta$  (as defined in the Appendix A) along the merger.

system strongly couples longitudinal and transverse emittance and causes the normalized emittance growth at the level few mm mrad even for high (9.1 MeV) injection energy. Again, rather low charge per bunch, very high injection energy and rather high initial beam emittance allowed the use of such merger without significant performance degradation.

Lattice of JAERI’s ERL merger has a dog-leg configuration with parallel-edge dipoles and a strongly focusing triplet in the middle, which makes the system achromatic. This system also has achromatic matching for a ‘frozen case’ of space charge forces. Emittance growth in the merger in this system comes from its incompatibility with emittance compensation technique (see next section). As the result, in the JAERI injection merger LSCF causes significant emittance growth [8]. The best simulation result showed normalized RMS emittances of 35 and 26 mm mrad for horizontal and vertical planes correspondently for a 9.4 ps, 0.5 nC bunches [8].

#### 4. Merger design for low emittance ERL

As we had seen in previous section, all existing ERLs operate with electron beam emittances 10–30 times larger compared with those generated by best linac photo-injectors [5,17]. At the same time most of the future ERL projects plan to use very low beam emittances comparable or even better than those achieved in the best linac photo-injectors [2,3,9].

In contrast with the ERLs, the low-emittance linacs are based on emittance compensations scheme utilizing axial symmetric elements till the end of the main linac, i.e. there is no dipoles or mergers or any kind seen by low energy beam.

In order to attain low beam emittance operation in ERLs it is essential to combine a merger, which decouples the longitudinal and transverse degrees of freedom, with emittance compensation schemes. One of very important requirements for emittance compensation scheme to work is that the motion of the electron in the section remains laminar (i.e. electron trajectories do not cross) [4]. The laminar flow conditions are clearly violated in systems shown in Figs. 4 and 5 which rely on strong focusing to make the conventional dispersion (and space charge

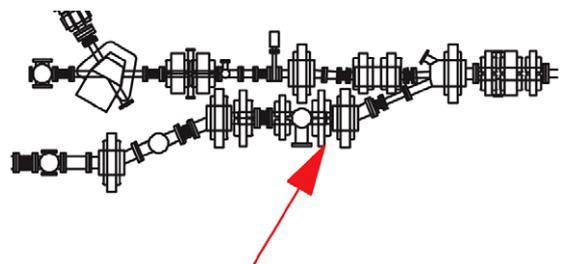


Fig. 5. Lattice of JAERI’s ERL merger.

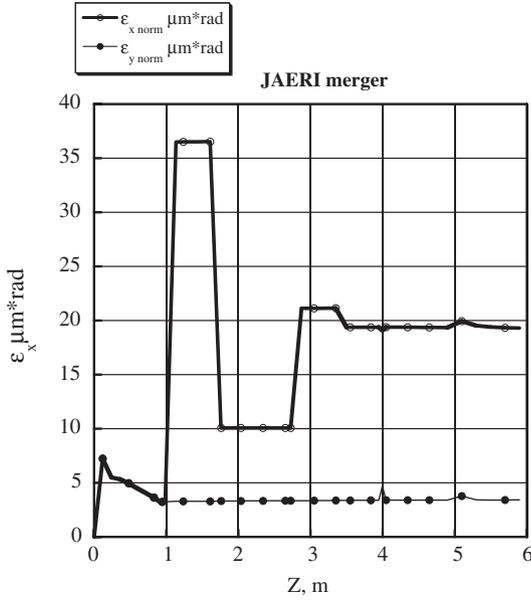


Fig. 6. Emittance evolution of normalized transverse emittances of 1 nC bunch in an ERL based on SRF 2.5 MeV gun and 20 MeV and 700 MHz SRF linac [10] equipped with a JAERI-type merger. Normalized horizontal emittance grows to about 20 mm mrad in such a system.

dispersion, in the case of JAERI design) to be zero. In these systems the focal length of the elements (like the focusing of the second dipole in the TJNAF scheme or triplet in the JAERI scheme) is much shorter than the length of the merger, which inevitably causes the crossing of the electrons trajectories (in other words a very small value of  $\beta$ -function) and a strong violation of the laminar flow conditions. As the result the electron beam is focused into an extremely small spot where the conditions for emittance compensation are violated and the beam emittance grows significantly (see Fig. 6).

