Multiple Spin Resonance Crossing

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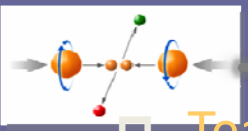
Spin Echo Experiment



- Team: L. Ahrens, M. Bai, A. Chao, V. Litvinenko, H. Huang, V. Ptitsyn, T. Roser, K. Yip
- Goal: observe spin echo and study resonance interference
- Experiment description
  - Machine: AGS
  - Energy: $G\gamma = 4.6$
  - Part I: to measure the spin echo
    - Drive the ac dipole adiabatically at a tune near the spin precession tune 0.6. Cross the resonance by jumping the ac dipole tune through the spin tune. Wait a time of τ and jump the ac dipole tune back to its start value
    - Start the CNI polarimeter measurement right after the second jump for 2τ. In order to observe the echo, the data need to be binned in < 1ms window. Detailed studies/simulation is in working progress
  - Part II: to study the resonance interference

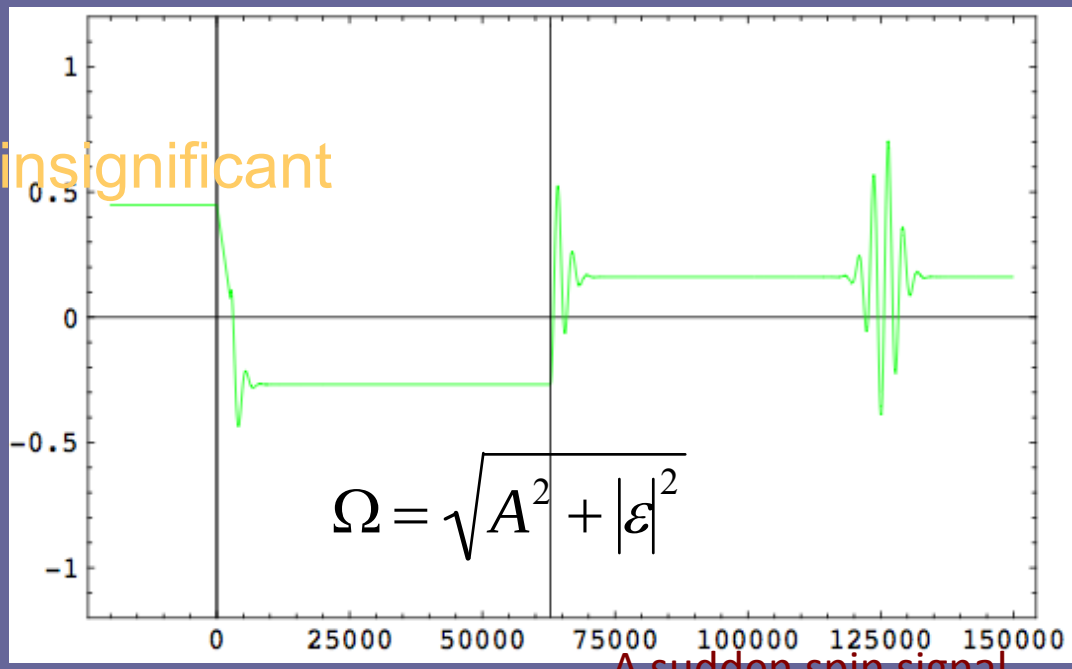
Spin Echo

- Condition:
  - interference terms are insignificant

$$\theta_2 - \theta_1 \geq \frac{\sqrt{2}\Omega}{|A|\kappa\sigma_\delta}$$

  - Jump Length

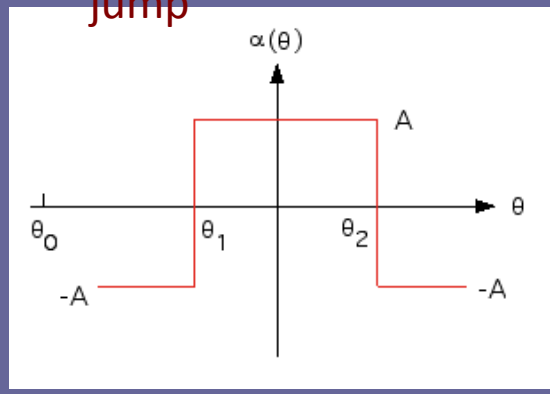
$$\frac{1}{\kappa} \leq N_{jump} \leq \frac{\sqrt{2}\Omega}{2\pi|A|\kappa\sigma_\delta}$$



