β* and s* measurements

$$\Delta Q = \frac{\langle \beta \rangle \Delta K l}{4\pi}$$

Change Q1 gradient -> measure betatron tune change -> extract quad beta function -> find β* and s* (using measurements both Q1 quads

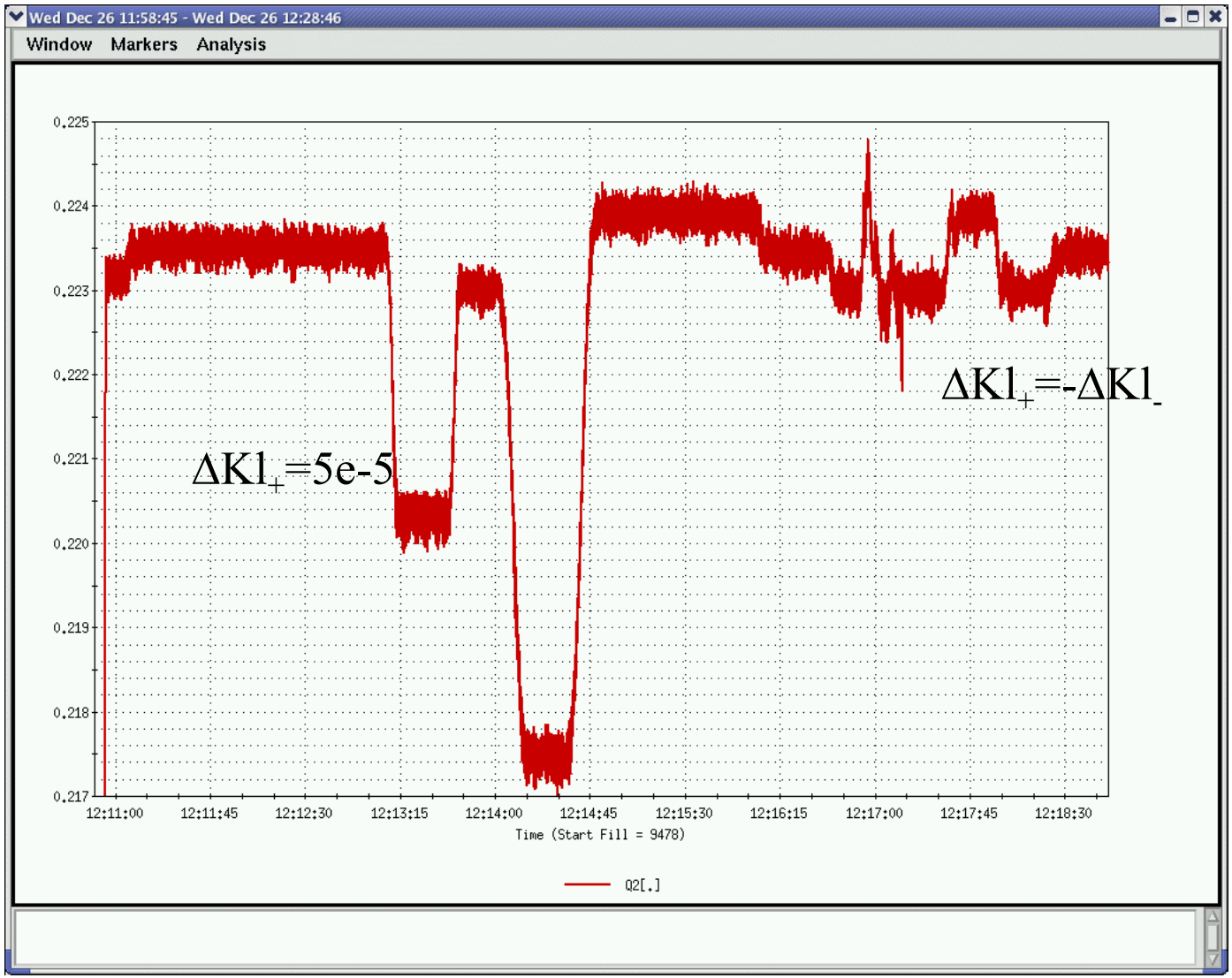
$$\langle \beta \rangle = \langle m_{11}^2 \rangle \beta_0 - 2\langle m_{11} m_{12} \rangle \alpha_0 + \langle m_{12}^2 \rangle \gamma_0$$

$$\beta_0 = \beta^* + \frac{(L - s^*)^2}{\beta^*}; \quad \alpha_0 = -\frac{L - s^*}{\beta^*}; \quad \gamma_0 = \frac{1}{\beta^*}$$