There are two effects which are important for design of a merger for space charge dominate e-beam:

- (a) the space charge de-focusing must be taken into account in the design of the achromatic merger. Defocusing caused by space charge can modify significantly the achromatic conditions;
- (b) lattice of the merger must be designed with the use of only weakly focusing elements with focal lengths larger or of the order of the merger length.

If one of these conditions is not satisfied, the space charge causes significant emittance growth in a merger, often irreversible by practical means (see Refs. [4,11] for details).

Natural way of designing a merger for a low emittance ERL should include a dipole scheme, which provide for these conditions by its geometry, i.e. without use of any strong focusing elements. One of such system is described below.

### 5. Zigzag merger

The idea of Zigzag merger [9–11] came from a simple observation of the energy variation in space charge dominated electron beam from 1.5 cell SRF gun studied for BNL’s ERL [10,11] using PARMELA [18]. Our studies did show that for a large number of cases we can use following approximate formulae for particles energy

$$\delta(s) \cong \delta_0 + f(\zeta_0)(s + \alpha s^2) \tag{22}$$

where functional dependence on the initial longitudinal position in the bunch,  $\zeta_0$ , can be rather accurately approximated by a analytical formula for the field of the homogeneously charged cylinder [10,19]:

$$f(\zeta_0) = a \left( 2\zeta_0 - \sqrt{r^2 + (\zeta_0 + l)^2} + \sqrt{r^2 + (\zeta_0 - l)^2} \right)$$

where  $a$  as an coefficient,  $r$  is the beam radius and  $l$  is the bunch length.

Fig. 7 shows a typical  $s$ -dependent energy variations which is very close to linear, i.e. as in so-called frozen case. It implies two additional simple conditions on the merger lattice (21b). One of general approaches for developing merger lattices satisfying the conditions (21) can be the using of lattice symmetries.

Fig. 8 shows one of such system, which is symmetrical in the focusing ( $K_1$ ) and asymmetrical in the curvature ( $K_0$ , i.e. in the dipole field). It is well known that elements of transport matrix in such system satisfy following conditions:

$$m_{11}(-s) = m_{11}(s), \quad m_{12}(-s) = m_{12}(s). \tag{23}$$

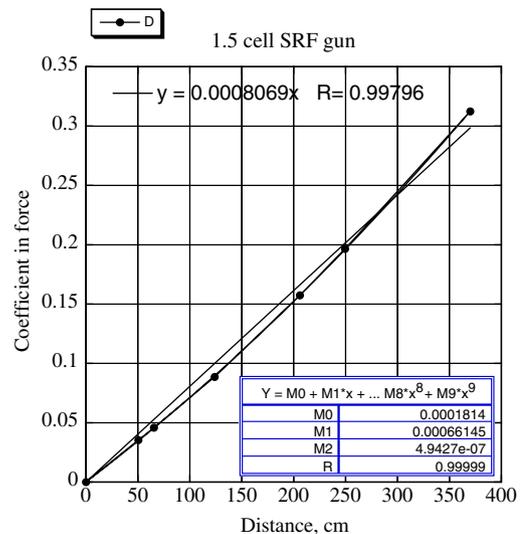


Fig. 7. Dependence of the energy gain on the azimuth  $s$ . Dots are the results of simulations; the lines are linear and second-order polynomial fits.

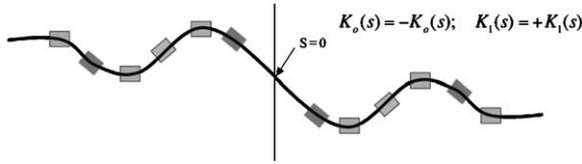


Fig. 8. Schematic of a Zigzag based on the symmetry: light-gray boxes are the dipoles, dark-gray boxes are focusing and defocusing lenses.