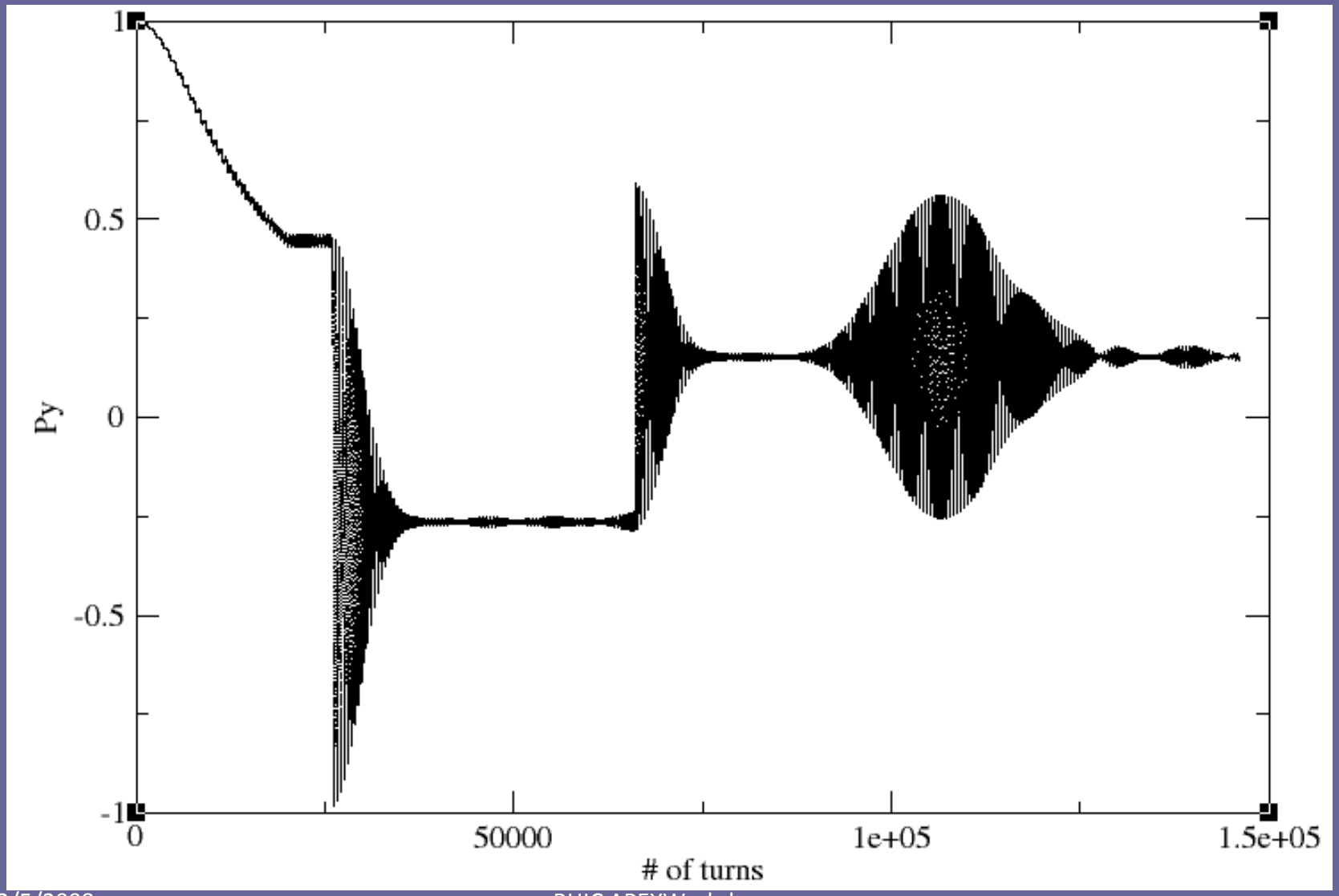
A sudden spin signal occurs at $\theta_2 - \theta_1$ after the second resonance jump

- The magnitude of echo signal

$$P_{y,echo} = \frac{2A|\varepsilon_0|^4}{\Omega^5} \quad when\ A = \frac{|\varepsilon_0|}{2},\ P_{y,echo,\max} = 57\%$$



Simulation results



Spin Echo Smearing

- The echo signal can be smeared out due to high order dependence of omega on the momentum spread, i.e.

