Orbit effects on spin tune shift

V.Ptitsyn, M.Bai, W.MacKay, T.Roser

Calculation in first order perturbation theory

$\vec{n}$ - periodical spin solution on the design orbit (stable spin direction)

$$\delta\nu_{sp} = \frac{1}{2\pi}\oint_C \vec{w}\vec{n}\,ds$$ - spin tune shift in first order on perturbations

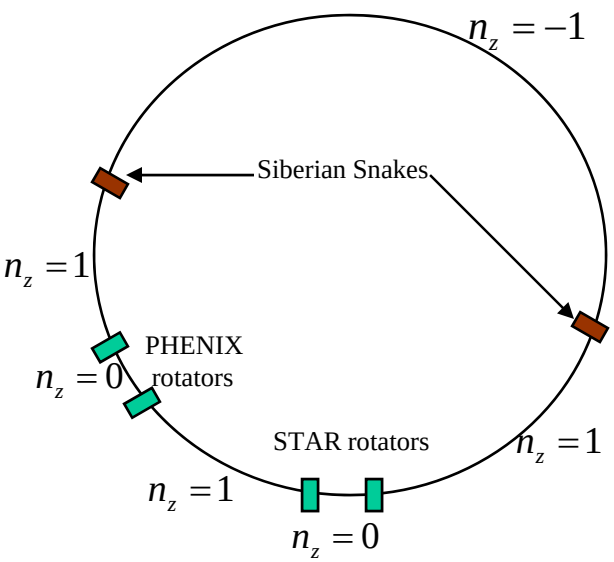
$$\nu_0 = G\gamma\,; \quad K_z = \frac{B_z}{B\rho}; \quad \Delta K_y = \frac{\Delta B_y}{B\rho}$$

$$w_x = (1+\nu_0)\,z''$$

$$w_y = (1+G)(K_z' z + \Delta K_y) - (\nu_0 - G)K_z \frac{\Delta p}{p}$$

$$w_z = -(1+\nu_0)\,x'' + \left(\nu_0 + \frac{G}{\gamma}\right)K_z \frac{\Delta p}{p}$$

z-vertical
y-longitudinal coordinate



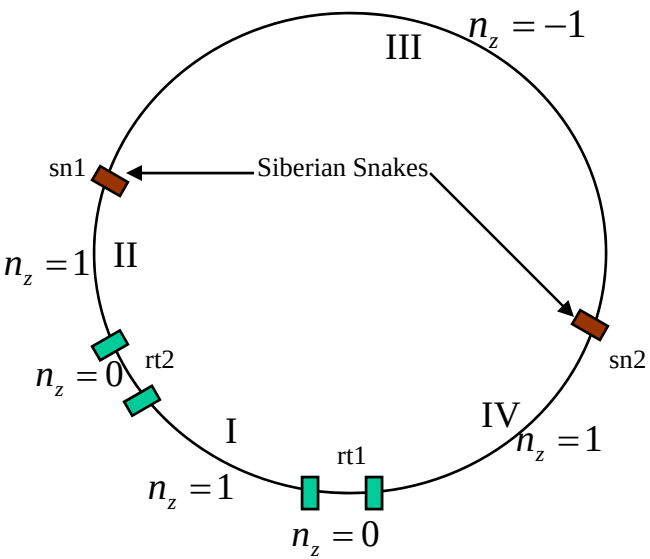
$$\delta\nu_{sp} = \frac{1}{2\pi}\cdot\int w_z n_z\,ds + \frac{1}{2\pi}\cdot\int (w_x n_x + w_y n_y)\,ds$$

outside rotator insertions — *inside rotator insertions*

Effects of horizontal orbit

integrates to 0

$$\delta\nu_{sp} = \frac{1}{2\pi}\oint_C \left[-(1+\nu_0)\, x'' + \left(\nu_0 + \frac{G}{\gamma}\right) K_z \frac{\Delta p}{p} \right] n_z ds = -\frac{1+\nu_0}{2\pi}\left(\int_{I,II,IV} x'' ds - \int_{III} x'' ds \right)$$



$$x = x_{co} + D\frac{\Delta p}{p}$$

-two parts of periodical orbit distortion: closed orbit and average momentum error (non-zero mean orbit)

For closed orbit: $\delta\nu_{sp} = \frac{1+\nu_0}{2\pi}\left(\Delta x'_{co_rt1} + \Delta x'_{co_rt2} + 2\Delta x'_{co_sn}\right)$

$$\Delta x'_{co_rti} = x'_{co_out} - x'_{co_in}\,; \quad \Delta x'_{co_sn} = x'_{co_sn2} - x'_{co_sn1}$$

measured in Q3-Q4 sections

measured in Q7-Q8 sections

Full solution was found: $\sin(\Delta Q_s \pi) = -\cos[\frac{1+G\gamma}{2}\theta_{32}]\cos[\frac{1+G\gamma}{2}\theta_{54}]\sin[(1+G\gamma)\theta_{61} - \frac{1+G\gamma}{2}(\theta_{32}+\theta_{54})]$