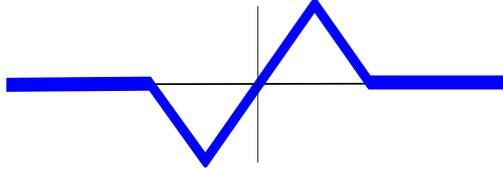


Fig. 9. Schematic trajectory in a simplest Zigzag based on four dipole magnets with parallel edges.

It automatically makes two of integrals zero

$$K_o(-s)m_{11}(-s) = -K_o(s)m_{11}(s)$$

$$\Rightarrow \int_{-L}^L K_o(s')m_{11}(s') ds' \equiv 0$$

$$K_o(-s)(-s)m_{12}(-s) = -K_o(s)(s)m_{12}(s)$$

$$\Rightarrow \int_{-L}^L K_o(s')m_{12}(s')s' ds' \equiv 0$$

(where  $2L$  is the length of the merger) and only two conditions remain:

$$\int_0^L K_o(s') \cdot m_{12}(s') ds' = 0, \quad \int_0^L K_o(s') \cdot s \cdot m_{11}(s') ds' = 0. \quad (24)$$

Usually, the remaining conditions can be satisfied by moving dipoles within this Zigzag merger.

As an oversimplified example of a Zigzag merger let us consider a system consists of  $2K$  short dipoles (with bending angle  $\theta_k$  and position  $s_k$  each) without focusing in horizontal direction. In this case the elements of transport matrix are:  $m_{11} = 1, m_{12} = s$  and only one condition remains:

$$\sum_{k=1}^K s_k \cdot \theta_k = 0. \quad (25)$$

For  $K = 2$  (a four dipole Zigzag) the condition (25) gives a simplest Zigzag with  $s_2 = 2s_1, \theta_1 = -2\theta_2$  [10] (Fig. 9).

Our simulation tests demonstrated that this simple concept of Zigzag combined with optics typical for emittance compensation schemes (i.e. a couple of solenoids located between the gun and the main linac in axially symmetric system (see Fig. 10)) provided for almost ideal preservation of emittances both in and out of the bending planes for magnets with small bending angles. To our surprise this simple concept, some version of which were

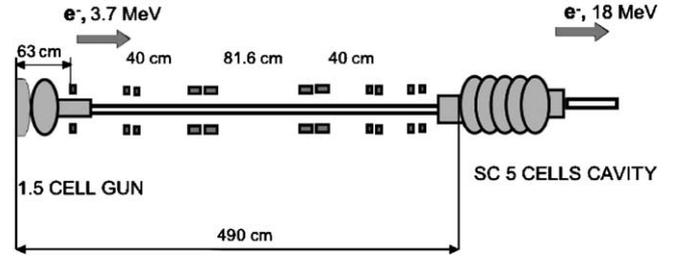


Fig. 10. A schematic geometry system with axial symmetry comprising of a 1.5 cell 3.7 MeV electron SC 703.75 MHz RF gun and 5-cell SC 703.75 MHz RF linac with emittance compensation solenoids between them.