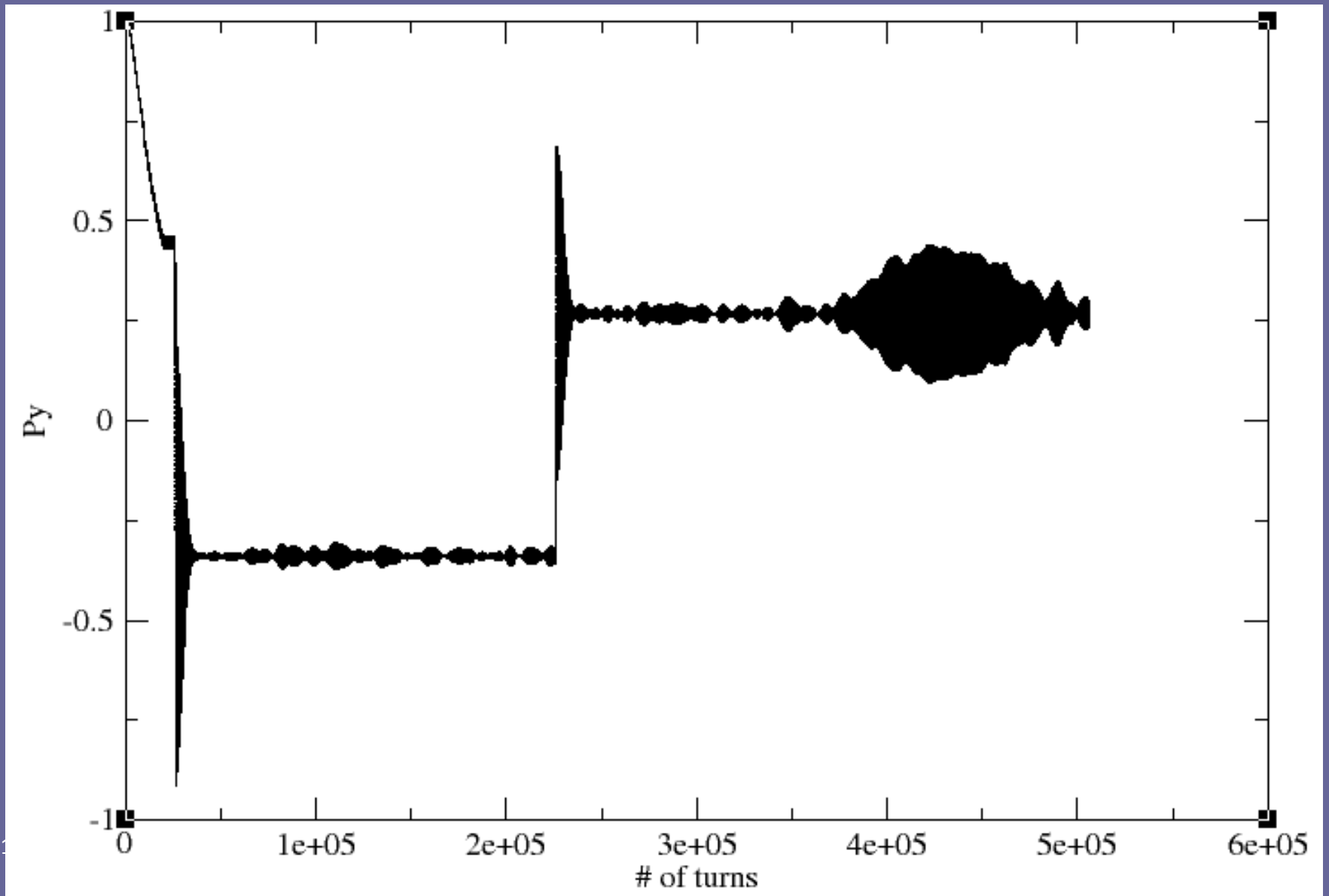
$$\Omega = \sqrt{(A+\kappa\delta)^2 + |\varepsilon_0|^2} \approx$$