(CA/AP Note 334,
M.Bai,V.Ptitsyn, T.Roser)

For momentum error: $\delta\nu_{sp} = \frac{1+\nu_0}{2\pi}\left(\Delta D'_{rt1} + \Delta D'_{rt2} + 2\Delta D'_{sn}\right)\frac{\Delta p}{p}$

Typical scales for horizontal orbit effects

Closed orbit error:

	Orbit angle error, mrad	$\Delta\nu_{sp}$, 10^{-3}	Tolerance (in mm) on BPMs to achieve 1e-3 spin tune shift
Snakes	0.1	15.4	0.042
Rotator	0.1	7.7	0.234

All calculations are done for 250 GeV energy

Momentum error dp/p = 5e-4 (~0.8mm radius offset):

	Design $\Delta D'$, mrad	$\Delta\nu_{sp}$, 10^{-3}
Snakes	-53.6	-4.2
Rotator	5.3	0.2

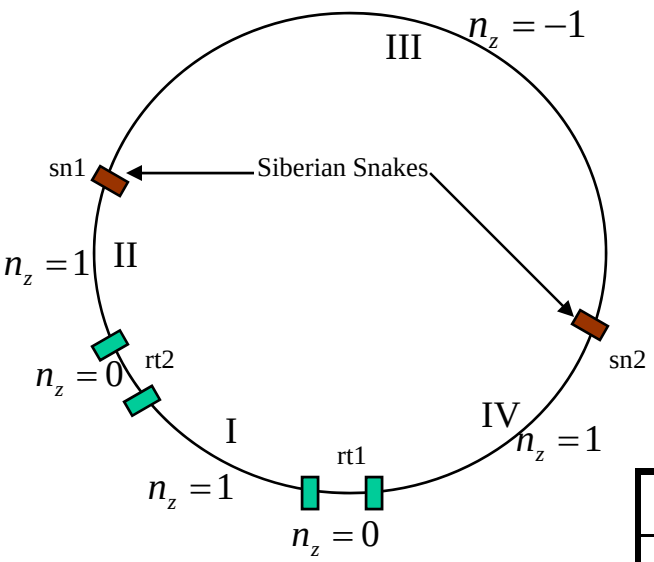
At store (after ramping RF to target frequency) the radius offset was (Run-8) less than 0.2mm in both rings.

The same number characterizes tune spread.
May be largest factor contributing to the spin tune spread.

Vertical orbit effects

$$\delta\nu_{sp} = \frac{1+\nu_0}{2\pi}\oint_C z'' n_x ds = \frac{1+\nu_0}{2\pi}\left(\int_{rt1} z'' n_x ds + \int_{rt2} z'' n_x ds\right)$$

n_x should be taken in the IR triplets. It depends on the beam energy and rotator settings



$z = z_{co}$-closed orbit distortion only (no dispersion)

Maximum tune shift (assuming n_x=1 in the IR triplets) from each rotator:

$$\boxed{\delta\nu_{sp} = \frac{1+\nu_0}{2\pi}\left(z'_{out} - z'_{in}\right)}$$

measured in Q3-Q4 sections

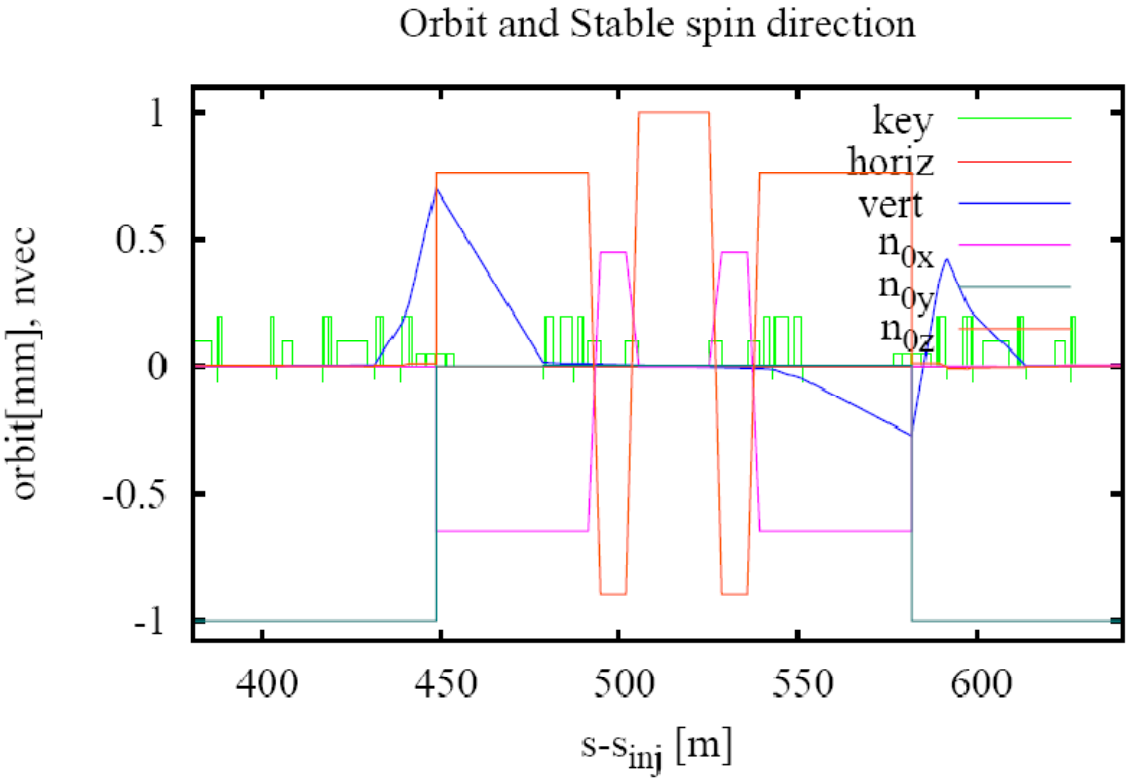
Per rotator at 250 GeV (assuming n_x=1)