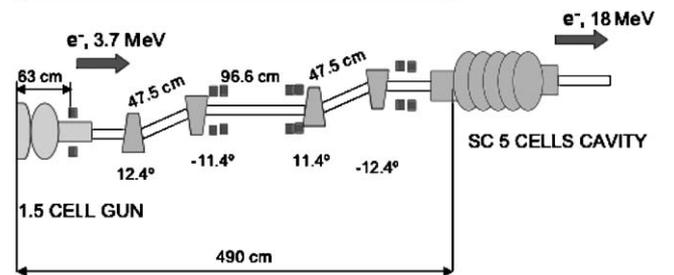
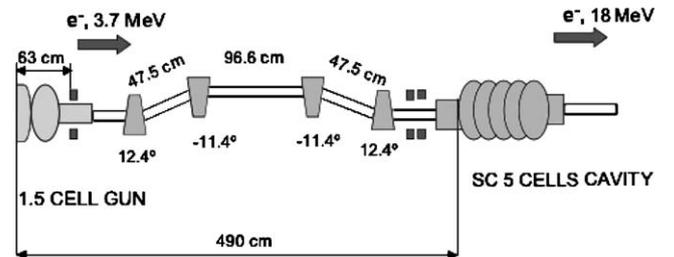
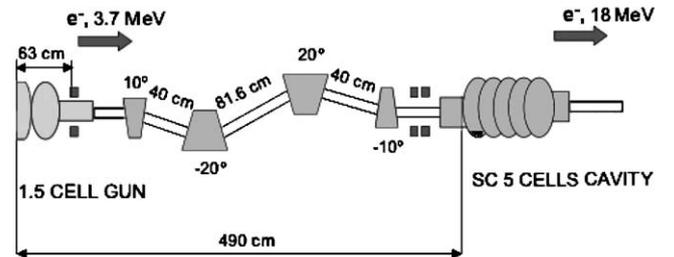


Fig. 11. Three merger schemes (from top to bottom: the Zigzag, the chicane and the Dog-leg) installed into the emittance compensations system, shown in Fig. 10.

intuitively used previously [20], works very well for many processes, including space charge dominated magnetized beams [21,22] and coherent synchrotron radiation [8,20].

Increase of the bending angles causes additional focusing, which should be almost evenly distributed between horizontal and vertical directions using edge focusing (so-called chevron magnets), and the strength of solenoidal focusing should be adjusted in order to preserve the emittance compensation mechanism. Furthermore, the geometry of the Zigzag should be slightly adjusted (i.e. the angles and distances between the magnets (see Refs. [10,11])) to take into account the focusing from the dipole magnets and the defocusing from the beam space charge

forces in the matrix elements and to satisfy the decoupling conditions (21).

In order to compare performance of Zigzag with axially symmetric scheme (without a merger) and with another merger schemes we developed a following test, illustrated in Fig. 11, by adding dipole magnets into the scheme shown in Fig. 10 to form achromatic mergers in form of a Zigzag, a chicane and a “Dog-leg”, while keeping the length of the path the same. All configurations are achromatic for particle with constant energy, i.e. in the conventional sense that regular dispersion is compensated (21a). In the Zigzag this condition was satisfied by a slight increase of the center straight section to 81.6 from 80 cm. In the chicane it was reached by modification of the angles. And in the dog-leg the achromaticity was achieved by introducing two focusing solenoids to make a minus unit matrix, i.e. the recipe similar to that used in JAERI and Jlab mergers, where strong focusing is used to make the system achromatic.

To make a fair comparison, all systems have the same focusing strength and are made of chevron dipoles with 86 cm radii or curvature. The strength of the solenoids was adjusted for the best emittance compensations in all cases.

In the numerical test performed with PARMELA, a 1 nC electron bunch from the 1.5-cell RF gun was propagated through the above systems followed by a 15 MeV, 703.75 MHz linac. The electron beam at the gun exit has total energy of 4.2 MeV (kinetic energy of 3.7 MeV). Initial beam has “beer-can distribution” with duration of  $12^\circ$  and radius 4 mm at the cathode of the gun. Comparison of the emittance evolution in the system is shown in Fig. 12. In two merging systems (the chicane and the Zigzag), vertical emittance evolution is very similar to that of the axially