$$\sqrt{A^2 + |\varepsilon_0|^2} + \frac{A\kappa\delta}{\sqrt{A^2 + |\varepsilon_0|^2}} + \frac{|\varepsilon_0|^2 (\kappa\delta)^2}{2(A^2 + |\varepsilon_0|^2)^{\frac{3}{2}}}$$

- The echo signal is significant damped if

$$\frac{1}{\theta_2 - \theta_1} < \frac{|\varepsilon_0|^2 (\kappa\delta)^2}{2\left(A^2 + |\varepsilon_0|^2\right)^{\frac{3}{2}}}$$

Simulation results



Difficulty of spin echo experiment at AGS

- Beam energy: Gγ=4.6
- AGS: a signle ac dipole at frequency ~100kHz and 20 Gauss-m
  - ε_0 = 0.0001
  - This requires:
    - A=0.00005
    - Momentum spread: σ_δ~ 4×10^{-6}
    - This is very difficult to obtain
  - In addition, the oscillating field of single ac dipole drives two resonances instead of one. This introduces additional motions

Spin Echo Experiment at RHIC ?

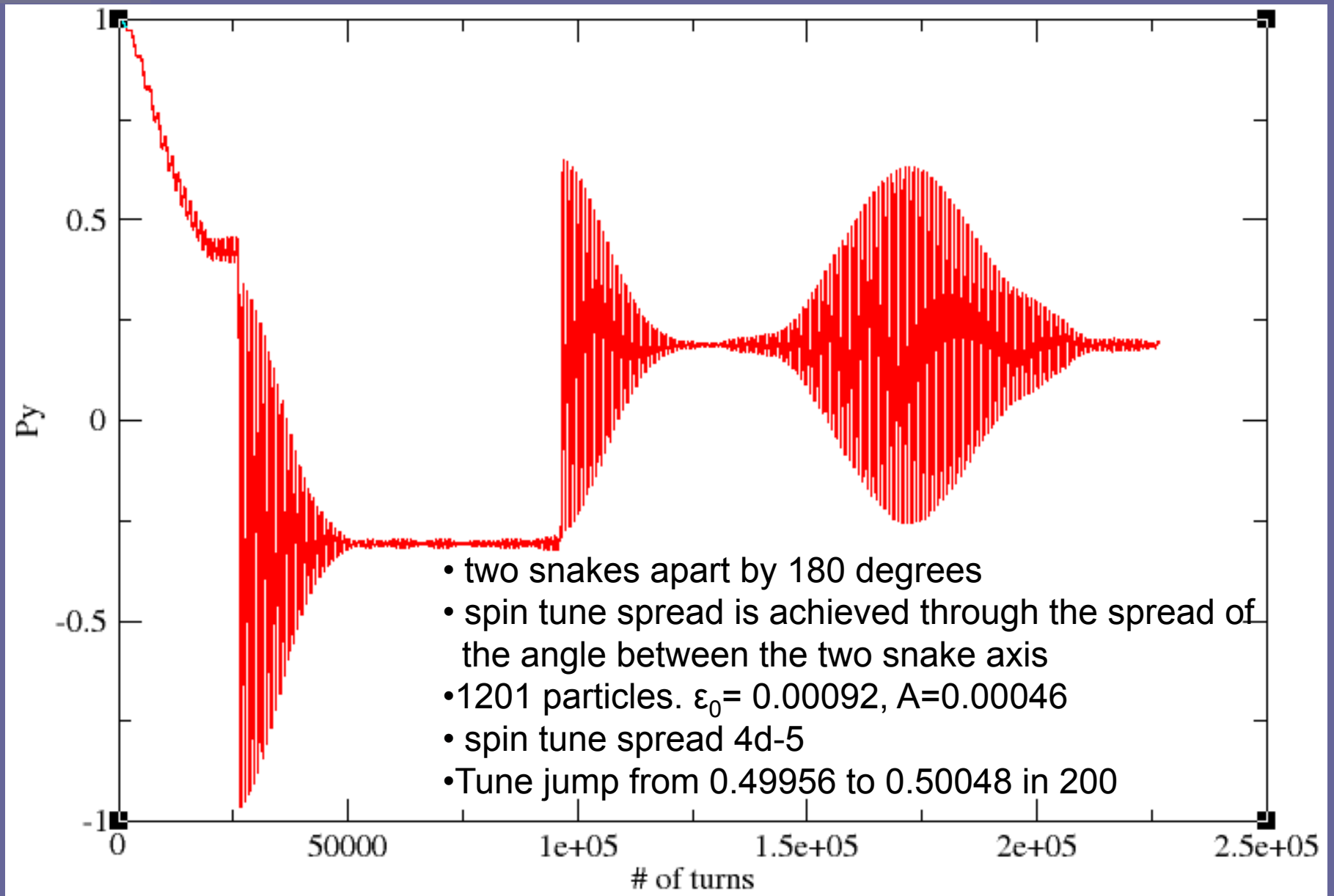
- RHIC spin flipper provides a rotating field with a maximum strength of $\varepsilon_0 = 0.00092$
- Small spin tune spread because the two full snakes which removes the spin tune dependence on the beam energy
- Source of Spin tune spread
  - difference of horizontal divergence of the beam at the two snakes. This causes a spread of horizontal angle between the two snakes. The spin tune spread is given by

