Measure Twiss and Coupling at IP

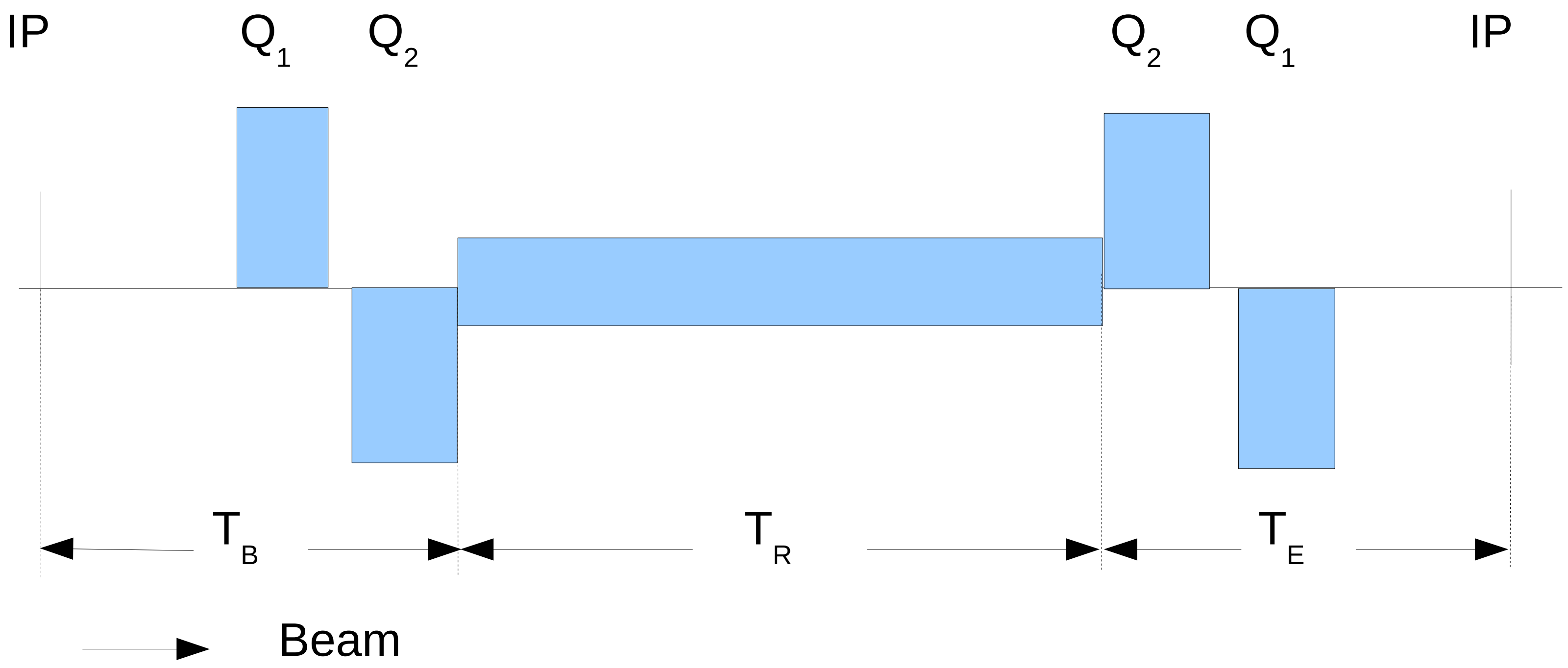

Figure 1: Schematic of the synchrotron from an IP through two of the triplet quadrupoles and back to the IP. Thus the beam transfer matrix is: $T = T_E T_R T_B$.

$$T = T_E T_R T_B$$

$$T_B = T_{Q_2} T_{Drift} T_{Q_1} T_{Drift} \qquad T_E = T_{Drift} T_{Q_1} T_{Drift} T_{Q_2}$$

The uncoupled transfer matrix:

$$\boldsymbol{M}_{x|y}=\begin{bmatrix}\cos(\mu_{x|y})+\alpha_{x|y}\sin(\mu_{x|y}) & \beta_{x|y}\sin(\mu_{x|y})\\ -\dfrac{1+\alpha_{x|y}^2}{\beta_{x|y}}\sin(\mu_{x|y}) & \cos(\mu_{x|y})-\alpha_{x|y}\sin(\mu_{x|y})\end{bmatrix} \qquad \boldsymbol{U}=\begin{bmatrix}\boldsymbol{M}_x & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{M}_y\end{bmatrix}$$

$$\boldsymbol{G}=\begin{bmatrix}a & b\\ c & d\end{bmatrix} \qquad \bar{\boldsymbol{G}}=\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$$

$$\boldsymbol{H}=\frac{1}{\sqrt{1+det(\boldsymbol{G})}}\begin{bmatrix}\boldsymbol{I} & \bar{\boldsymbol{G}}\\ -\boldsymbol{G} & \boldsymbol{I}\end{bmatrix} \qquad \boldsymbol{H}^{-1}=\frac{1}{\sqrt{1+det(\boldsymbol{G})}}\begin{bmatrix}\boldsymbol{I} & -\bar{\boldsymbol{G}}\\ \boldsymbol{G} & \boldsymbol{I}\end{bmatrix}$$