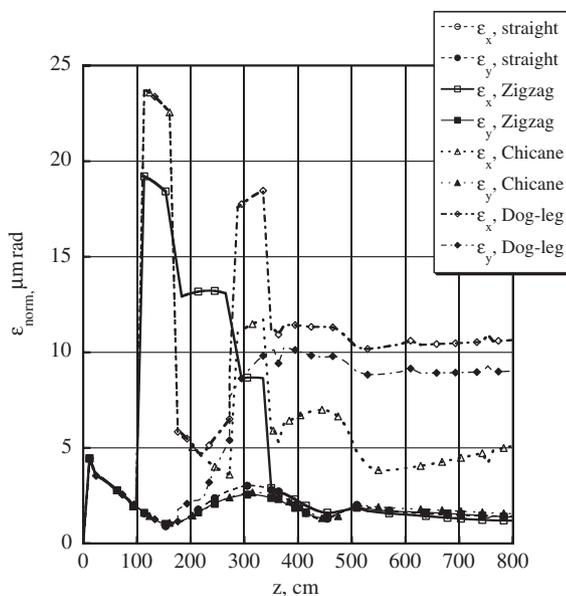


Fig. 12. Evolution of horizontal and vertical normalized emittances in the four systems: axially symmetric emittance compensation scheme, the Zigzag, the chicane and the Dog-leg.

symmetric system with final value of about 1.4 mm mrad (normalized). In the chicane horizontal emittance remain very large ( $\sim 5$  mm mrad, normalized) after the merger and main linac. Detailed study shows that this is direct result of energy variation along the path and the violation of the decoupling condition (21b) for this case.

At the exit of the Zigzag merger, horizontal emittance is practically identical to the vertical and reaches 1.4 mm mrad (normalized) at the end of the system, i.e. when the emittance compensation process is completed. Hence, in this case the Zigzag provided both for the decoupling conditions (21) and for emittance compensation.

In contrast with both the chicane and the Zigzag, the Dog-leg merger causes significant increases in both horizontal and vertical emittance. Even though the Dog-legs formally satisfies the decoupling conditions (21) for a low charge beams, it violates the laminar conditions via use of a strong focusing elements necessary for achromaticity of this system. Detailed studies of the distributions show that the focusing elements cause a very sharp pinching of the beam at and around  $z = 2.7$  m, which causes blow-up of both emittances to about 10 mm mrad (normalized).

Further studies of the Zigzag merger showed that it preserves its qualities (i.e. equal emittances and compatibility with emittance compensation schemes) for a wide range of the beam parameters (charges from 0.1 to 10 nC per bunch, energies as low as 2.5 MeV in the merger, energy spread of  $\pm 10\%$  in the e-beam, etc.) and also helps with emittance preservation even in magnetized beams. Slight modifications of the Zigzag geometry allow also compensating for slightly nonlinear functional dependencies on  $s$  (see Fig. 7). These issues further explored in Ref. [11].

Performance of Zigzag merger, which is based on a very simple idea, exceeded our expectations. Even though it is obvious that the assumption on which the Zigzag merger concept is based should fall apart in the case of strong longitudinal motion or in the case of very short bunches. Still, this concept provides interesting opportunities of preserving emittances for ERL operating with rather impressive bunch charges (up to 10 nC) and with low or modest energy of injection.

Furthermore, detailed studies of the correlations in the electron beam can provide for the use of the above analytical approach for creating more advanced and more elaborate mergers (both in the geometry and the focusing features [11]) preserving the emittances and compatible with the emittance compensation.

## 6. Conclusions

Merger is one distinct element of any ERL, which makes it different from standard axi-symmetric low emittance linear accelerators. Desire to operate electron beams with significant charges per bunch and to lower energy of injection into ERL requires mergers compatible with emittance compensation in space-charge dominated beams.

In addition, variation of particles energies along the pass of a merger, caused by the space charge forces of the bunch, introduce additional conditions on the merger lattice.

Mergers used in presently operating ERLs were not designed for operating with very low emittance electron beam, and, therefore, cannot be used for ERL operating beams with normalized emittances  $\sim 1$  mm mrad or lower.

The concept of a Zigzag merger, based on a rather simple assumption, promises (at least at the level of the 3-D simulations using PARMELA) to solve some of the challenges presented by future ERL operating with super-bright intense electron beams. Compatibility with emittance compensation schemes and simple geometry of Zigzag mergers promise to be useful in the next generation of ERL. The experimental validity of the Zigzag merger and its performance in ERL will be tested in 20 MeV, 0.5 A ERL which is under construction as Brookhaven national Laboratory [9].