$$\frac{G\gamma\,\Delta\sqrt{\frac{1+\alpha^2}{\beta}\varepsilon}}{\pi}$$

Beam emittance

Twiss parameter at snakes

Simulation of Spin Echo In RHIC



- two snakes apart by 180 degrees
- spin tune spread is achieved through the spread of the angle between the two snake axis
- 1201 particles. ε_0= 0.00092, A=0.00046
- spin tune spread 4d-5
- Tune jump from 0.49956 to 0.50048 in 200

Multiple Spin Resonance Crossing

- The interference between each resonance crossing depends on the spin phase advance in between. It is a function of the speed of each resonance crossing as well as the time between each resonance crossing
  - Chao crossing
  - Kondratenko crossing

- Application:
  - Spin flipping
  - Avoid polarization losses

Chao Crossing

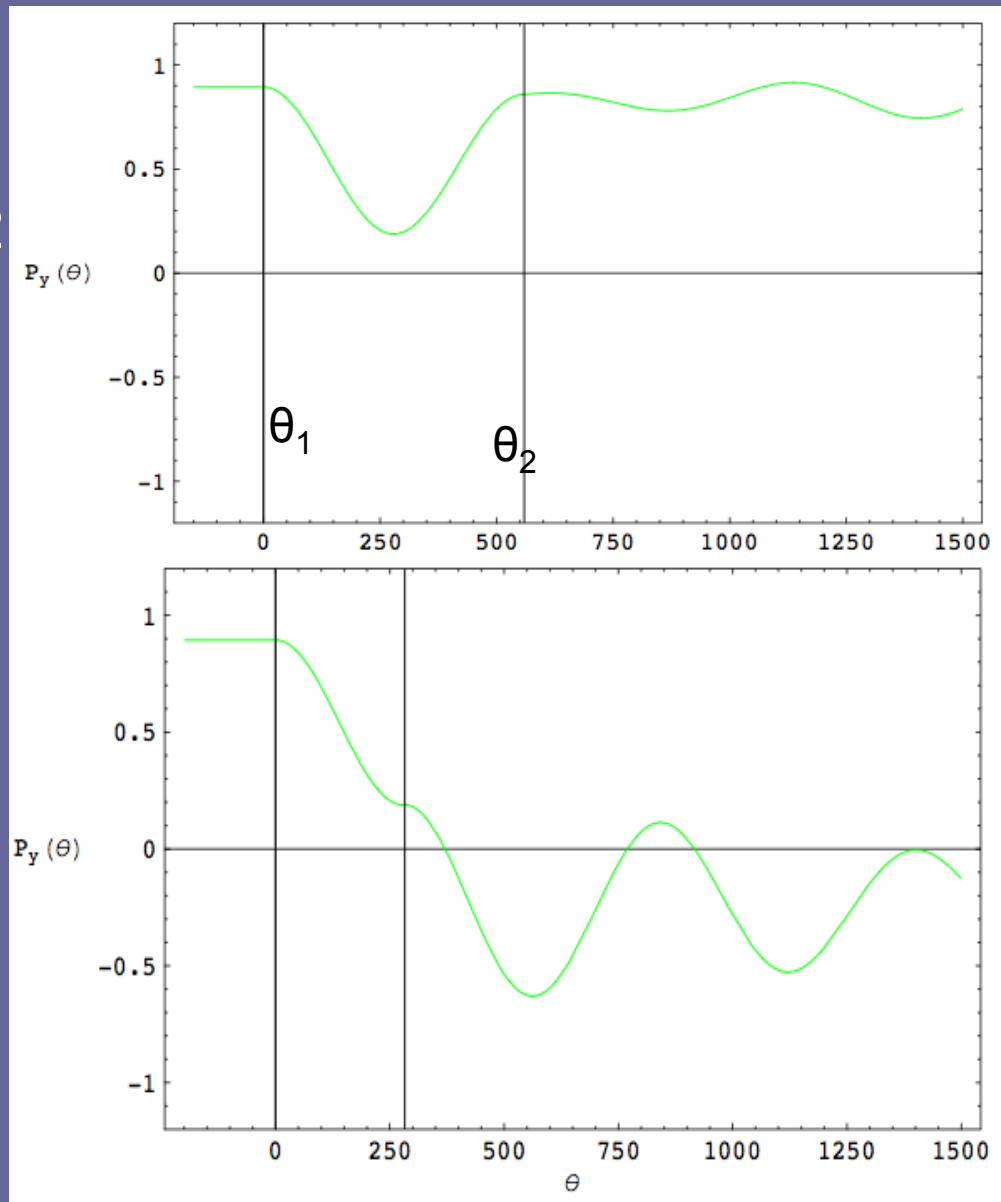
- multiple particles with Gaussian distributed momentum spread
- $\sigma_E/E=0.0002$
- $\varepsilon_0=0.005$, A=0.01, jump size:0.02