Adding coupling:

$$\boldsymbol{T}=\begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}\\ \boldsymbol{C} & \boldsymbol{D}\end{bmatrix}=\boldsymbol{H}\,\boldsymbol{U}\,\boldsymbol{H}^{-1}=\boldsymbol{H}\begin{bmatrix}\boldsymbol{M}_x & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{M}_y\end{bmatrix}\boldsymbol{H}^{-1}$$

$$\boldsymbol{A}=\frac{1}{1+det(\boldsymbol{G})}(\boldsymbol{M}_x+\bar{\boldsymbol{G}}\boldsymbol{M}_y\boldsymbol{G}) \qquad \boldsymbol{B}=\frac{1}{1+det(\boldsymbol{G})}(\bar{\boldsymbol{G}}\boldsymbol{M}_y-\boldsymbol{M}_x\bar{\boldsymbol{G}})$$

$$\boldsymbol{C}=\frac{1}{1+det(\boldsymbol{G})}(\boldsymbol{M}_y\boldsymbol{G}-\boldsymbol{G}\boldsymbol{M}_x) \qquad \boldsymbol{D}=\frac{1}{1+det(\boldsymbol{G})}(\boldsymbol{M}_y+\boldsymbol{G}\boldsymbol{M}_x\bar{\boldsymbol{G}})$$

10 parameters: $[\mu_x, \alpha_x, \beta_x, \mu_y, \alpha_y, \beta_y, a, b, c, d]$

The eigen-tunes from the transfer matrix:

$$Q_{\pm} = Tune_{\pm}(\boldsymbol{T}) = \frac{1}{2\pi}\arccos\left(\frac{1}{2}(Tr(\boldsymbol{A}) + Tr(\boldsymbol{D})) \pm \sqrt{\frac{1}{4}(Tr(\boldsymbol{A}) - Tr(\boldsymbol{D}))^2 + det(\bar{\boldsymbol{B}} + \boldsymbol{C})}\right)$$

The ΔQ_{min} from the transfer matrix:

$$\Delta Q_{min} = DtuneMin(\boldsymbol{T}) = \frac{\sqrt{det(\bar{\boldsymbol{B}} + \boldsymbol{C})}}{\pi[\sin(2\pi Q_+) + \sin(2\pi Q_-)]}$$

BBQ measures the eigen-tunes and the ΔQ_{min}. Using the above equations, we solve for the 10 parameters that describe the transfer matrix *T*.

Constructing the transfer matrices: T_E and T_B:

$$\boldsymbol{M}(k,l)=\begin{cases}\begin{bmatrix}\cos(\sqrt{k}\,l) & \frac{1}{\sqrt{k}}\sin(\sqrt{k}\,l)\\ -\sqrt{k}\sin(\sqrt{k}\,l) & \cos(\sqrt{k}\,l)\end{bmatrix} & 0<k\\ \begin{bmatrix}1 & l\\ 0 & 1\end{bmatrix} & k=0\\ \begin{bmatrix}\cosh(\sqrt{-k}\,l) & \frac{1}{\sqrt{-k}}\sinh(\sqrt{-k}\,l)\\ \sqrt{-k}\sinh(\sqrt{-k}\,l) & \cosh(\sqrt{-k}\,l)\end{bmatrix} & k<0\end{cases}$$

$$\boldsymbol{U}_Q=\begin{bmatrix}\boldsymbol{M}(k,l) & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{M}(-k,l)\end{bmatrix}$$

$$\boldsymbol{R}=\begin{bmatrix}\cos(\theta) & 0 & \sin(\theta) & 0\\ 0 & \cos(\theta) & 0 & \sin(\theta)\\ -\sin(\theta) & 0 & \cos(\theta) & 0\\ 0 & -\sin(\theta) & 0 & \cos(\theta)\end{bmatrix}$$

$$\boldsymbol{R}^{-1}=\begin{bmatrix}\cos(\theta) & 0 & -\sin(\theta) & 0\\ 0 & \cos(\theta) & 0 & -\sin(\theta)\\ \sin(\theta) & 0 & \cos(\theta) & 0\\ 0 & \sin(\theta) & 0 & \cos(\theta)\end{bmatrix}$$

$$\boldsymbol{T}_Q=\boldsymbol{R}\,\boldsymbol{U}_Q\,\boldsymbol{R}^{-1}$$

The perturbed transfer matrices as a function of the unperturbed transfer matrix:

$$\boldsymbol{T}_{\Delta_i} = \boldsymbol{T}_{E+\Delta_i} \boldsymbol{T}_R \boldsymbol{T}_B = (\boldsymbol{T}_{E+\Delta_i} \boldsymbol{T}_E^{-1}) \boldsymbol{T}_E \boldsymbol{T}_R \boldsymbol{T}_B = (\boldsymbol{T}_{E+\Delta_i} \boldsymbol{T}_E^{-1}) \boldsymbol{T}$$