		$\Delta\nu_{sp}$, 10^{-3}
Orbit angle error	0.1 mrad	7.7
Design bump (IR separation)	3 mm	12.8

Bumps are present on the rotator ramp

If both PHENIX and STAR rotators are set for same IP polarization, the tune shift shift doubles

For better precision (~10^{-3}) of the spin tune shift calculation orbit kick produced by the rotators should be included into the consideration

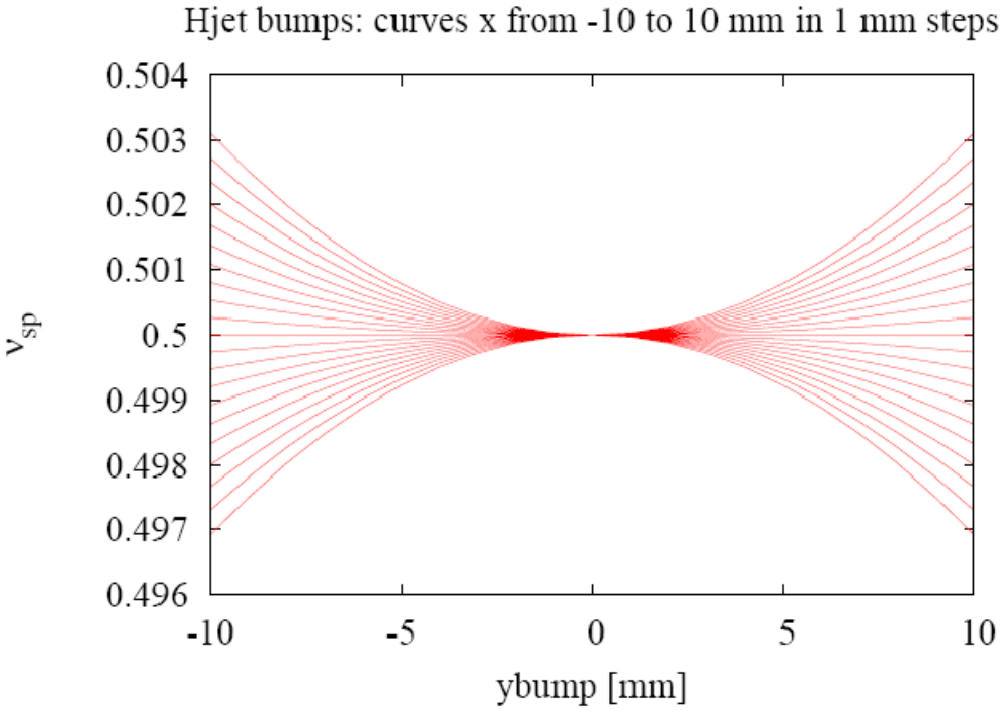


W.Mackay
Simulation results

Each helix twist $359.6° < 360° \Rightarrow$ vertical kick

Higher-order orbit effects

Considerable spin tune shift from combined horizontal and vertical separation bumps at IR12. Combined bumps are used to manage the beams around and through the H-jet.



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Simulation results

Third order effect: ~ xy^2

Becomes important for bump amplitudes larger than 5mm

Possible studies

- Several effects related with the closed orbit lead to the spin tune shifts at 10^{-2}-10^{-3} level:
  - Horizontal orbit angles at Snakes
  - Horizontal orbit angles at Rotators
  - Vertical orbit angles at Rotators
  - Combined orbit bumps at IR12
  - Mean horizontal orbit (radius) offsets
- Measurements of those effects:
  - should improve our understanding of polarization dynamics
  - reduce the possibility of the beam depolarization on the ramp and at the store
  - provide more reproducibility in the beam polarization from store to store and from run to run.
- The measurements are based on the spin tune measurements by the spin flipper, at least at 10^{-2} level.