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### Appendix A. Analysis of emittance growth in an ERL merger using first-order theory of a beam motion

Although the analysis of emittance growth is a general subject in accelerator physics, the emittance growth of a low-energy beam passing through bending magnets has not been well studied. The emittance growth due to transverse space charge force (TSCF) has been studied in the development of a high-brightness electron beam from a photocathode RF-gun for free-electron lasers. [4]. In some ERL mergers, transverse focusing is applied by quadrupole magnets instead of solenoid, because dipole magnets introduces asymmetric beam envelopes in horizontal and vertical planes. The compensation of emittance growth due to TSCF in a merger is possible in a similar manner to the case of an RF gun by using quadrupole focusing field.

The emittance growth is also induced by LSCF [23]. This is based on the same physics as CSR-induced emittance growth [24], and can be explained as the violation of achromatic condition due to electron-energy redistribution in a bunch traveling through a bending path.

The emittance growth due to LSCF can be calculated by first-order beam transport theory as well as the CSR case [16]. If an electron bunch does not change its longitudinal

and transverse size largely in a merger, we can assume that the longitudinal space charge potential keeps a constant profile. Under this assumption, a first-order equation of electron horizontal motion in a beam transport element is given by

$$x'' = -\frac{x}{\rho^2} + \frac{1}{\rho}(\delta_o + \delta_{sc} + \kappa(s - s_0)) \quad (\text{A.1})$$

where  $\delta_{sc}$  is accumulated energy deviation caused by LSCF upstream of the element,  $\kappa$  is the normalized LSCF potential in the element. Given a vector to specify the motion of an electron in a horizontal plane:  $\vec{x} \equiv (x, x', \delta_o, \delta_{sc}, \kappa)$ , one can solve the electron trajectory through a beam path using a first-order transfer matrix:

$$\vec{x}^T(s_1) = R(s_1|s_0)\vec{x}^T(s_0). \quad (\text{A.2})$$

The transfer matrix for each element is derived by Green's function method (see Ref. [15, p. 107]):

$$R = \begin{bmatrix} R_{11} & R_{12} & R_{12} & R_{14} & R_{15} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} \\ R_{31} & R_{32} & R_{33} & 0 & 0 \\ 0 & 0 & 0 & 1 & L \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{A.3})$$

where  $R_{ij}(i, j = 1; 2; 3)$  are same as those of a conventional  $3 \times 3$  matrix [15],  $L$  is the length of the element. For a sector magnet with bending radius  $\rho$  and bending angle  $\theta$ :  $R_{14} = \rho(1 - \cos \theta)$ ,  $R_{15} = \rho^2(\theta - \sin \theta)$ ,  $R_{24} = \sin \theta$  and  $R_{25} = \rho(1 - \cos \theta)$ .

For other elements such as a drift and a quadrupole, these elements are equal zero.

We define the space-charge dispersion function to track a linear off-axis motion due to LSCF:

$$\begin{aligned} &(\zeta_x(s_1), \zeta'_x(s_1), 0, L(s_0), 1)^T \\ &= R(s_1|s_0)(\zeta_x(s_0), \zeta'_x(s_0), 0, L(s_0), 1)^T \end{aligned} \quad (\text{A.4})$$

where  $L(s) = s$  is the total path length. Each bunch slice has an off-axis motion correlated to the LSCF potential. In the linear regime, each bunch slice aligns on the line  $\zeta_x x' - \zeta'_x x = 0$  in the  $(x, x')$  phase space. The emittance growth due to LSCF, which is displacement of bunch slices in the  $(x, x')$  phase space, can be minimized by matching the displacement to the orientation of the phase ellipse at the merger exit. Given the LSCF potential, we can calculate emittance growth in the same manner as the CSR case [25]:

$$\varepsilon^2 = (\varepsilon_o \beta_x + D^2)(\varepsilon_o \gamma_x + D'^2) - (\varepsilon_o \alpha_x + DD')^2 \quad (\text{A.5})$$

where  $\varepsilon_o$  and  $\varepsilon$  are the initial and the final emittance as un-normalized values, respectively,  $(D, D') \equiv \kappa_{\text{rms}}(\zeta_x, \zeta'_x)$  is rms spread of bunch slice displacement in  $(x, x')$  plane, and  $\kappa_{\text{rms}}$  is rms spread of the normalized LSCF potential.