- Destructive case

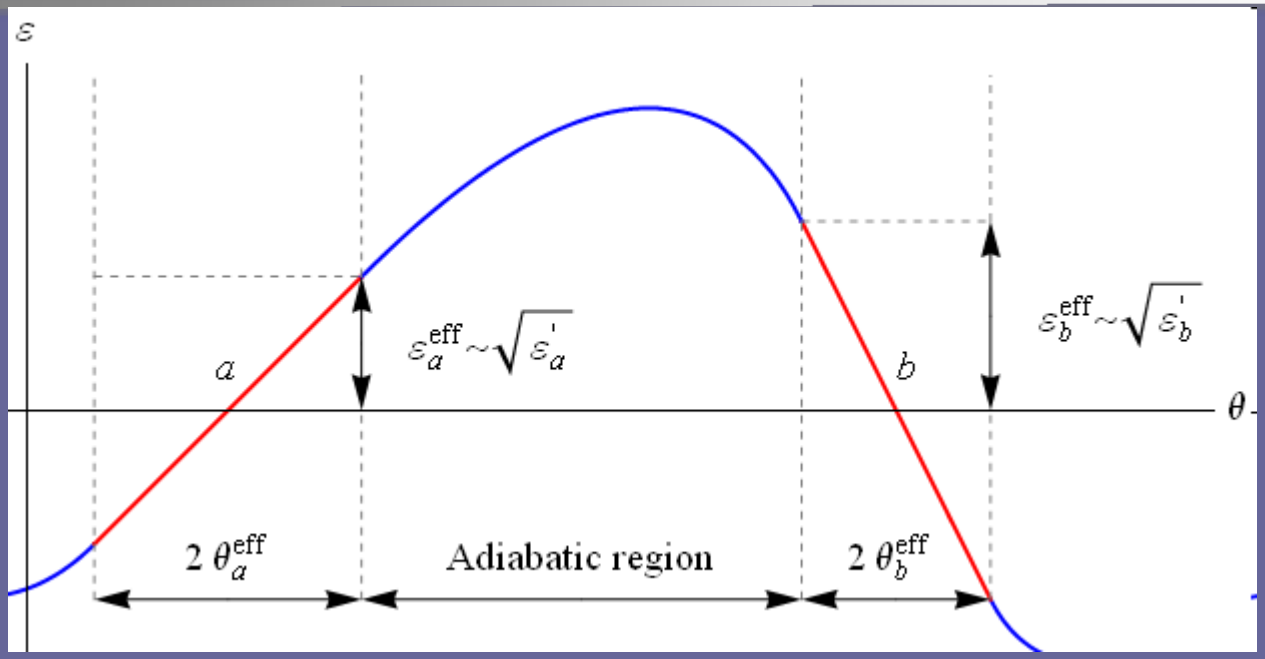
$$\Omega(\theta_2-\theta_1)=2k\pi$$

- Constructive case

$$\Omega(\theta_2-\theta_1)=k\pi+\frac{\pi}{2}$$



Kondratenko crossing



The spinor transformation matrix is

$$M(\theta_f,\theta_i) = M_z(\Psi_f)\, M_y(-2\beta)\, M_z(\Psi_{ab})\, M_y(2\alpha)\, M_z(\Psi_i)$$

$$\sin(\alpha) = \exp\left(-\frac{\pi\,æ_a^2}{4}\right), \qquad \sin(\beta) = \exp\left(-\frac{\pi\,æ_b^2}{4}\right),$$