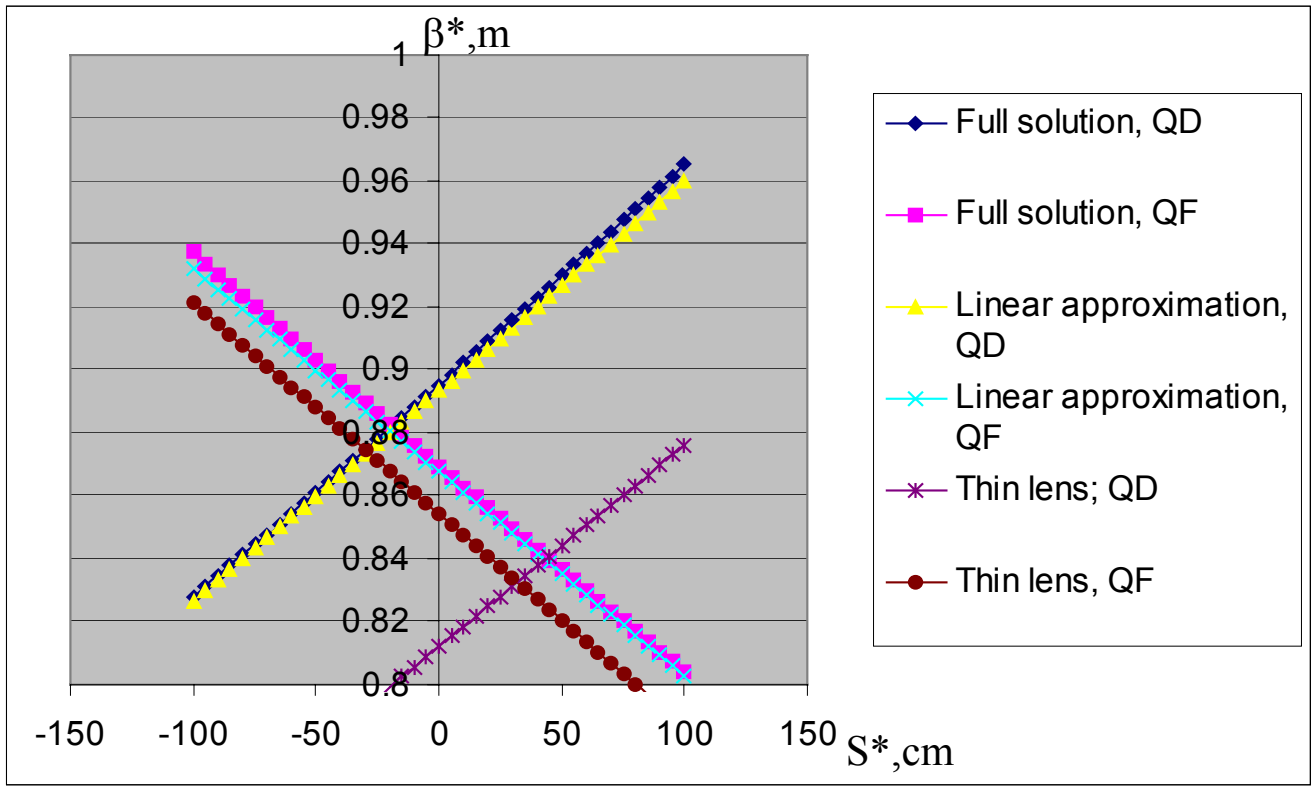
$$\frac{L^2}{\beta^{*2}}; \quad \frac{L}{\beta^*}; \quad 1 \rightarrow$$

Terms hierarchy. Neglecting terms ~1 leads to linear equations (in β* and s*)