As an example, we consider the emittance growth and its compensation in a JLab-type three-dipole merger as shown in Fig. 4. The bending radius and angle are 1 m and  $15^\circ$ , respectively. The second magnet has edge rotation of  $-21^\circ$ ,

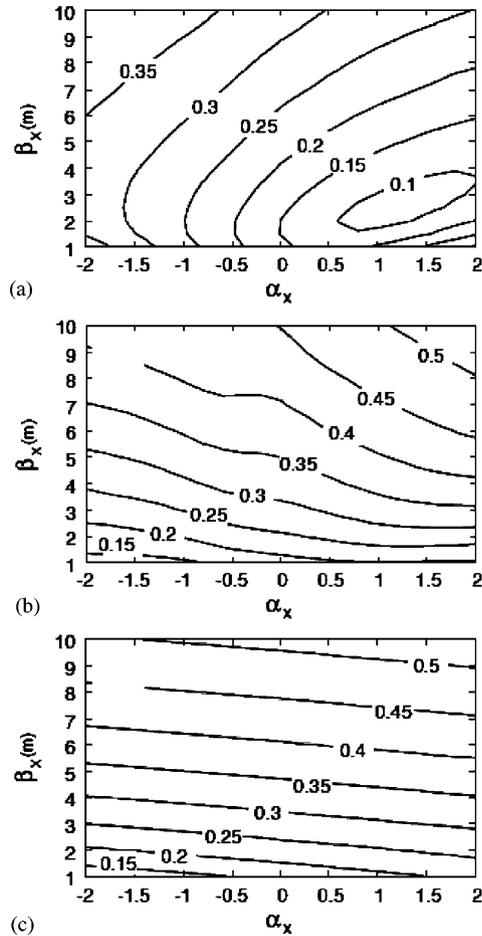


Fig. A1. Emittance growth in the three-dipole merger as a function of  $\alpha_x$  and  $\beta_x$  at the merger entrance. The contours represent  $\Delta\epsilon$  (mm-mrad): (a) growth by TSCF (numerical); (b) growth by LSCF (numerical); and (c) growth by LSCF (analytical).

and the drift between magnets is 0.82 m. The momentum dispersion and the space charge dispersion functions along the merger are also shown in Fig. 4.

The emittance growth is calculated by a particle tracking code JPP [26], with scanning Courant–Snyder parameters at the merger entrance:  $2 : 0 < \alpha_x < 2 : 0$ ,  $1 \text{ m} < \beta_x < 10 \text{ m}$ . Envelope for the vertical plane is fixed as  $\alpha_y = 2 : 76$  and  $\beta_y = 8 \text{ m}$  at the merger entrance. The bunch parameters are chosen as energy of 10 MeV, charge of 77 pC, initial normalized emittance of 1 mm mrad, bunch length of 6 ps (rms of Gaussian profile). The calculated emittance growth by TSCF and LSCF is plotted in Fig. A1, where the emittance growth is defined as  $\Delta\epsilon_n^2 = \epsilon_{n,f}^2 - \epsilon_{n,i}^2$ .

It can be seen that the emittance growth depends on the beam envelope in the merger as predicted above. However, we cannot eliminate the total emittance growth in the merger, because the optimum envelope for two sources of emittance growth is not consistent with each other.

We also plot the emittance growth due to LSCF obtained by linear analysis in Fig. A1. The result is in good agreement with the numerical result, but shows some

deviation for large  $\beta_x$ , where the assumption of constant LSCF potential is not valid, and higher-order aberrations may exist.

In the three-dipole merger, we have residual emittance growth not canceled by envelope matching. The emittance growth, however, is reasonably small, if both the bending angle and the total path length of merger are small enough [27].

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