$$\boldsymbol{T}_{\Delta_i} = \boldsymbol{T}_E \boldsymbol{T}_R \boldsymbol{T}_{B+\Delta_{i-2}} = \boldsymbol{T}_E \boldsymbol{T}_R \boldsymbol{T}_B (\boldsymbol{T}_B^{-1} \boldsymbol{T}_{B+\Delta_{i-2}}) = \boldsymbol{T} (\boldsymbol{T}_B^{-1} \boldsymbol{T}_{B+\Delta_{i-2}})$$

where: $i = 1, 2, 3, 4$

15 equations with 10 unknowns:

$$\mu_x = 2\pi Q_+ \qquad \mu_y = 2\pi Q_- \qquad \Delta Q_{min} = DtuneMin(\boldsymbol{T}) \qquad \mu_x > \mu_y$$

$$Q_\pm^{(1)} = Tune_\pm(\boldsymbol{T}_{\Delta_1}) \qquad \Delta Q_{min}^{(1)} = DtuneMin(\boldsymbol{T}_{\Delta_1}) \qquad Q_\pm^{(2)} = Tune_\pm(\boldsymbol{T}_{\Delta_2}) \qquad \Delta Q_{min}^{(2)} = DtuneMin(\boldsymbol{T}_{\Delta_2})$$

$$Q_\pm^{(3)} = Tune_\pm(\boldsymbol{T}_{\Delta_3}) \qquad \Delta Q_{min}^{(3)} = DtuneMin(\boldsymbol{T}_{\Delta_3}) \qquad Q_\pm^{(4)} = Tune_\pm(\boldsymbol{T}_{\Delta_4}) \qquad \Delta Q_{min}^{(4)} = DtuneMin(\boldsymbol{T}_{\Delta_4})$$

Preliminary Error Analysis

IBS-suppression optics with rolls in the triplets

Model	ALFX	BETX	ALFY	BETY
Case #1	-0.1883	0.7655	0.8647	0.6219
Case #2	-0.2964	1.2880	0.4687	1.2437

	Quadrupole Errors				**Case #1**				**Case #2**			
Row	**Q2I**	**Q1I**	**Q1O**	**Q2O**	**ALFX**	**BETX**	**ALFY**	**BETY**	**ALFX**	**BETX**	**ALFY**	**BETY**
0	0%	0%	0%	0%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1	-1%	0%	0%	0%	1.17%	0.02%	6.42%	5.20%	2.09%	0.30%	1.88%	0.69%
2	1%	0%	0%	0%	1.20%	0.02%	6.24%	5.44%	2.12%	0.29%	1.93%	0.72%
3	0%	-1%	0%	0%	2.25%	0.11%	6.64%	5.37%	1.35%	0.17%	2.37%	0.84%
4	0%	1%	0%	0%	2.21%	0.11%	6.41%	5.59%	1.36%	0.16%	2.39%	0.88%
5	0%	0%	-1%	0%	4.91%	0.45%	7.72%	7.04%	2.43%	0.35%	2.71%	1.10%
6	0%	0%	1%	0%	5.29%	0.46%	8.56%	7.11%	2.45%	0.37%	2.84%	1.11%
7	0%	0%	0%	-1%	2.78%	0.31%	6.59%	5.97%	3.12%	0.43%	2.14%	0.87%
8	0%	0%	0%	1%	2.97%	0.32%	7.23%	6.05%	3.10%	0.46%	2.24%	0.87%

Measure Twiss and Coupling at IP

- No approximations were made.
  - I simulated this in MADX to high accuracy.
- Possible issues to be resolved:
  - How good is our model for T_E and T_B?
    - Quadrupole strength, DX model, etc.
  - The equations may have more than one solution.
    - Depends on initial guess.
    - May not be able to find an adequate solution.
  - How well does BBQ measure ΔQ_{min}?
  - Doing the Yellow ring, with opposite beam direction, correctly.

Title: Measure the ring Transfer and Coupling Matrix at the IP

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Team: M. Minty, V. Ptitsyn, S. Tepikian

Experiment Goal: Measure the twiss parameters alpha and beta in both planes along with the coupling matrix.

Benefits: The full 4x4 transfer matrix can be measured at the IP.
Can be used for establishing beam sizes.

Experiment Description: This is an extenstion of V. Ptitsyn's method of measuring the Beta*. We will vary the strengths four quadrupoles (rolled, skew and reqular quadrupoles). After each magnet is tweaked, the eigen-tunes and DQmin are measured with BBQ. Thus, we have 15 measured values and 10 unknowns, from which the twiss matrix in both planes and the coupling matrix can be deduced.

Resources:

- Instrumentation: BBQ
- Application: Specialized application
- Time: 2 * 2Hrs
- Personnel: team + operation crew