$$\Psi_i = \int_{\theta_i}^{a} h_k\, d\theta + \varphi_a \qquad \Psi_f = \int_{b}^{\theta_f} h_k\, d\theta + \varphi_b \qquad \Psi_{ab} = \int_{a}^{b} h_k\, d\theta + \varphi_a + \varphi_b$$

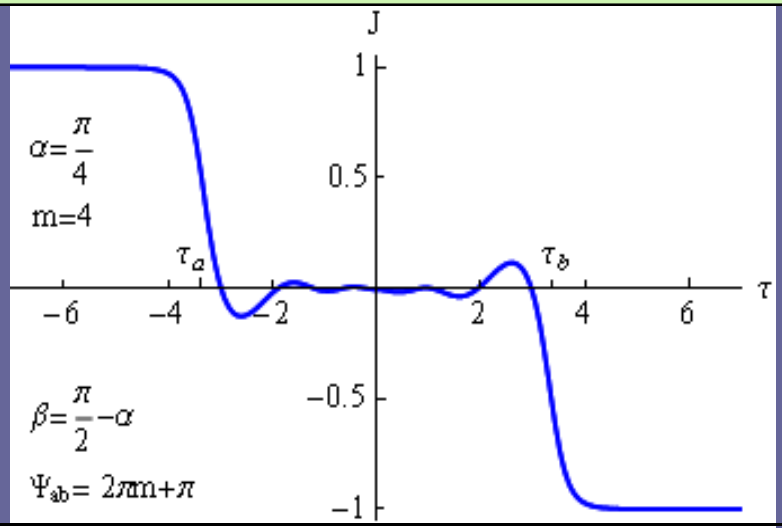
RHIC spin flipper Layout

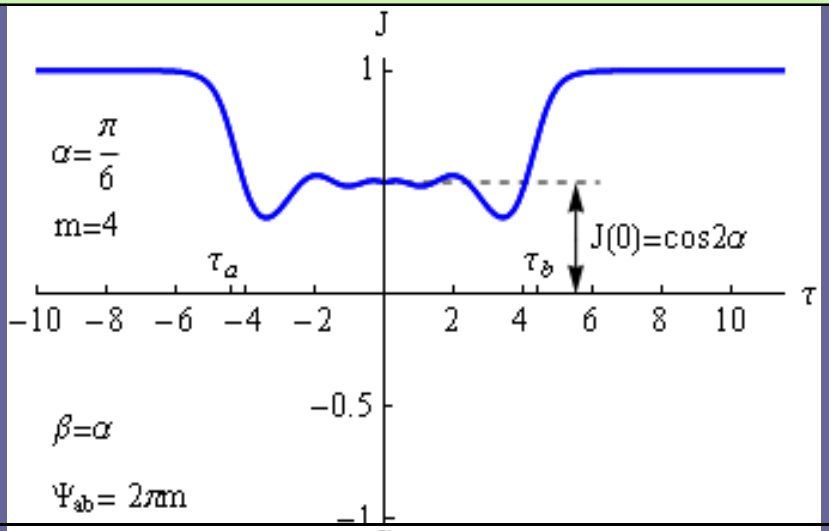
The case of SAI-flip $\left(J_f = -J_i\right)$

