Modulating tunes

Let $\omega_y(t)$ be the time dependent angular frequency in yellow and similarly for blue. Consider a particle in center of the yellow bucket colliding with beam in blue. Assume the vertical bump is many sigma. Let the extent of the IR be $\pm\hat{\theta}$, where θ increases by 2π each turn. One has

$$\ddot{y} + \omega_\beta^2 y = \Delta\ddot{y}(t)$$

$$\Delta\ddot{y}(t) = K\int_{-\hat{\theta}}^{\hat{\theta}} \beta(\theta)d\theta I_b(\theta,t)\delta_p(\theta - \omega_y t)$$

$$I_b(\theta,t) = \sum_j \exp(-(\theta + \omega_b t - 2\pi j)^2/(2\sigma^2))$$

$$= \sqrt{2\pi}\sigma \sum_{k=-\infty}^{\infty} \exp(ik(\theta + \omega_b t) - k^2\sigma^2/2)$$

So,

$$C(t) = K\sum_{j,k}\int d\theta\left[\theta^2+\theta_0^2\right]\exp(j[\theta-\omega_y t]+k[\theta+\omega_b t]-\sigma^2k^2/2)$$

$$\approx K'\sum_{j,k}\exp(-j\omega_y t+k\omega_b t-\sigma^2k^2/2)F(|\,j+k\,|)$$

where

$$F(z)=\int_{-\hat\theta}^{\hat\theta}\left[\theta^2+\theta_0^2\right]\cos(z\theta)d\theta$$

In the above θ_0 is β_*/R and $\sigma=\omega\sigma_t$. If the yellow and blue frequencies are the same then C(t) is periodic at the revolution frequency. When they differ it has sum and difference harmonics which can drive resonances.

The function F(z) has very long tails, like 1/z, so many resonances will be excited.

The expression above is for one bunch in blue, with 110 we need to change

$$\omega_b t \to \omega_b t - mT_{rev}/120$$

And sum m=1,…110. This will affect the magnitude of the driving terms but not the frequencies.