Example of Yellow IR6 vertical tune change during Q1 gradient variations



Solution for Yellow IR6 horizontal data



Data from linear approximation for Yellow ring measurements

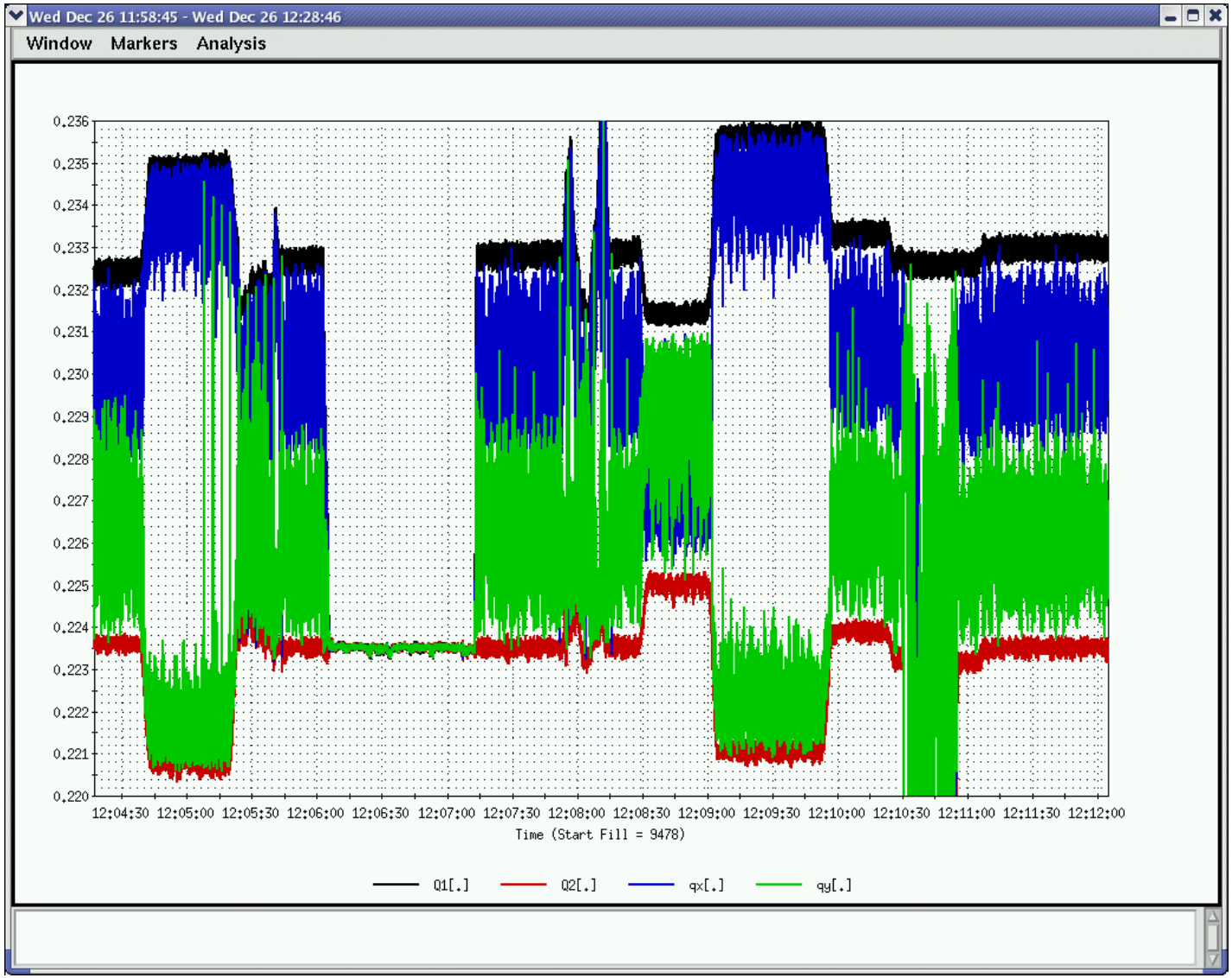
	IR6 H	IR6 V	IR8 H	IR8 V
β*,m	0.88	0.91	1.09	1.04
s*,cm	-20	-47	-0.53	0.49

$$\beta_+ = \langle \beta(L) \rangle; \quad \beta_- = \langle \beta(-L) \rangle$$

$$a_\pm = \langle m_{11}^2 \rangle_\pm; \quad b_\pm = \langle m_{11} m_{12} \rangle_\pm; \quad c_\pm = \langle m_{12}^2 \rangle_\pm$$

$$s^* = -\frac{L^*(\beta_+ a_- - \beta_- a_+) + 2(\beta_+ b_- - \beta_- b_+)}{2(\beta_+ a_- + \beta_- a_+)}; \qquad \beta^* = \frac{1}{\beta_+}\left(a_+ L^2 + 2b_+ L - 2a_+ L s^*\right)$$

Example of IR6 tune changes during Q1 gradient variations for mode and uncoupled tunes



Measurement using simultaneous Q1 gradient change on both side of the IR

$$\beta_+ - \beta_- = \frac{1}{\beta^*}\left((a_+ - a_-)L^2 + 2(b_+ - b_-)L - 2(a_+ + a_-)L\,s^*\right)$$

$$\beta_+ + \beta_- = \frac{L}{\beta^*}\left((a_+ + a_-)L + 2(b_+ + b_-)\right)$$

$$\Delta Q = \left(4.32 - 8.08\,s^*\right)\frac{\Delta Kl}{\beta^*}; \qquad \Delta Kl_+ = -\Delta Kl_- = \Delta Kl$$

$$\Delta Q = \frac{108.2\,\Delta Kl}{\beta^*}; \qquad \Delta Kl_+ = \Delta Kl_- = \Delta Kl$$