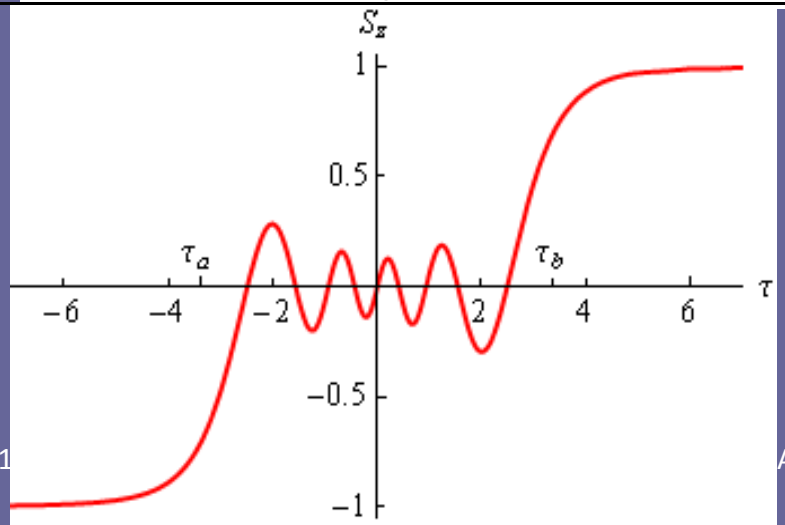
$$M_{11} = 0 \Rightarrow \alpha = \frac{\pi}{2} - \beta, \quad \Psi_{ab} = 2\pi m + \pi$$

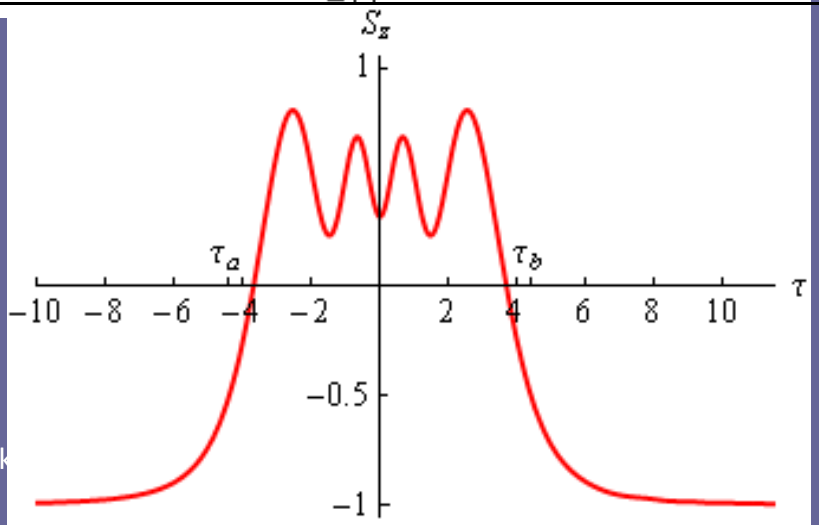
The SAI keeps sign $\left(J_f = J_i\right)$

$$M_{12} = 0 \Rightarrow \alpha = \beta, \quad \Psi_{ab} = 2\pi m$$









Multiple Spin Resonance Crossing Study at RHIC

- Goal
  - To study the interference of multiple spin resonance crossing
  - To drive and measure spin echo
- Experiment plan
  - RHIC spin flipper at injection
  - With DC spin rotator in between ac dipoles at 10 degrees to reduce the orbit excursion
  - Study the polarization transmission efficiency as a function of resonance crossing speed as well as time between the two resonance crossing